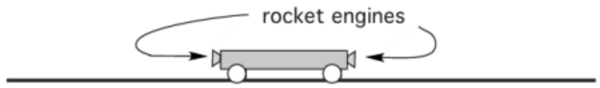
Introduction

Imagine a railroad car powered by rocket engines on each side.



We introduce the variables

- $x(t)$ is the position of the rocket railroad car on the train track at time t
- $v(t)$ is the velocity of the rocket rail road car at time t
- The force from the rocket engines at time t, where we only consider $F(t) \in [-1, 1]$, and the sign of $F(t)$ depends on which engine is firing. firing.

One might ask, starting at a given location A, whether there exists a choice F so that the car stops at a predetermined location B. If so, is there a way to do so with minimal time or energy?
This constitutes the basic problem of optimal control theory. Here is a quote from Wikipedia regarding Richard E. Bellman:

> *In 1949, Bellman worked for many years at RAND corporation, and it was during this time that he developed dynamic programming.*

The so-called Dynamic Programming Principle connects the control problem above to a partial differential equation known as the Hamilton–Jacobi–Bellman equation. We will provide an introduction to this topic and explore the fascinating connection between dynamics and partial differential equations.

Instructor

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Course Description

We will delve into the foundational aspects of the following related subjects:

- An introduction to the optimal control problem and the first-order Hamilton-Jacobi-Bellman equation.
- An overview of viscosity solutions and fundamental techniques.
- (If time allows) Engaging in projects centered around one-dimensional examples and enhancing research-level findings from the existing literature.

The anticipated commitment for this reading plan is around 1 credit, involving approximately 1-1.5 hours of weekly meetings, complemented by additional reading at home. Our main aim is to have fun exploring and appreciate the beauty of viscosity solutions in optimal control theory.

Additional topics might be covered

- Homogenization theory.
- Numerical computation of optimal control and viscosity solution.

Required background

- Linear algebra
- Some basic real analysis and measure theory, Ordinary differential equations will be sufficient.

References

In case you want to have some references, I suggest some followings (in no particular order):

1. A lecture note by Alberto Bressan and Benedetto Piccoli:
   Introduction to the Mathematical Theory of Control.
   It is available free of charge at
   `https://www-aimsciences-org.proxy2.cl.msu.edu/book/AM/volume/27`
2. A book by Hung V. Tran:
   Hamilton–Jacobi Equations: Theory and Applications, volume 213 of Graduate studies in Mathematics. American Mathematical Society, 2021.
   It is available free of charge at
   `http://math.wisc.edu/ hung/HJ-equations-Tran-AMS.pdf`.
3. Chapter 10 of Lawrence C. Evans's PDE book:
   Partial Differential Equations: Second Edition
4. A lecture note by Khai T. Nguyen
   Topics on optimal control and PDEs
   It is available free of charge at
   `https://tnguye13.math.ncsu.edu/course1.pdf`.