$$f(X;\pi,u)=\prod_{n=1}^{N}\sum_{i=1}^{I}\pi_i \,\mathrm{Dir}(X_n,u_i)$$

$$\beta_N=\beta_0-\ln X+\mathbb{1}_N$$

$$v_N=v_0+N$$

$$f(u;\beta_0,v_0)=\frac{1}{C(\beta_0,v_0)}\left[\frac{\Gamma\left(\sum_{k=1}^{K+1}u_k\right)}{\prod_{k=1}^{K+1}\Gamma(u_k)}\right]^{v_0}e^{-\beta_0^T(u-1_{K+1})}$$

$$f(u|X,\beta_N,v_N)=\frac{\mathrm{Dir}(X|u)\,f(u;\beta_0,v_0)}{\int \mathrm{Dir}(X,u)\,f(u;\beta_0,v_0)\,du}=\frac{1}{C(\beta_N,v_N)}\left[\frac{\Gamma\left(\sum_{k=1}^{K+1}u_k\right)}{\prod_{k=1}^{K+1}\Gamma(u_k)}\right]^{v_N}e^{-\beta_N^T(u-1_{K+1})}$$

$$f(X,Z|\pi,u)=f(X|Z,u)\,f(Z|\pi)$$

$$f(X|Z,u)=\prod_{n=1}^{N}\prod_{i=1}^{I}\left[\mathrm{Dir}(X_n|u_i)\right]^{Z_{ni}}$$

$$f(Z|\pi)=\prod_{n=1}^{N}\prod_{i=1}^{I}\pi_i^{Z_{ni}}$$

$$f(\pi)=\mathrm{Dir}(\pi;c_0)$$

$$f(u)=\prod_{i=1}^{I}\prod_{k=1}^{K+1}\mathrm{Gam}(u_{ki};\mu_{k0},\alpha_{k0})$$

$$f(X,\cancel{Z})=f(X,u,\pi,Z)=f(X|Z,u)\,f(Z|\cancel{\pi})\,f(\pi)\,f(u)$$

$$=\prod_{n=1}^{N}\prod_{i=1}^{I}\left[\pi_i\frac{\Gamma\left(\sum_{k=1}^{K+1}u_{ki}\right)}{\prod_{k=1}^{K+1}\Gamma(u_{ki})}\prod_{k=1}^{K+1}X_{kn}^{u_{ki}-1}\right]^{Z_{ni}}\times\frac{\Gamma\left(\sum_{i=1}^{I}c_{i0}\right)}{\sum_{i=1}^{I}\Gamma(c_{i0})}\prod_{i=1}^{I}\pi_i^{c_{i0}-1}$$

$$\times\prod_{i=1}^{I}\prod_{k=1}^{K+1}\frac{\alpha_{k0}^{\mu_{k0}}}{\Gamma(\mu_{k0})}\mu_{ki}^{\mu_{k0}-1}e^{-\alpha_{k0}\mu_{ki}}$$

$$\mathcal{L}=E_Z[\ln f(X,Z)]-E_Z[\ln f(Z)]$$

$$\cancel{E[\ln f(X,Z)]=E\left[\prod_{n=1}^{N}\prod_{i=1}^{I}\left[\pi_i\frac{\Gamma\left(\sum_{k=1}^{K+1}u_{ki}\right)}{\sum_{k=1}^{K+1}\Gamma(u_{ki})}\prod_{k=1}^{K+1}X_{kn}^{u_{ki}-1}\right.\right.}$$

$$E[\ln g(X,Z)] = E\Big\{ \sum_{n=1}^{N} \sum_{i=1}^{I} z_{ni} \Big[\ln \pi_i + \ln \Big(\frac{\Gamma(\sum_{l=1} u_{li})}{\prod_{l=1}^{k+1} \Gamma(u_{li})} \Big) + \sum_{l=1}^{k+1} (u_{li} - 1) \ln (X_{ln}) \Big] +$$

$$\ln \frac{\Gamma(\sum_{i=1}^{I} c_{i0})}{\sum^{I} \Gamma(c_{i0})} + \sum_{i=1}^{I} (c_{i0} - 1) \ln (\pi_i) +$$

$$\sum_{i=1}^{I} \sum_{l=1}^{k+1} \Big[\ln \frac{\lambda_{li0}^{\mu_{li0}}}{\Gamma(\mu_{li0})} + (\mu_{li} - 1) \ln (u_{li}) - \lambda_{li} u_{li} \Big] \Big\}$$

u_{li}

$$\sum_{n=1}^{N} E[z_{ni}] \cdot E\Big[\ln \frac{\Gamma(\sum_{l=1}^{k+1} u_{li})}{\prod_{l=1}^{k+1} \Gamma(u_{li})} \Big] + \sum_{l=1}^{k+1} E[u_{li}] \sum_{n=1}^{N} E[z_{ni}] \ln X_{ln} + \sum\sum (\mu_{li} - 1) E[\ln(u_{li})] - \sum\sum \lambda_{li} E[u_{li}]$$

$$\sum_{n=1}^{N} \pi_n \qquad \sum_{l=1}^{k+1} \frac{\mu_{li}}{\lambda_{li}} \qquad \sum_{n=1}^{N} \pi_n \qquad \sum\sum \Psi(\mu_{li}) - \ln(\lambda_{li}) \qquad \sum\sum \frac{\mu_{li}}{\lambda_{li}}$$

$$\Gamma(x) \approx \sqrt{\frac{2\pi}{x}} \Big(\frac{x}{e} \Big)^x$$

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$$\int u_i \cdot \log \sqrt{\frac{2\pi}{u}} + u_i \cdot \log \frac{u_i}{e} \, d u_i$$

$$\ln \frac{\Gamma\left(\sum_{k=1}^{K+1} u_{ki}\right)}{\prod_{k=1}^{K+1} \Gamma(u_{ki})} = \ln\left(\Gamma\left(\sum_{k=1}^{K+1} u_{ki}\right)\right) - \sum_{k=1}^{K+1} \ln\left(\Gamma(u_{ki})\right) \quad f(x) = f(a) + f'(a)(x-a) \Rightarrow f(\ln u_{1i}) = f(\ln \bar{u}_{1i}) + f'(\ln \bar{u}_{1i})(\ln u_{1i} - \ln \bar{u}_{1i})$$

$$\geq \ln \frac{\Gamma\left(\bar{u}_{1i} + \sum_{k=2}^{K+1} u_{ki}\right)}{\Gamma(\bar{u}_{1i}) \prod_{k=2}^{K+1} (u_{ki})} + \left[\psi\left(\bar{u}_{1i} + \sum_{k=2}^{K+1} u_{ki}\right) - \psi(\bar{u}_{1i})\right] \bar{u}_{1i} (\ln u_{1i} - \ln \bar{u}_{1i})$$

$$\geq \ln \frac{\Gamma\left(\sum \bar{u}_{ik}\right)}{\prod_{k=1}^{K+1} \Gamma(\bar{u}_{ik})} + \sum_{k=1}^{K+1} \left[\psi\left(\sum_{m=1}^{k} \bar{u}_{mi} + \sum_{l=k+1}^{K+1} u_{li}\right) - \psi(\bar{u}_{ik})\right] \bar{u}_{ik} (\ln u_{ik} - \ln \bar{u}_{ik})$$

$$\int \left(u_i \log \sqrt{\frac{2\pi}{u_i}} + u_i \log \frac{u_i}{e}\right) du_i = \int \left(u_i \log \frac{\sqrt{2}\sqrt{\pi}}{\sqrt{u_i}} + u_i \ln(e u)\right) du$$

$$\int u_i \log\left(\frac{2\pi}{u}\right)^{1/2} + u_i \log \frac{u_i}{e} \, du = \int u_i \frac{\log \frac{2\pi}{u}}{2} + u \log \frac{u}{e} \, du = \int \frac{u \log \frac{2\pi}{u} + 2u \log \frac{u}{e}}{2} \, du$$

$$= \frac{1}{2} \int u \log \frac{2\pi}{u} + 2u(\log u - 1) \, du = \frac{1}{2} \int u\left(\log \frac{2\pi}{u} + 2(\log u - 1)\right) du$$

$$\int f'g = fg + \int fg' \quad f' = u \quad g = \log \frac{2\pi}{u} + 2(\log u - 1) \rightarrow f = \frac{u^2}{2}, \quad g' = \frac{2}{u} + \frac{u}{2\pi}\left(\frac{2\pi}{u}\right)'$$

$$= \frac{2}{u} + \frac{u}{2\pi} \cdot \frac{-2\pi}{u^2} = \frac{2}{u} - \frac{1}{u}$$

$$\frac{u^2}{2} \cdot \left(\log \frac{2\pi}{u} + 2(\log u - 1)\right) + \int \frac{u^2}{2} \cdot \frac{1}{u} \, du = \frac{u^2}{2}\left(\log \frac{2\pi}{u} + 2(\log u - 1)\right) + \int \frac{u}{2} \, du = \frac{u^2}{2}\left(\log \frac{2\pi}{u} + 2(\log u - 1)\right) + \frac{u^2}{4}$$

$$E_{u_{ki}}\left[\ln \frac{\Gamma\left(\sum_{k=1}^{K+1} u_{ki}\right)}{\prod_{k=1}^{K+1}\Gamma(u_{ki})}\right] \geq E_{u_{p\neq k,i}}\left[\ln \frac{\Gamma\left(\sum_{k=1}^{K+1} \bar{u}_{ki}\right)}{\prod_{k=1}^{K+1}\Gamma(\bar{u}_{ki})} + \sum_{k=1}^{K+1}\left[\Psi\left(\sum_{m=1}^{k}\bar{u}_{mi} + \sum_{l=k+1}^{K+1} u_{li}\right) - \Psi(\bar{u}_{ik})\right]\bar{u}_{ik}(\ln u_{ik} - \ln \bar{u}_{ik}\right]$$

$$\geq \ln \frac{\Gamma\left(\sum_{k=1}^{K+1} \bar{u}_{ki}\right)}{\prod_{k=1}^{K+1}\Gamma(\bar{u}_{ki})} + \sum_{k=1}^{K+1} E\left[\Psi\left(\sum_{m=1}^{k}\bar{u}_{mi} + \sum_{l=k+1}^{K+1} u_{li}\right) - \Psi(\bar{u}_{ik})\right]\bar{u}_{ik}\left(E[\ln u_{ik}] - \ln \bar{u}_{ik}\right)$$

$$E_x[\Psi(x+y)] \leq \Psi(\bar{x}+y) \qquad E_x[\ln(x)] \leq \ln \bar{x}$$

$$\geq \ln \frac{\Gamma\left(\sum_{k=1}^{K+1} \bar{u}_{ki}\right)}{\prod_{k=1}^{K+1}\Gamma(\bar{u}_{ki})} + \sum_{k=1}^{K+1}\left[\Psi\left(\sum_{m=1}^{k}\bar{u}_{mi} + \sum_{l=k+1}^{K+1} \bar{u}_{li}\right) - \Psi(\bar{u}_{ik})\right]\bar{u}_{ik}\left(E[\ln u_{ik}] - \ln \bar{u}_{ik}\right)$$

$$\geq \ln \frac{\Gamma\left(\sum_{k=1}^{K+1} \bar{u}_{ki}\right)}{\prod_{k=1}^{K+1}\Gamma(\bar{u}_{ki})} + \sum_{k=1}^{K+1}\left[\Psi\left(\sum_{k=1}^{K+1}\bar{u}_{ki}\right) - \Psi(\bar{u}_{ik})\right]\bar{u}_{ik}\left(E[\ln u_{ik}] - \ln \bar{u}_{ik}\right)$$

$$\ln q^*(u_{ki}; \mu_{ki}, \alpha_{ki}) \simeq E_{u_{ki}}[\ln \tilde{q}(X, Z)]$$

$$\simeq \sum_{n=1}^{N} E[z_{ni}]\left[\Psi\left(\sum_{k=1}^{K+1}\bar{u}_{ki}\right) - \Psi(\bar{u}_{ik})\right]\bar{u}_{ik}\left(E[\ln u_{ik}] - \ln \bar{u}_{ik}\right) + E[u_{ik}]\sum_{n=1}^{N} E[z_{ni}]\ln x_{kn}$$

$$+ \sum\sum(\mu_{ki} - 1)E[\ln(u_{ki})] - \alpha_{ki}E[u_{ik}]$$