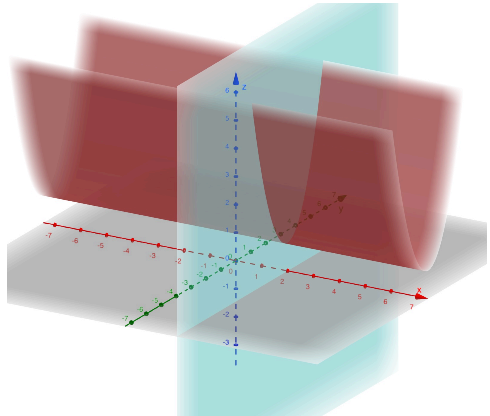
Cylinder and Quadriceps Surfaces

• To graph a surface in 3D, it is useful to determine the curves of intersection of the surface with the planes parallel to the coordinate planes.
These curves are called traces (or cross sections)

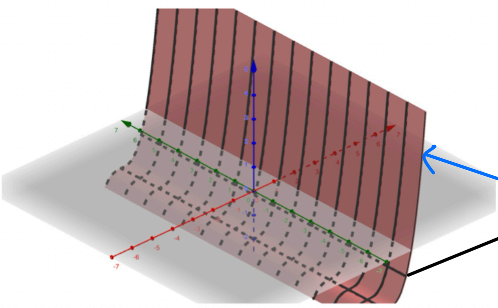
Example with

$z = y^2$



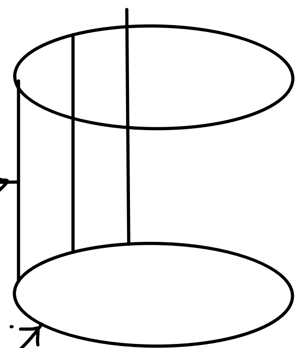
• A cylinder is a surface that consists of all lines (called rullings) that are parallel to a given line and pass through a given planar

$z = y^3$



rulling

planar curve



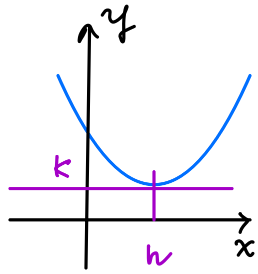
$x^2 + y^2 = 1$

→ usually missing a variable is a cylinder (but not all)

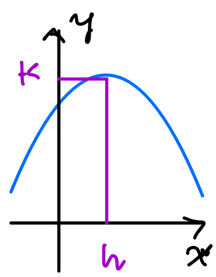
Review 2D calculus

• Parabolas $y = ax^2 + bx + c = a(x-h)^2 + k$

↳ 1 square term



$a > 0$



$a < 0$

• Ellipses : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

↳ 2 square terms , same signs

• Hyperbolas : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

↳ 2 square terms , opposite signs

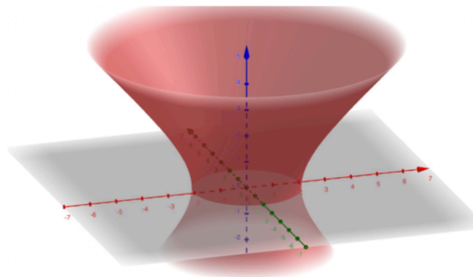
Quadric surfaces : are all of the form

$$Ax^2+By^2+Cz^2+Dxy+Eyz+Fxz+Gx+Hy+Iz+J=0$$

Standard form :

$$\boxed{Ax^2+By^2+Cz^2+J=0} \quad \text{or} \quad \boxed{Ax^2+By^2+Iz=0}$$

How to classify? $\longrightarrow$ next page



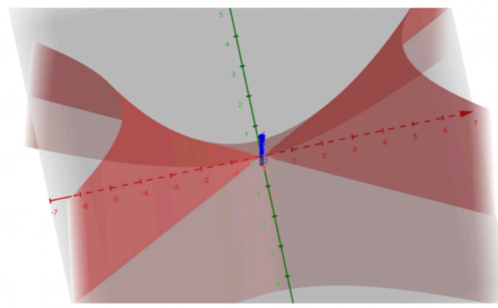
Ex 1. $x^2+y^2-z^2=4$

all power = 2, alternate signs $\Rightarrow$ hyperboloid

$z=0$: $x^2+y^2=4$

has solution $\Rightarrow$ hyperboloid of one sheet

Ex 2. $x^2-4y^2=z$



Power of z is 1 $\Rightarrow$ paraboloid

The square terms have different signs

$\Rightarrow$ hyperbolic paraboloid

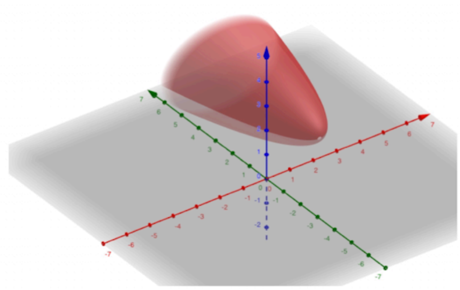
Ex 3. $x^2+2z^2-6x-y+10=0$

$$(x^2-6x+9)+2z^2-y+10=9$$

$$(x-3)^2+2z^2+1=y$$

similar to

$$x^2+2z^2=y$$



power of y is 1 $\Rightarrow$ paraboloid

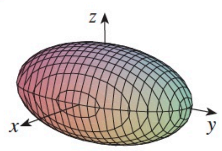
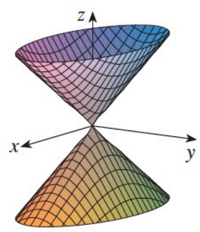
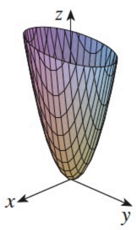
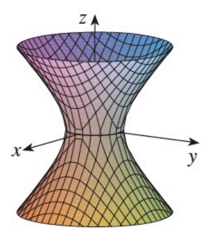
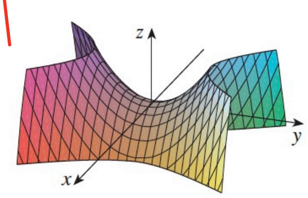
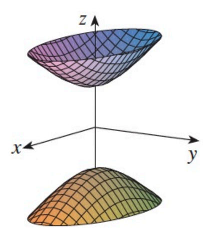
The square terms have the same signs

$\Rightarrow$ elliptic paraboloid

How to classify:

- Check all the power, if all are power 2:
    - If no constant: Cone
    - There is a constant
        - If all the sign are the same then ellipsoid
        - If sign are alternate: hyperboloid (then check by letting z = 0 to see if 1 sheet or 2 sheet)
- If there is a power 1: paraboloid
    - If all sign are the same: elliptic paraboloid
    - else: hyperbolic paraboloid

TABLE 1 Graphs of quadric surfaces

Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ All traces are ellipses. If $a=b=c$, the ellipsoid is a sphere.	Cone 	$\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x=k$ and $y=k$ are hyperbolas if $k\neq 0$ but are pairs of lines if $k=0$.
Elliptic Paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c<0$ is illustrated.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Horizontal traces in $z=k$ are ellipses if $k>c$ or $k<-c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.