

## PROBLEM OF THE MONTH

DEPARTMENT OF MATHEMATICS  
ELMHURST UNIVERSITY  
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**Problem.** Let  $x, y$  be two positive integers such that  $\frac{x+1}{y} + \frac{y+1}{x}$  is an integer. Show that

$$\gcd(x, y) \leq \sqrt{x+y}.$$

**Bonus question.** Write a program to find all pairs of  $1 \leq x, y \leq 10^6$  with the required property. Convince yourself that there are infinitely many  $x, y$  with that property.

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Scan the following QR code for submission instructions.

