AAE6102 Assignment
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GitHub page: https://github.com/tungtungyan/21063325r-LeungYanTung-AAE6102-Assignment

This report uses the single-epoch data sets rcvr.dat and eph.dat to set up the linearized navigation equations and solve for user position and clock bias. Appendix A shows the data file format of rcvr.dat and eph.dat.
The linearized navigation equation:

$$pseudorange: \rho_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2} + ct_u$$
$$x_u = \hat{x}_u + \Delta x_u$$
$$y_u = \hat{y}_u + \Delta y_u$$
$$z_u = \hat{z}_u + \Delta z_u$$
$$t_u = \hat{t}_u + \Delta t_u$$

The required corrections for the satellite clock bias and relativity are referred to ICD. This report will skip the ionospheric corrections because we do not have access to the parameter values of the Klobuchar model for this data set. This report will consider the tropospheric correction based on the standard atmosphere model.
The initial position to start the iteration using $\begin{bmatrix} -2694685.473 \\ -4293642.366 \\ 3857878.924 \end{bmatrix}$ (WGS 84 XYZ, in meters).
The algorithm uses zero clock bias for Initialize. Terminate the iteration when the change in the estimate is suitably small.

GPS Constants:
Speed of light: c = 299792458.0 (m/s)
WGS 84 value of earth's rotation rate: Wedot= 7.2921151467e-5 (r/s)
WGS 84 value of earth's universal gravitation constant: mu= 3.986005e+14 (m^3/s^2)
Relativistic correction term constant: F= -4.442807633e-10

Tropospheric constant:
Pressure: Pr = 1013.25(mbar)
Temperature: Tr = 291.15(K)
Temperature at sea-level: temp_0 = 15

## Calculation processing:

Using the data of rcvr.dat and eph.dat, it can calculate the receiver's position at the time of week 440992 using the following process.

1. Calculate the XYZ positions for all valid satellites at time 440992.

Calculate the satellite Earth-centered, Earth-fixed coordinate system (ECEF) position vector.

The square root of semi-major axis a (Sqrta) was saved in Column 10 of eph.dat.
Therefore, the semimajor axis (a) is

$$a = sqeta^2$$

Corrected mean motion(n):

$$n = n_0 - \Delta n$$

Computed mean motion ($n_0$):

$$n_0 = \sqrt{\frac{mu}{a^3}}$$

Time from ephemeris epoch ($t_k$):

$$t_k = t - \Delta t_{oe}$$

Where $t_{oe}$ is reference time of ephemeris parameters (s).

Mean anomaly:

$$M_k = M_0 + nt_k$$

Assume that the maximum number of iterations is 10.
Then, using Kepler's equation of eccentric anomaly is solved by iteration.

$$E_k = E_0 - e \sin E_k$$

Calculate the true anomaly:

$$\nu_k = \tan^{-1}\left\{\frac{\frac{\sqrt{1-e^2}\sin E_k}{1-e\cos E_k}}{\frac{\cos E_k - e}{1-e\cos E_k}}\right\}$$

Argument of latitude:

$$\Phi_k = \nu_k + \omega$$

Use the Second Harmonic Perturb to find to Correction value:
Argument of latitude Correction:

$$\delta u_k = c_{us}\sin 2\Phi_k + c_{us}\cos 2\Phi_k$$

Radius Correction:

$$\delta r_k = c_{rs}\sin 2\Phi_k + c_{rs}\cos 2\Phi_k$$

Inclination correction:

$$\delta i_k = c_{is}\sin 2\Phi_k + c_{is}\cos 2\Phi_k$$

Use the Correction value for Corrected:
Corrected argument of latitude:

$$u_k = \Phi_k + \delta u_k$$

Corrected radius:

$$r_k = a(1 - e\cos \mathrm{E}_k) + \delta r_k$$

Corrected Inclination:

$$i_k = i_o + \delta i_k + (IDOT)t_k$$

Where IDOT in Column 17 of eph.dat.

Corrected longitude of ascending node:

$$\Omega_k = \Omega_o + (\Omega - \Omega_e)t_k - \Omega_e t_{oe}$$

Position in orbital plane:
x-coordinate:

$$x_k' = \mathrm{r}_k \cos u_k$$

y-coordinate:

$$y_k' = \mathrm{r}_k \sin u_k$$

Therefore, the Earth-fixed coordinates calculate by

$$x_k = x_k'\cos\Omega_k - y_k'\cos i_k\, x \sin\Omega_k$$
$$y_k = x_k'\sin\Omega_k + y_k'\cos i_k\cos\Omega_k$$
$$z_k = y_k'\sin i_k$$

However, the coordinates have some rotation error when transforming the coordinate from ECEF coordinate to Earth-Centered, Inertial (ECI) coordinate system.

The rotational matrix for coordinate transform correction:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_k \\ y_k \\ z_k \end{pmatrix}$$

2. Determine the broadcast satellite clock error.

Calculate the satellite clock offset ($t$).

$$t = t_{SV} - \Delta t_{SV}$$
$$\Delta t_{SV} = a_0 + a_1(t - t_{0c}) + a_2(t - t_{0c})^2 + \Delta t_r$$
$$\Delta t_r = F * e * sqeta \sin(E_k)$$

$t_{SV}$: The individual satellite times.
$a_0$: clock correction coefficient – group delay (s)
$a_1$: clock correction coefficient (s/s)
$a_2$: clock correction coefficient (s/s/s)

3. Estimate the tropospheric delay for each satellite.

First, resolve the tropospheric error of each iteration.
Using iterative algorithm to transform the initial position of ECEF coordinates to Geodetic coordinates ($\phi, \lambda, h$).

$$\text{ECEF coordinates} = \begin{bmatrix} -2694685.473 \\ -4293642.366 \\ 3857878.924 \end{bmatrix}$$

Then, Transform the Geodetic coordinates to east, north, up (ENU) coordinates.
Using Saastamoinen model to enable tropospheric error estimation.

$$z = \frac{\pi}{2} - \sin^{-1}\frac{h}{(\phi^2 + \lambda^2)^2}$$

Tropospheric delay includes two parts which are the hydrostatic delay (trph) and wet delay (trpw).

$$trph = \frac{0.0022768}{cos(z)} * \frac{p}{1 - 0.00266 * cos(2.0 * \phi) - 0.00028 * H / 1e3)}$$
$$trpw = 0.002277 * \left(\frac{1255.0}{T} + 0.05\right) * \frac{e}{cos(z)}$$

where p, T and H are pressure, temperature and humidity at height h.
where e is the partial water vapor pressure.

4. Use the linearized GPS measurement equation developed in class to estimate the vector $\Delta x$

The geometric range ($\rho$):

$$\rho = \sqrt{\sum_{i=1}^{n}\left[\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_i - \begin{bmatrix} -2694685.473 \\ -4293642.366 \\ 3857878.924 \end{bmatrix}\right]^2}$$

Where i is the number of satellites.
Approx pseudorange (p):

$$p = \rho + c * (\Delta t_{rec} - \Delta t_{SV}) + (trph + trpw)$$

Using least squares method to position:

$$\Delta x = H^{-1}\Delta p$$
$$\Delta x = (H^T H)^{-1} H^T \Delta p$$
$$H = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{xn} & a_{yn} & a_{zn} & 1 \end{bmatrix}$$
$$\Delta p_i = (pr_i - p_i) = a_{xi}\Delta x_u + a_y \Delta y_u + a_z \Delta z_u + c\Delta t_u$$

Where pr is the pseudorange in Column 3 of rcvr.dat, $\hat{x}_u, \hat{y}_u, \hat{z}_u$ is the approx position, $\hat{t}_u$is the offset and a is a function of the elevation angles.

$$x_u = \hat{x}_u + \Delta x_u$$

$$y_u = \hat{y}_u + \Delta y_u$$
$$z_u = \hat{z}_u + \Delta z_u$$
$$t_u = \hat{t}_u + \Delta t_u$$
$$a_{xi} = \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial x_u} + \Delta x_u$$
$$a_{yi} = \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{y}_u} + \Delta y_u$$
$$a_{zi} = \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{z}_u} + \Delta z_u$$

5. Update the estimate of the user position:

Update the position of ECEF and receiver clock offset:

$$\mathrm{X}_{0new} = \mathrm{X}_{0old} + \Delta x$$
$$= \begin{bmatrix} -2694685.473 \\ -4293642.366 \\ 3857878.924 \\ 0 \end{bmatrix} + \Delta x$$

6. if δx <10−4 m, then we have successfully converged on a valid position solution (Some of the MATLAB functions in the folder will be useful in solving this problem.)

```
if norm(dx(1:3)) < 1e-4 || ...  % check delta position solution smaller than threshold
        iter > 10  % check iteration number
    break;
```

## Result:

This report uses MATLAB as a tool to calculate all the processes. Below shows all the results of the process.

The following picture shows that the initial position in Latitude, Longitude, Altitude coordinates system (LLA) and Earth-centered, Earth-fixed coordinates system (ECEF).

```
Time of Iteration: 0 ----(initial)
Initial position: ECEF(m):
X                   Y                   Z
-2694685.4730(m), -4293642.3660(m), 3857878.9240(m)

WGS84 LLA:
Latitude                 Longitude                Altitude
37.458376433(degree), -122.112338996(degree), -31.4557(m))

Total position error:6374.4662(m)
```

The following table shows that the result of all the iterations result.

| | Time of Iterations | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| | $\Delta x$ | | | |
| X (m) | -5684.2133 | -0.1630 | -0.0002 | -0.0000 |
| Y (m) | 1073.3790 | -0.1773 | -0.0004 | -0.0000 |
| Z (m) | -2614.5187 | 0.1874 | 0.000 | 0.0000 |
| T (s) | 0.0017327 | -0.0000000 | 0.0000000 | 0.0000000 |

| | | | | |
|---|---|---|---|---|
| Distance (t*c) (m) | 519450.0754 | -0.4423 | 0.0005 | 0.0000 |
| Updated position: | | | | |
| ECEF(m) | | | | |
| X (m) | -2700369.6863 | -2700369.8493 | -2700369.8496 | -2700369.8496 |
| Y (m) | -4292568.9870 | -4292569.1644 | -4292569.1647 | -4292569.1647 |
| Z (m) | 3855264.4053 | 3855264.5927 | 3855264.5930 | 3855264.5930 |
| WGS84 LLA | | | | |
| Latitude (°) | 37.428085511 | 37.428085555 | 37.428085555 | 37.428085555 |
| Longitude (°) | -122.173180707 | -122.173181199 | -122.173181199 | -122.173181199 |
| Altitude (m) | 58.4526 | 58.7546 | 58.7552 | 58.7552 |
| Updated receiver clock offset: | | | | |
| R_t(s) | 0.0017327 | 0.0017327 | 0.0017327 | 0.0017327 |
| Distance (R_t*c) (m) | 519450.0754 | 519449.6331 | 519449.6336 | 519449.6336 |
| Total position error: | | | | |
| (m) | 32.1090 | 31.9732 | 31.9730 | 31.9730 |
| LS residual, squared error: | | | | |
| (m^2) | 2021.6757 | 2039.0026 | 2038.9986 | 2038.9986 |

To summarize,
The initial position of ECEF is X= -2694685.4730(m), Y=-4293642.3660(m) and Z=3857878.9240(m).
The final position of ECEF is X= -2700369.8496(m), Y= -4292569.1647(m) and Z= 3855264.5930(m).
The target position of ECEF is X= -2700400.0000(m), Y=-4292560.0000(m) and Z=3855270.0000(m).

The final position error is 31.9730(m).

As the time of the week of this report is 440992 which is the time that receiver thinks. However, it has a different error between the true time and the time that receiver thinks such as tropospheric error, satellite clock error, and rotation error which this report has mentioned. The previous result finds that the estimate of the user clock bias b is 0.0017327(s), 519449.6336(m). Therefore, the reported receiver clock time at this epoch (Column 1 of the rcvr matrix) is 440992.00173454 seconds. However, this report has some of the errors that did not mention. Therefore, the result still has a 0.0000127 (s) error.

Appendix A
Data file format of rcvr.dat and eph.dat

Data File Format of rcvr.dat:

rcvr.dat is an 8x7 matrix containing raw ranging information. Each of the 8 rows contains independent measurements for each of 8 satellites in view at the current epoch (an epoch is simply a term refers to a single discrete time; since our receivers provide data at approximately 1 sec. intervals, each epoch occurs approximately 1 sec. after the prior epoch. The columns of this matrix in clued the following data:

| Column | Symbol | Meaning |
|---|---|---|
| Column 1: | rcvr_tow | receiver time of week(s) |
| Column 2: | Svid | satellite PRN number (1 – 32) |
| Column 3: | pr | pseudorange (m) |
| Column 4: | cycles | number of accumulated cycles |
| Column 5: | Phase | to convert to (0 – 359.99) mult. by 360/2048 |
| Column 6: | slp_dtct | 0 = no cycle slip detected; non 0 = cycle slip |
| Column 7: | snr_dbhz | signal to noise ratio (dB-Hz) |

Data File Format of eph.dat:

eph.dat is an 8 x 24 matrix containing the ephemeris data from a GPS receiver. This data is used to estimate the orbital position of each satellite at any given time. Each row contains ephemeris data for a single satellite. The columns of this matrix include the following data:

| Column | Symbol | Meaning |
|---|---|---|
| Column 1: | rcvr_tow | receiver time of week(s) |
| Column 2: | Svid | satellite PRN number (1 – 32) |
| Column 3: | toc | reference time of clock parameters (s) |
| Column 4: | toe | reference time of ephemeris parameters (s) |
| Column 5: | af0 | clock correction coefficient – group delay (s) |
| Column 6: | af1 | clock correction coefficient (s/s) |
| Column 7: | af2 | clock correction coefficient (s/s/s) |
| Column 8: | ura | user range accuracy (m) |
| Column 9: | e | eccentricity (-) |
| Column 10: | sqrta | square root of semi-major axis a (m**1/2) |
| Column 11: | dn | mean motion correction (r/s) |
| Column 12: | m0 | mean anomaly at reference time (r) |
| Column 13: | w | argument of perigee (r) |
| Column 14: | omg0 | right ascension (r) |
| Column 15: | i0 | inclination angle at reference time (r) |
| Column 16: | odot | rate of right ascension (r/s) |
| Column 17: | idot | rate of inclination angle (r/s) |
| Column 18: | cus | argument of latitude correction, sine (r) |
| Column 19: | cuc | argument of latitude correction, cosine (r) |
| Column 20: | cis | inclination correction, sine (r) |
| Column 21: | cic | inclination correction, cosine (r) |
| Column 22: | crs | radius correction, sine (m) |
| Column 23: | crc | radius correction, cosine (m) |
| Column 24: | iod | issue of data number |