Michigan State University
Math 234 – Spring 2024

610642

Lecture 03
2024-01-12

Key takeaways:

- Cross-product $\mathbf{a} \times \mathbf{b}$ computations, it is $\perp$ both $\mathbf{a}$ and $\mathbf{b}$.
- Area of triangles

The Cross Product of

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} = (a_1, a_2, a_3) \qquad \text{and} \qquad \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k} = (b_1, b_2, b_3)$$

is a vector that is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i} \underbrace{\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}}_{a_2b_3 - b_2a_3} - \mathbf{j} \underbrace{\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}}_{a_1b_3 - b_1a_3} + \mathbf{k} \underbrace{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}_{a_1b_2 - b_1a_2}.$$

Example Evaluate $(1, 2, 3) \times (-2, 1, 0)$.

Notes and Theorem $\mathbf{a} \cdot \mathbf{b}$ is a number, while $\mathbf{a} \times \mathbf{b}$ is a vector.

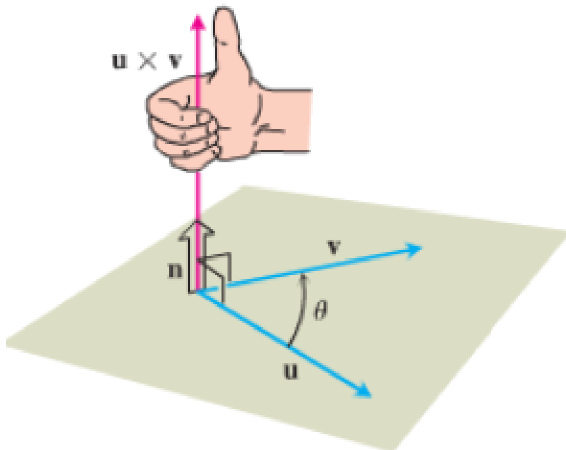
(a) $\mathbf{a} \times \mathbf{b}$ is both perpendicular to $\mathbf{a}$ and $\mathbf{b}$.

(b) If θ is the angle between $\mathbf{a} \cdot \mathbf{b}$ $(0 \le \theta \le \pi)$ then $|\mathbf{a} \cdot \mathbf{b}| = |\mathbf{a}|.|\mathbf{b}| \sin\theta$.

(c) Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if $|\mathbf{a} \times \mathbf{b}| = 0$.

Example Find a vector $\mathbf{u}$ that satisfies $\mathbf{u} \cdot (9, 3, 1) = 0$ and $\mathbf{u} \cdot (-2, 4, 0) = 0$.

Theorem - Direction of the Cross Product Take two non-zero non-parallel vectors **a** and **b**. Then the direction of $\mathbf{a} \times \mathbf{b}$ is determined by the right-hand rule. That is: the way your right thumb handrub right points when your right-hand fingers curl through the angle θ from **a** to **b**.



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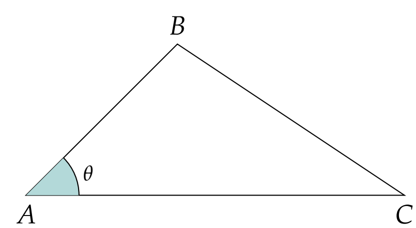
Computing area of triangle

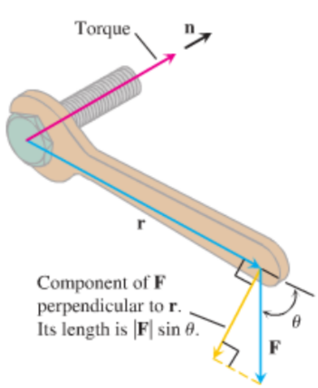
Given a triangle ABC, we have

$$S_{ABC} = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}|AB|.|AC|.\sin(\theta).$$



Example Find the area of the triangle with vertices $P(1,0,1)$, $Q(-2,1,3)$, and $R(4,2,5)$.

Torch



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$$\text{Torch} = \text{Force} \times \text{Distance from pivot}$$

- The magnitude of the torque is
  Magnitude of $\mathbf{F} \times \mathbf{r}$
- The torch vector is $\mathbf{F} \times \mathbf{r}$
  (note the direction with right hand thumb rule)