Suggestion/demonstration - the overall system is considered (processing plant and utilities) with fuel gas and ventilated gas. The inputs of this control volume are therefore the well-fluid (crude oil), the cooling water used on-site and the air used in the gas turbines. The outputs are the exhaust gases from the gas turbines, the oil and gas sent onshore, the produced water, the ventilated gas from the process units and the rejected cooling water. $\dot{I}$ denotes the internal thermodynamic irreversibilities and $\dot{E}^{Q}_{CW}$ the increase of physical exergy of the cooling water. i denotes the chemical specie and k the stream of interest.

$$\dot{E}_{feed} + \dot{E}_{air} = \dot{E}_{oil} + \dot{E}_{gas,export} + \dot{E}_{gas,ventilated} + \dot{E}_{produced\ water} + \dot{E}^{Q}_{CW} + \dot{E}_{exhaust} + \dot{I}$$

Decomposing each exergy term (except for the air) into its chemical and physical exergies...

$$\begin{aligned}
\dot{E}^{ph}_{feed} + \dot{E}^{ch}_{feed} + \dot{E}_{air} &= \dot{E}^{ph}_{oil} + \dot{E}^{ch}_{oil} + \dot{E}^{ph}_{gas,export} + \dot{E}^{ch}_{gas,export} \\
&\quad + \dot{E}^{ph}_{gas,ventilated} + \dot{E}^{ch}_{gas,ventilated} \\
&\quad + \dot{E}^{ph}_{produced\ water} + \dot{E}^{ch}_{produced\ water} \\
&\quad + \dot{E}^{Q}_{CW} + \dot{E}_{exhaust} + \dot{I} \qquad (1) \\
\dot{E}^{ph}_{feed} + \dot{E}_{air} &= \dot{E}^{ph}_{oil} + \dot{E}^{ch}_{oil} + \dot{E}^{ph}_{gas,export} + \dot{E}^{ch}_{gas,export} \\
&\quad + \dot{E}^{ph}_{gas,ventilated} + \dot{E}^{ch}_{gas,ventilated} \\
&\quad + \dot{E}^{ph}_{produced\ water} + \dot{E}^{ch}_{produced\ water} \\
&\quad + \dot{E}^{Q}_{CW} + \dot{E}_{exhaust} + \dot{I} \\
&\quad - \dot{E}^{ch}_{feed} \qquad (2)
\end{aligned}$$

Assuming ideal mixtures, with an activity coefficient equal to 1, the chemical exergy can be expressed as:

$$\dot{E}^{ch} = \sum_i \dot{n}_i \left(\bar{\epsilon}^0_i + \bar{R} T_0 \ ln \ x_i \right) \qquad (3)$$

Adding the term $\left(\sum_i \dot{n}_{i,fuel} \left(\bar{\epsilon}^0_i + \bar{R} T_0 \ ln \ x_{i,feed} \right) \right)$, which is the fraction of the feed sent as fuel to the gas turbines, the exergy balance becomes:

$$\begin{aligned}
\dot{E}^{ph}_{feed} + \dot{E}_{air} + \left(\sum_i \dot{n}_{i,fuel}\left(\bar{\epsilon}^0_i + \bar{R}T_0\ ln\ x_{i,feed}\right)\right) \quad = \quad & \dot{E}^{ph}_{oil} + \dot{E}^{ch}_{oil} + \dot{E}^{ph}_{gas,export} + \dot{E}^{ch}_{gas,export} \\
& + \dot{E}^{ph}_{gas,ventilated} + \dot{E}^{ch}_{gas,ventilated} \\
& + \dot{E}^{ph}_{produced\ water} + \dot{E}^{ch}_{produced\ water} \\
& + \dot{E}^{Q}_{CW} + \dot{E}_{exhaust} + \dot{I} \\
& + \left(\sum_i \dot{n}_{i,fuel}\left(\bar{\epsilon}^0_i + \bar{R}T_0\ ln\ x_{i,feed}\right)\right) \\
& - \dot{E}^{ch}_{feed} \qquad (4)
\end{aligned}$$

The difference of chemical exergies between the processing plant outlet and inlet is defined as:

$$\Delta\dot{E}^{ch} = -\dot{E}^{ch}_{feed} + \dot{E}^{ch}_{oil} + \dot{E}^{ch}_{gas,export} + \dot{E}^{ch}_{gas,ventilated} + \dot{E}^{ch}_{produced\ water} + \dot{E}^{ch}_{fuel} \qquad (5)$$

$$\begin{aligned}
&= \sum_i \left(\bar{\epsilon}^0_i\left(-\dot{n}_{i,feed} + \dot{n}_{i,oil} + \dot{n}_{i,gas,export} + \dot{n}_{i,gas,ventilated} + \dot{n}_{i,produced\ water} + \dot{n}_{i,fuel}\right)\right) \\
&+ \bar{R}T_0 \sum_i \left(-\dot{n}_{i,feed}\ ln\ x_{i,feed} + \dot{n}_{i,gas,export}\ ln\ x_{i,gas,export} + \dot{n}_{i,gas,ventilated}\ ln\ x_{i,gas,ventilated}\right) \\
&+ \bar{R}T_0 \sum_i \left(\dot{n}_{i,produced\ water}\ ln\ x_{i,produced\ water} + \dot{n}_{i,fuel}\ ln\ x_{i,fuel}\right) \qquad (6)
\end{aligned}$$

As no chemical species are produced or consumed within *the processing plant*:

$$\sum_i \left(\bar{\epsilon}^0_i\left(-\dot{n}_{i,feed} + \dot{n}_{i,oil} + \dot{n}_{i,gas,export} + \dot{n}_{i,gas,ventilated} + \dot{n}_{i,produced\ water} + \dot{n}_{i,fuel}\right)\right) = 0 \qquad (7)$$

and therefore:

$$\begin{aligned}
\Delta\dot{E}^{ch} &= \bar{R}T_0 \sum_i \dot{n}_i \left(-ln\ x_{i,feed} + ln\ x_{i,gas,export} + ln\ x_{i,gas,ventilated} + ln\ x_{i,produced\ water} + ln\ x_{i,fuel}\right) \\
&= \sum_{export\ oil+export\ gas+fuel\ gas} \Delta\dot{E}^{ch} + \sum_{ventilatedgas+producedwater} \Delta\dot{E}^{ch} \qquad (8)
\end{aligned}$$

Splitting the variation of chemical exergy between the inlet and outlet of the system into contributions for each outflow stream k of the processing plant:

$$\Delta\dot{E}^{ch}_k = \bar{R}T_0 \sum_i \dot{n}_{i,k}\left(ln\ x_{i,k} - ln\ x_{i,feed}\right) \qquad (9)$$

Assuming that produced water and ventilated gas are not useful (i.e. losses to the environment) and that exported oil, exported gas and fuel gas are useful, the exergy balance equation becomes:

$$\begin{aligned}
\dot{E}^{ph}_{feed} + \dot{E}_{air} + \left(\sum_{fuel} n_{i,fuel}\left(\bar{\epsilon}^0_i + \bar{R}T_0\ ln\ x_{i,feed}\right)\right) \quad = \quad & \sum_{k,useful} \Delta\dot{E}^{ch}_{k,useful} + \sum_{k,useful} \dot{E}^{ph}_{k,useful} \\
& + \sum_{k,waste} \Delta\dot{E}^{ch}_{k,waste} + \sum_{k,waste} \dot{E}^{ph}_{k,waste} + \dot{E}^{Q}_{CW} + \dot{E}_{exhaust} + \dot{I} \qquad (10)
\end{aligned}$$

We know that the exergy losses associated to streams of matter is defined as:

$$\dot{E}_l = \sum \dot{E}_{waste} = \sum_{k,waste} \dot{E}^{ph}_{k,waste} + \sum_{k,waste} \left(\sum_i \dot{n}_i \left(\bar{\epsilon}^0_i + \bar{R} T_0 \; ln \; x_{i,waste} \right) \right) \tag{11}$$

And that the chemical exergy variation for waste streams is:

$$\sum_{waste} \Delta \dot{E}^{ch} = \bar{R} T_0 \sum_{waste} \sum_i \dot{n}_{i,waste} \left(ln \; x_{i,waste} - ln \; x_{i,feed} \right) \tag{12}$$

Thus:

$$\begin{aligned} \dot{E}^{ph}_{feed} + \dot{E}_{air} + \left(\sum_{fuel} \dot{n}_{i,fuel} \left(\bar{\epsilon}^0_i + \bar{R} T_0 \; ln \; x_{i,feed} \right) \right) + \left(\sum_{waste} \dot{n}_{i,waste} \left(\bar{\epsilon}^0_i + \bar{R} T_0 \; ln \; x_{i,feed} \right) \right) = \\ \sum_{useful} \Delta \dot{E}^{ch}_{useful} + \sum_{useful} \dot{E}^{ph}_{useful} + \dot{E}^Q_{CW} + \dot{E}_{exhaust} + \dot{I} + \sum_{waste} \dot{E}_{waste} \end{aligned} \tag{13}$$

The general exergy loss/product/fuel/destruction balance is expressed as, according to Bejan et al.:

$$\dot{E}_p = \dot{E}_f - \dot{E}_l - \dot{E}_d \tag{14}$$

The desired product is identified as the separation exergy term and the physical flow exergy rate of the useful products:

$$\sum_{useful} \Delta \dot{E}^{ch}_{useful} + \sum_{useful} \dot{E}^{ph}_{useful}$$

The losses are the sum of the exergy discharged to the sea, the exergy contents of the exhaust gases and of the wasted streams of the processing plant:

$$\dot{E}^Q_{CW} + \dot{E}_{exhaust} + \dot{I} + \sum_{waste} \dot{E}_{waste}$$

The exergy destruction corresponds to the internal irreversibilities

$$\dot{I}$$

The exergetic fuel is therefore the physical exergy rate of the feed and air, as well as the chemical exergy of the fractions of the feed consumed on-site in combustion and the ones rejected into the environment:

$$\dot{E}^{ph}_{feed} + \dot{E}_{air} + \left(\sum_{fuel} n_{i,fuel} \left(\bar{\epsilon}^0_i + \bar{R} T_0 \; ln \; x_{i,feed} \right) \right) + \left(\sum_{waste} n_{i,waste} \left(\bar{\epsilon}^0_i + \bar{R} T_0 \; ln \; x_{i,feed} \right) \right)$$

In practice, the term $\left(\sum_{waste} n_{i,waste} (\bar{\epsilon}^0_i + \bar{R} T_0 \; ln \; x_{i,feed}) \right)$ means that an offshore plant which does not make use of the produced water or ventilates significant amounts of gas needs to process a larger feed to obtain the same exergetic product.

The overall exergetic efficiency of an offshore platform is thus:

$$\Psi = \frac{\sum_{useful} \Delta \dot{E}^{ch}_{useful} + \sum_{useful} \dot{E}^{ph}_{useful}}{\dot{E}^{ph}_{feed} + \dot{E}_{air} + \left(\sum_{fuel} n_{i,fuel} \left(\bar{\epsilon}^0_i + \bar{R}T_0 \ ln \ x_{i,feed}\right)\right) + \left(\sum_{waste} n_{i,waste} \left(\bar{\epsilon}^0_i + \bar{R}T_0 \ ln \ x_{i,feed}\right)\right)} \tag{15}$$

$$= 1 - \left(\frac{\left(\dot{E}^{Q}_{CW} + \dot{E}_{exhaust} + \dot{I} + \sum_{waste} \dot{E}_{waste}\right)}{\dot{E}^{ph}_{feed} + \dot{E}_{air} + \left(\sum_{fuel} n_{i,fuel} \left(\bar{\epsilon}^0_i + \bar{R}T_0 \ ln \ x_{i,feed}\right)\right) + \left(\sum_{waste} n_{i,waste} \left(\bar{\epsilon}^0_i + \bar{R}T_0 \ ln \ x_{i,feed}\right)\right)} \right) \tag{16}$$