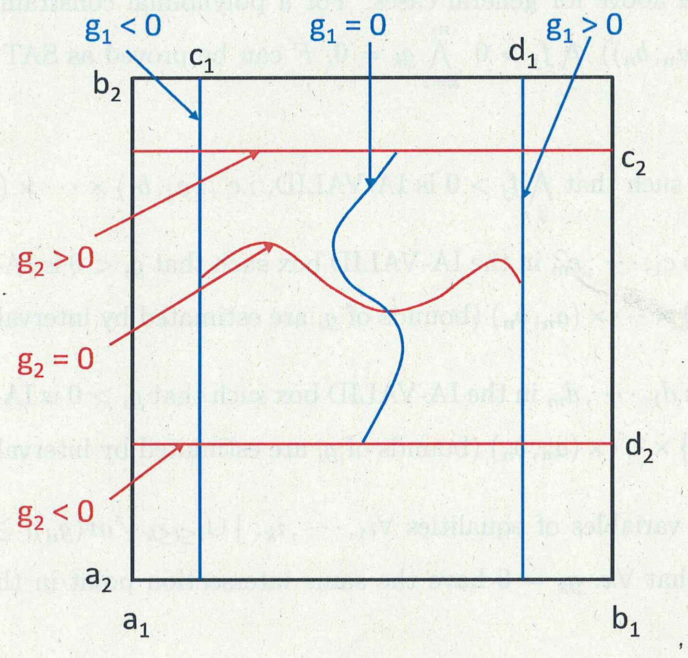

Figure 10.2: Solving multiple equalities

10.2.2 Extension to Multiple Equalities

We first present an approach for solving polynomial constraints with 2 equalities and then extend the same idea for general cases. We assume that constraints have two variables.

For a polynomial constraint $F = (x_1 \in (a_1, b_1) \wedge x_2 \in (a_2, b_2)) \bigwedge_j f_j > 0 \wedge g_1 = 0 \wedge g_2 = 0$, F can be proved as SAT by following steps:

- first, find a box such that $\bigwedge_j f_j > 0$ is IA-VALID, i.e., $(l_1, h_1) \times (l_2, h_2)$,
- find 2 instances $c_1, d_1 \in (l_1, h_1)$ such that $g_1 < 0$ on $\{c_1\} \times (l_2, h_2)$ and $g_1 > 0$ on $\{d_1\} \times (l_2, h_2)$ (values of g_1 are estimated by interval arithmetic),
- find 2 instances $c_2, d_2 \in (l_2, h_2)$ such that $g_2 < 0$ on $(l_1, h_1) \times \{c_2\}$ and $g_2 > 0$ on $(l_1, h_1) \times \{d_2\}$ (values of g_2 are also estimated by interval arithmetic).

Figure 10.2 demonstrates our approach when solving polynomial constraints with 2 equalities. The idea behind is finding a box such that IA-VALID for polynomial inequalities and proving that the line $g_1 = 0$ intersects the line $g_2 = 0$ insides the IA-VALID box.