7. $z = x^2+y^2$ and $2x - 4y - z - 1 = 0$ $\longrightarrow$ $\vec{r}(t) = (x(t), y(t), z(t))$

goal

$z = x^2+y^2$ $\qquad$ $z = 2x - 4y - 1$

$$x^2+y^2 = 2x - 4y - 1$$

$$x^2 - 2x + y^2 + 4y + 1 = 0$$

$$(x^2-2x+1) + (y^2+4y+4) + \cancel{1} = \cancel{5}\ 4$$

$$\underbrace{(x-1)}_{X}^2 + (y+2)^2 = 4$$

$$X^2 + Y^2 = 4 \Rightarrow \begin{cases} X = 2\cos t \\ Y = 2\sin t \end{cases} \Rightarrow \begin{cases} x-1 = 2\cos t \\ y+2 = 2\sin t \end{cases}$$

$$\Rightarrow \begin{cases} x = 2\cos t + 1 \\ y = 2\sin t - 2 \\ z = 2(2\cos t + 1) - 4(2\sin t - 2) - 1 \end{cases} \qquad t \in [0, 2\pi]$$

11. $x^2 - 2x + y^2 + z^2 = 0$ $\qquad$ cylindrical (r, θ, z)

spherical (ρ, θ, ϕ)

$$z^2 + (x^2 - 2x + y^2) = 0$$

polar $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\qquad$ $\underbrace{x^2+y^2}_{r^2} - 2x$

$r^2 - 2r\cos\theta$ $\qquad$ $\tan\phi = \frac{r}{z}$

$$\boxed{z^2 + r^2 - 2r\cos\theta = 0}$$

$$\boxed{\begin{aligned} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{aligned}}$$

Spherical : $\underbrace{x^2+y^2+z^2}_{\rho^2} - 2x = 0$

$$\rho^2 - 2(\rho\sin\phi\cos\theta) = 0$$

$$\rho(\rho - 2\sin\phi\cos\theta) = 0 \qquad \boxed{\rho = 2\sin\phi\cos\theta}$$

$$\rho - 2\sin\phi\cos\theta = 0$$

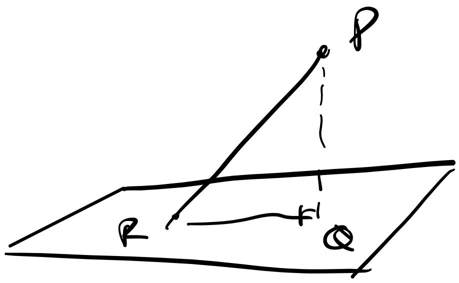
3, 5.

5. distance from $(1, -2, 4)$ to $3x + 2y + 6z = 5$

$P(x_0, y_0, z_0)$

$ax + by + cz + d = 0$

$$\boxed{\text{dist}(P,) = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}}$$



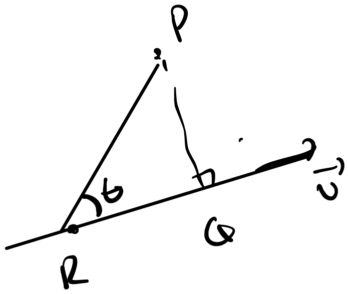
$|PQ| = \text{proj}_{\overrightarrow{PQ}} (\overrightarrow{RP})$

↓

normal of plane

$P(x_0, y_0, z_0)$

$\vec{r}(t) = \vec{r_0} + t\vec{v}$



$|PQ| = \underbrace{|\overrightarrow{RP}| \sin\theta}_{\frac{|\overrightarrow{RP} \times \vec{v}|}{|\overrightarrow{RP}| \cdot |\vec{v}|}}$

$\text{dis}(P, \vec{r}) = \dfrac{|\overrightarrow{RP} \times \vec{v}|}{|\vec{v}|} \longrightarrow$ #6.

6 $\quad \underbrace{(4, 1, -2)}_{P}$ to $(1+t, 3-2t, 4-3t)$

$\underbrace{(1, 3, 4)}_{R} + t\underbrace{(1, -2, -3)}_{\vec{v}}$

$\text{dist} = \dfrac{|\overrightarrow{RP} \times \vec{v}|}{|\vec{v}|}$

Note: $\overrightarrow{RP} = P - R$
$= (4, 1, -2) - (1, 3, 4)$
$= (3, -2, -6)$

3. Start from $(0,0,3)$ move 5 units along $x = 3\sin t$
$y = 4t$
$z = 3\cos t$

$r(t) = (3\sin t, 4t, 3\cos t) \overset{?}{=} (0,0,3)$

$\Rightarrow$ Start at $t = 0$

arclength function $\underline{s(t)} = \displaystyle\int_0^t |\vec{r}\,'(s)|\, ds$

$= \displaystyle\int_0^t \sqrt{(3\cos u)^2 + 4^2 + (-3\sin u)^2}\, du$

$\underbrace{9\sin^2 u + 9\cos^2 u}_{9(\sin^2 u + \cos^2 u)}$ $\underbrace{(\sin^2 u + \cos^2 u)}_{1}$

$= \displaystyle\int_0^t \sqrt{9 + 16}\, du = 5t$

5 units $\Rightarrow s(t) = 5 \Rightarrow 5t = 5$
$\Rightarrow t = 1$

the point we are at is $r(1) = (3\sin 1, 4, 3\cos 1)$ (radian)

2. angle between $x + 4y - 3z = 1$ and $-3x + 6y + 7z = 0$

$\cos\theta = 0$

- angle between planes = angle between normals
- lines ______ direction vectors

$(1,4,-3)$ $\qquad$ $(-3,6,7)$

$$\cos\theta = \frac{(1,4,-3)\cdot(-3,6,7)}{\sqrt{1^2+4^2+3^2}\cdot\sqrt{3^2+6^2+7^2}} =$$

$-3+24-21=0$

$\cos\theta = 0 \to \pi/2$

or 90°

1. $r(t) = (t^2, t^3, t^4)$ $\qquad 0 \le t \le 2$

$s(t) = \int_0^t$

$$s(2) = \int_0^2 |r'(u)|\,du$$

$$= \int_0^2 \sqrt{(2u)^2 + (3u^2)^2 + (4u^3)^2}\,du$$

$$= \int_0^2 \sqrt{4u^2+9u^4+16u^6}\,du$$

$$\sqrt{4u^2+9u^4+16u^6} = \sqrt{u^2(4+9u^2+16u^4)}$$

$$\int_0^2 u\sqrt{4+9u^2+16u^4}\,du$$

$v = 4+9u^2+16u^4$

$dv = (18u+64u^3)\,du$

$$\int_4^{4+9\cdot 2^2+16\cdot 2^4} \sqrt{v}\,.$$

8. line $\vec r(t) = \vec r_0 + t\vec v$

$(0,1,2)$

go $(0,1,2)$

$//$ $x+y+z=2$

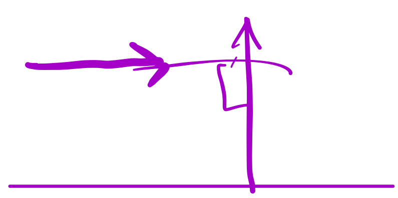
$\perp$ $(1+t, 1-t, 2t)$

↳ direction

$(1,1,0) + t(1,-1,2)$

$\vec v \;//\; x+y+z=2$ $\quad (1,1,1)$

$\vec v \perp (1,-1,2)$

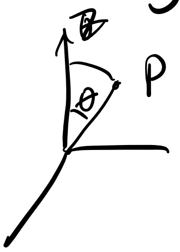


$$\begin{cases} \vec v \perp (1,1,1) \\ \vec v \perp (1,-1,2) \end{cases}$$

$\vec v = (1,1,1)\times(1,-1,2)$

$$\Rightarrow \quad \vec{r}(t) = (0,1,2) + t\vec{v} \quad , t \in \mathbb{R}$$

10. $\phi = \frac{\pi}{3}$ in spherical



$$\boxed{\tan\phi = \frac{r}{z}}$$

$$z^2 = x^2 + y^2$$

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$\Rightarrow \quad \underbrace{x^2 + y^2}_{r^2} = \rho^2 \sin^2\phi$

$r = \rho \sin\phi$

$z = \rho \cos\phi$

$\frac{r}{z} = \tan\phi$

$$\tan\frac{\pi}{3} = \frac{\sqrt{x^2+y^2}}{z} = \sqrt{3}$$

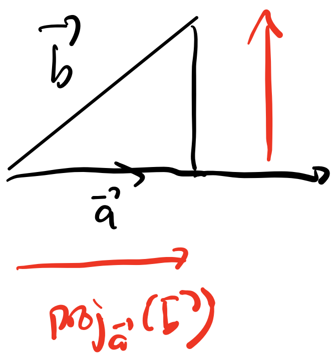
$$\sqrt{x^2+y^2} = \sqrt{3}\,z \quad \Rightarrow \quad x^2 + y^2 = 3z^2$$

9. $\text{proj}_{\vec{a}}(\vec{b}) = \left(\frac{\vec{b}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a}$ $\qquad \text{comp}_{\vec{a}}(\vec{b}) = |\text{proj}_{\vec{a}}(\vec{b})| = \frac{|\vec{b}\cdot\vec{a}|}{|\vec{a}|}$

$\text{orth}_{\vec{a}}(\vec{b})$

$\vec{a} = (1,1,-2)$

$\vec{b} = (3,-2,1)$



$$\boxed{\text{orth}_{\vec{a}}(\vec{b}) = \vec{b} - \text{proj}_{\vec{a}}(\vec{b})}$$

$$\text{proj}_{\vec{a}}(\vec{b}) = \frac{3-2-2}{\sqrt{6}\,\sqrt{14}}(1,1,-2) = \frac{-1}{\sqrt{84}}(1,1,-2)$$

$$\text{orth}_{\vec{a}}(\vec{b}) = (3,-2,1) - \frac{-1}{\sqrt{84}}(1,1,-2)$$