Fully-Decentralised Reinforcement Learning in Fog-based Internet-of-Things

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Abstract—**Reinforcement learning (RL) algorithms offers more insights about the overall futuristic functionalities for intelligent Internet-of-Things (IoT) devices. With the explosive growth in the number of IoT devices, as well as the highly-distributed deployments of these devices today, managing the IoT devices centrally becomes infeasible. As such, several disruptive paradigms have emerged, one of which is the fog computing-based IoT, which aim towards shifting computation, control, and decision-making closer to the network edge. However, mobility and power-constrain of these fog devices remains an issue of concern. In this paper, we aim to minimize the global outage in communication within a fog-based IoT network, by optimizing the power-control parameter of each potential mobile fog-relay agent (MFRA), as well as optimizing the physical position. As such, each MFRA is compelled to take certain actions that may influence its environment. We optimize the communication performance by applying a decentralized q-learning approach, with each MFRA acting independently, but contributing towards a global local-optimal policy. Furthermore, we show that our algorithm is scalable even with very large number of devices.**

Index Terms—**Reinforcement learning (RL), Internet-of-Things (IoT), Fog-based IoT, Markov decision process (MDP), machine learning (ML).**

I. Background

THE fog computing-based IoT paradigm aims at moving computation, control, and decision-making within the IoT ecosystem closer to the network edge [1]. The key driver of this paradigm are fog devices, which may be energy-constrained or not, and can either be mobile or static. The deployment and efficient utilization of these fog devices will contribute to the success of future IoT systems [2], one of which is serving as relays to overcome communication outages due to obstacles or long distances between a source node and a remote destination node where IoT services may be rendered.

However, in order for devices to communicate efficiently with minimal outage (loss of transmitted packets), several bottlenecks may arise, one of which is the efficient utilization of energy by power-constrained IoT devices. Energy can be used up when these devices unnecessarily increase their power-level in order to communicate with neighbouring devices within the network, conversely, energy can be saved when the devices regulate their transmission power, especially in situations when they are relatively close to the communicating parties. For example, an IoT end-device that transmits at high power irrespective of the channel conditions or proximity of its neighbours may drastically deplete its energy and die-out, hence, resulting in communication breakdown due to a

point-of-failure within the network. Another challenge to be addressed is the energy consumed due to mobility. In order to deliver on some acceptable quality-of-service (QoS) within the IoT ecosystem, it is imperative for devices to be mobile, however, these devices are at risk of draining most of their energy on movement. For instance, energy will be inefficiently used if a power-constrained fog device decides to move closer to the communicating party only to relay very few numbers of packets, though having a guarantee that all the packets are delivered to the destination. However, if the size of the number of packets to be relayed is large, it may be optimal for the fog device to move closer, hence, conserving the energy of the IoT end-device, who can now transmit at a lower power level.

Moreover, since power-control mechanisms and smart mobility are critical in minimizing outages in communication, these devices should be able to learn when to increase, decrease, or maintain their power-levels or to move efficiently in order to increase the long-term performance of the network. More so, near-optimal actions from these devices are required to drive several smart cities applications, most importantly the Industrial IoT (IIoT), where industrial robots are deployed to act intelligibly in a dynamic industrial setting, and intelligent monitoring applications, where surveillance drones are deployed in militarised zones to meet stringent quality-of-service (QoS) requirements [3].

The work in [3] used an iterative algorithm based on the steepest descent method to address the problem in a multi-tier fog-based IoT architecture with a fog device which could adjust its position and power-control parameter in order to minimize outage in communication. However, the work did not take into consideration the performance of the approach in a highly-decentralised IoT environment. In [5], a clustering algorithm was used on eight (8) unmanned aerial vehicle (UAVs) to collect data from ground IoT devices with an objective of minimizing the energy consumed by the IoT devices during uplink communications. However, this work considered a centralized network where the locations of every IoT device and UAV were known to the controller. Considering the highly-distributed nature of deployed IoT devices, it becomes infeasible to manage devices centrally [4]. As such, reinforcement learning can be effectively deployed on fog devices to allow them to act independently based on their local experiences in the environment, i.e. each fog device should be able to learn independently without a central entity.

A decentralised stateless Q-learning approach was proposed in [4] to improve aggregate throughput in four coexistent wireless networks (WN). Each WN was considered to be an agent running the stateless Q-learning algorithm with agents having action space as channel number, and transmit
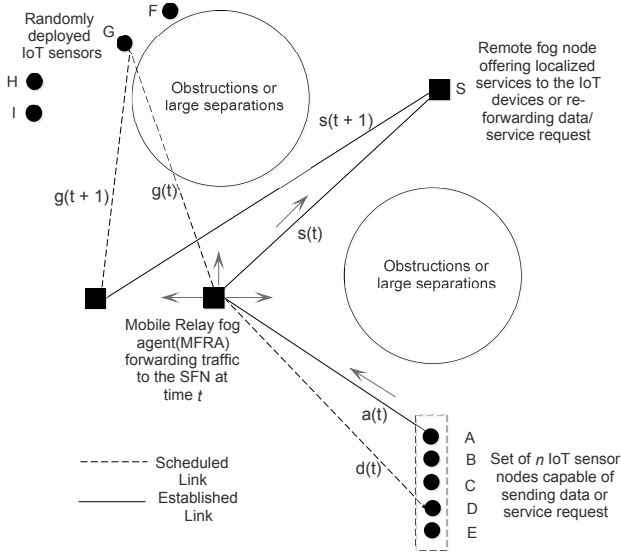
Fig. 1. Micro-view of the proposed fog-based IoT system.

power (dBm). A lightweight distributed learning approach was proposed in [7] to increase energy efficiency and reliability of IoT communications. There was significant performance improvement when the proposed algorithm was compared to a centralized optimized strategy. Transmit power, sub-channel, and spreading factor made up the action space. However, the system model in both works was rather hierarchical than distributed, ie. each WN was assumed to be an independent central entity with no specifications to what is learnt within each sub-network. Though IoT is defined as a large-scale network where various sub-networks coexist [1], applying RL to end-devices within sub-network may bring about meaningful performance improvement in the overall IoT network.

The main contribution of this paper is to propose a fully-decentralised multi-agent-based reinforcement learning approach as in [6] that addresses communication performance within a fog-based IoT architecture. First, we assumed a realistic scenario of possible state-action pair for each communication scenario as seen in Fig. 1, where each fog-agent is faced with a unique topology and environment. Next, in order to guarantee energy-efficient communications for the fog devices in the dynamic environment, we apply a decentralised q-learning algorithm, where each agent observes its position with respect to the communicating party, likewise the size of the number of packets to be transmitted and learns to take actions that improve the long-term communication performance. The fog-agents will actively try to communicate with neighbouring IoT devices in a manner where computing and processing are fully-decentralised, as well as achieving energy balance within the system.

The remainder of this work is organized as follows. In Section II, we reviewed related works, and present our proposed approach in Section III. In Section IV, we evaluate the proposed fog-based IoT system, and present the results in Section V. Section V concludes the paper and outlines future directions.

II. PROBLEM DEFINITION

In this section, we provide full description of the system model, as well the RL approach used to address the problem. The Mobile Fog Relay Agent (MFRA) and its environment are discussed below.

A. MFRA environment

States: The states are defined as a tuple, $\langle$Outage communication cost ($\mathcal{P}_{out}$) /Energy status of the fog relay (J) /Energy status of the IoT sensor (J)$\rangle$.

- Outage communication cost: Outage observations from the environment is estimated using (1) from [3], which gives an estimate of the communication outage when the agent takes an action, such as changing power levels or location, or both.
- Energy expended by fog relay: This observation gives the agent insight on how much energy by the fog agent when following policy $\omega_i \in \omega_{fog}$. If the fog agent continues to take sub-optimal actions, it depletes its energy and dies out.
- Energy expended by IoT sensor: This observation gives the agent insight on how much energy the IoT sensor has used up by following a policy $\omega_i \in \omega_{IoT}$. If the IoT sensor continues to take sub-optimal actions, it depletes its energy and dies out.

$$\mathcal{P}_{out} = 1 - (1 + 2\Psi^2 \ln \Psi) \exp\left(-\frac{N_0 \tilde{\kappa}}{P_I(D_I + \delta)^{-\sigma}}\right), \quad (1)$$

where $\Psi = \sqrt{(N_0\tilde{\kappa})/(P_R(D_S + \delta)^{-\sigma})}$, and $\mathcal{P}_{out}$ is an expression for the outage probability with values between 0 and 1. We assume a predefined threshold $\tilde{\kappa}$ which determines the outage in communication, P_I is transmit power of the IoT sensor, P_R is transmit power of the fog relay agent, D_I is the distance between IoT sensor and fog relay agent, and D_S is the distance between fog relay agent and destination node. We assume a small change in the position of the fog relay agent, $\delta = \pm 0.25m$, N_0 to be the channel noise, and σ to be the path-loss exponent.

B. MRFA agent

We apply a the Q-Learning algorithm, an RL approach which requires no prior knowledge of the environment by the agent. In Q-learning, the agent interacts with the environment over periods of time according to a policy ω. At every time-step $k \in N$, the environment produces an observation $s_k \in \mathbb{R}^{D_s}$. By sampling, the agent then picks an action a_k over $\omega(s_k)$, $a_k \in \mathbb{R}^{D_a}$, which is applied to the environment. The environment consequently produces a reward $r(s_k, a_k)$ and may end the episode at state s_N or transits to a new state s_{k+1}. The agent's goal is to minimize the expected cumulative cost, $\min_\omega \mathbb{E}_{s_0,a_0,s_1,a_1,...,s_N} \left[\sum_{i=0}^{N} \gamma^i \mathcal{C}(s_i) \right]$, where $0 \leq \gamma \leq 1$ is the discount factor, and $\mathcal{C}$ is the overall cost function of our model.

First, the agent takes an initial random action a_k and gets observations from the environment which corresponds

to that action, as well as a reward. It then discretizes the continuous observations emanating from the environment into a $50 \times 10 \times 10$ state space corresponding to the tuple, $\langle$Outage communication cost $(\mathcal{P}_{out})$ /Energy status of the fog relay (J) /Energy status of the IoT sensor (J)$\rangle$. The agent then updates it's Q-values at each time-step k following (3).

$$
\begin{aligned}
Q(s_k, a_k) := &\, Q(s_k, a_k) \\
&+ \alpha \Big[r_{k+1} + \gamma \max_a Q(s_{k+1}, a) - Q(s_k, a_k) \Big],
\end{aligned} \quad (2)
$$

where α is the learning rate, which determines the impact of new experience on the Q-value, r_{k+1} is the reward the agent receives by being in s_{k+1} from s_k. Based on the policy followed by the agent, it gets observations and rewards from the environment.

Action space: The actions are move and transmit by fog relay, and select a power-level and transmit by IoT end-device, which make up eight possible actions.

- Mobility: Move by $\pm\delta$ and transmit, where $\delta = \pm 0.25m$ and mobility range (m) = [-30, 30]
- Power-level: Choose power-level and transmit, P, where transmit power ranges (W) = [0.001, 0.01, 0.15, 0.2, 0.25, 0.3]

Goal: The goal is for the agent to learn to minimize the overall cost $\mathcal{C}$ in the tuple, $\langle$Outage communication cost $(\mathcal{P}_{out})$ /Energy status of the fog relay (J) /Energy status of the IoT sensor (J)$\rangle$, by keeping all nodes within the link alive while ensuring that the packets received in each transmission does not fall below the pre-defined threshold, which was set at 95%.

Rewards: The reward function used is given in (3) as

$$
R = \begin{cases} 100, & \text{if } goal == Reached \\ 0, & \text{otherwise.} \end{cases} \quad (3)
$$

Metrics: Outage probability, i.e. the ratio of the number of packet lost to those transmitted, which we measure in percentages, and energy status of the fog relay agent and the IoT sensor, i.e. the ratio of depleted energy to the initial capacity in Joules, which we measure as a percentage.

The MFRA's learning process is summarized in Algorithm 1. A new learning episode is terminated when the agent attains the pre-defined goal of minimizing the communication outage in the link, or when either the fog relay or the IoT sensor dies out due to taking sub-optimal actions without getting to the goal. When an fog relay moves closer to the communicating parties, the IoT sensor uses a lower power level as compared to when it is far away, hereby saving IoT sensor energy. However, mobility have some cost and if the fog relay continues to move in order to minimize the communication outage, it may die out soon, hereby causing a point-of-failure to the network. As each episode is completed, a reward of 100 points is given to the agent if it reaches its goal and a 0 points otherwise. The reward is updated in the Q-learning table, with environmental information updated as well.

Algorithm 1 MFRA Learning Process

1: **Initialize:** Power levels (W) = [0.001, 0.01, 0.15, 0.2, 0.25, 0.3], $\delta = \pm 0.25m$ and mobility range (m) = [-30, 30]
2: *top*:
3:    ResetEnvironment()
4:    *state* $\leftarrow$ MapLocalObservationToState(*env*)
5:    *action* $\leftarrow$ QLearning.SelectAction(*state*)
6:    **if** *action* == "move close and Tx" **then**
7:      Env.EstimateOutage (1)
8:      Env.EstimateFogEnergyStatus
9:    **else if** *action* == "move away and Tx" **then**
10:      Env.EstimateOutage (1)
11:      Env.EstimateSensorEnergyStatus
12:    **else if** *action* == "choose power level and Tx" **then**
13:      Env.EstimateOutage (1)
14:      Env.EstimateSensorEnergyStatus
15:    **endif**
16:    InvokePolicy(ExponentialDecay)
17:    UpdateQLearningProcedure() (3)
18:    CurrentState $\leftarrow$ NewState
19:    **if** *goal* == "Reached" **then return** Reward = 100,
20:    **else if** *goal* != "Reached" or Agent == Death **then return** Reward = 0
21:    **endif**
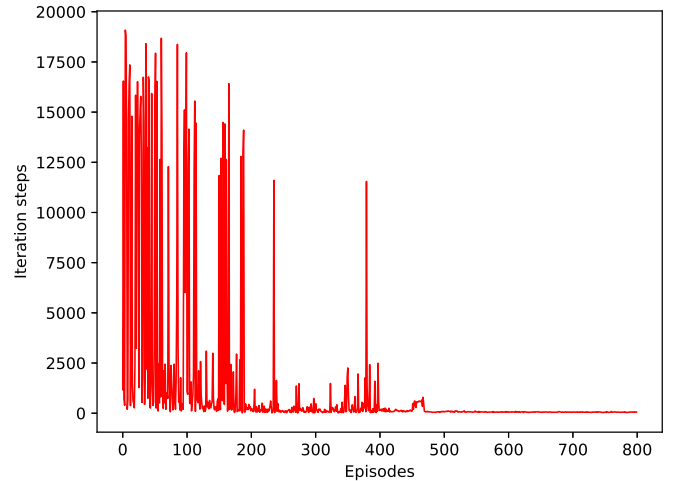22: EndEpisode **goto** *top*.

Fig. 2. Number of iteration over episodes prior to the point of failure.

III. EXPERIMENTAL SETUP

A. General settings

We carried out experimentation with 150 fog relays and 1000 IoT sensors randomly deployed.

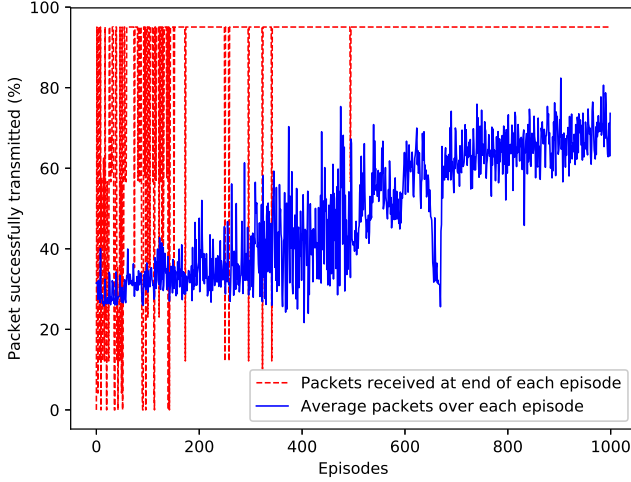Table I shows a summary of the parameters used in simulating

Fig. 3. Percentage of packets successfully transmitted via the fog agent.
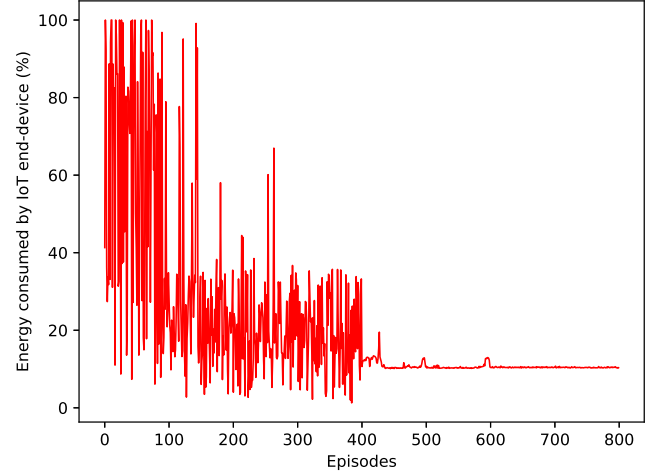

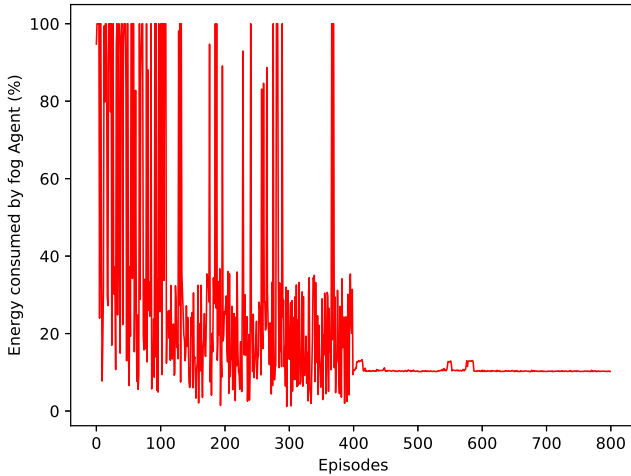Fig. 5. Average energy consumed by IoT sensor over episodes.

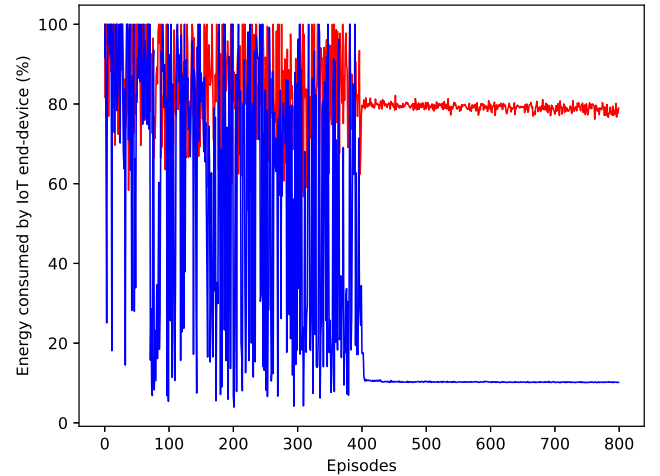Fig. 4. Average energy consumed by fog agent over episodes.

Fig. 6. Energy consumption for fixed mobility vs. learning over episodes.

B. Baselines

C. MFRA scenarios

D. Indicators

IV. RELATED WORKS

Considering the dynamics and heterogeneity within the ultra-distributed IoT environment, it is important for communication devices which are mobile to move seamlessly without degrading quality of service (QoS). More so, the constrained devices should communicate efficiently without depleting all their energy by transmitting at high-power, which may have long-term consequences to the network. Several non-RL-based approaches have been proposed to optimize the communication performance in IoT based networks, however, when some parameters in the network are changed, these approaches may fail. The work in [3] considered a multi-tier fog-based IoT architecture where a mobile/static fog node

acts as an amplify and forward relay that transmits received information from a IoT sensor node to a higher hierarchically-placed fog device, which offers some localized services. In order to minimize the outage in communication, an iterative algorithm based on the steepest descent method (SDM) was proposed to jointly optimize the mobility pattern and power-control parameters. However, the single-agent based work did not take into consideration the performance of the approach in a highly-decentralised IoT environment. Furthermore, each time the topology is changed, the agent will need to recompute the model in other to act optimally.

RL can be applied to a new environment, since the it allows the agent to learn, by take actions that will improve its long-term return. Furthermore, RL can be applied to multiple agents. In [4], a decentralised, multi-agent-based stateless Q-learning approach was proposed, where no information about neighbouring nodes are available to agents. Though the paper

Parameter	Values
D_I	40 metres
P_I	[0.001, 0.3] Watts
D_S	35 metres
P_R	0.3 Watts
δ	± 0.25 metres
Mobility bound	[-35, 35] metres
Noise power N_0	2×10^{-7} Watts
Path-loss exponent σ	3
Pre-defined threshold κ	1
Discount factor γ	0.9
Learning rate α	0.1
Episodes N	1000
Iteration runs	100000
Policy ϵ	$e^{-0.0015N}$

was able to improve aggregate throughput in the network, by allowing the networks modify both the transmission power and the channel used, however, high variability was observed in the throughput of the individual networks. Only eight agents were observed using a stateless Q-learning approach. The work did not adequately depict the distributed nature of the IoT network. Moreover, mobility of agents was completely ignored.

Several works in WSN have applied the decentralised RL technique. In [10], a multi-agent reinforcement learning-based multi-hop mesh cooperative (MRL-CC) mechanism for the improvement of some QoS metrics in the WSNs. Though the mobility of the cooperative nodes were taken into account when learning the optimal policy, the MRL-CC failed to consider the power dissipated by the power-constrained devices, as well as the overall outage in communication.

In [9], a reliable and energy-efficient routing (REER) protocol was proposed using a geographic routing approach. The work considered the idea of a central entity, called a reference node (RN), which is assumed to be situated at an ideal location between source and destination. Several other cooperative nodes, which contend to relay data, are assumed to be situated around the RN. The work was able to examine the trade-off between reliability and energy-efficiency when the distances between RNs was adjusted.

We present a fully-decentralised multi-agent reinforcement learning approach that adequately addresses some performance issues in IoT. The main task of our work is to minimize global outage in communication within a fog-based IoT network, by optimizing the power-control parameter of the potential mobile fog-relay agent (MFRA), as well as optimizing the position of each relaying agents in the network. As such, each MFRA is compelled to take certain actions that may influence its environment. However, the duration it takes the MFRA to learn is significantly influenced by the state space, as well as the possible set of actions [8]. The variables for the state, action and reward of an agent may be discrete or continuous, with the former represented as small interval of values which imply distinct levels [11], and can easily be represented in a tabular form. However, it is difficult to represent continuous

space using Q-learning tables. The work in [12] considered a RL agent that explores continuous state and action space using Gaussian unit search behaviour. Other works [8], [14] considered the reduction of states by eliminating states that are unlikely to occur. However, this may pose a big risk especially in a highly dynamic environment. RL can be effective for learning action policies in discrete stochastic environments, but its efficiency can decay exponentially with increasing state space [13]. Our proposed problem can be observed to have continuous state-action pairs, and is approached by discretizing the state and action space.

It is noteworthy that agents in a multi-agent system (MAS) may take actions that can have direct consequences on neighbouring agents, which may have further impact on other agents within the network. For instance, if a relaying agent decides to increase the transmit power beyond some threshold value, in order to boost its communication capabilities, its action may result in channel interference to its immediate neighbours, and worst, it may deplete its energy fast, and die-out, leading to link failure that can affect the performance of the entire network. Issues like this may arise in a typical multi-agent IoT network, as such, we present a fully-decentralised MAS where each agents learn to follow a local policy that improves the global objective. To the best of our knowledge, this is the first approach that employs a decentralised RL technique to minimize global outage in communication within a fog-based IoT network, taking into consideration the wireless channel conditions, by jointly optimizing the location of each relaying agent and the power-control parameter.

In this paper, we assume the following.

1) The MFRA is completely oblivious of its environment, and as such, has no prior knowledge of the overall cost function.
2) The MRFA may change its position in order to ensure better communication. Also, the MFRA may change its position (2D/3D) depending on the scenario considered.
3) The MFRA has an objective of learning to make actions that yield better outcomes within its local view of the environment.
4) The states are be divided into discrete levels to overcome the exponential decay in the efficiency of the proposed approach due infinite state space.
5) Each MFRA independently tries to optimize power usage and moves in a direction that maximizes the communication outage.

In Fig. 1, we present a situation where different IoT end-devices, inclusive of smart things embedded with sensors (smart-meters, smart-watches, traffic lights, washing machine, dish-washers, herds, or even a sick patient being monitored, etc.), which can send data/service request to a remote fog service provider via a MFRA. However, there may be change in the topology of the network or even in the environment, making it difficult for data collection or service provision. Furthermore, the environment may be too hazardous for human control and as such, some level of intelligence is expected from the MFRA. The MFRA learns to optimize its position and adjust it's power-level to minimize the communication
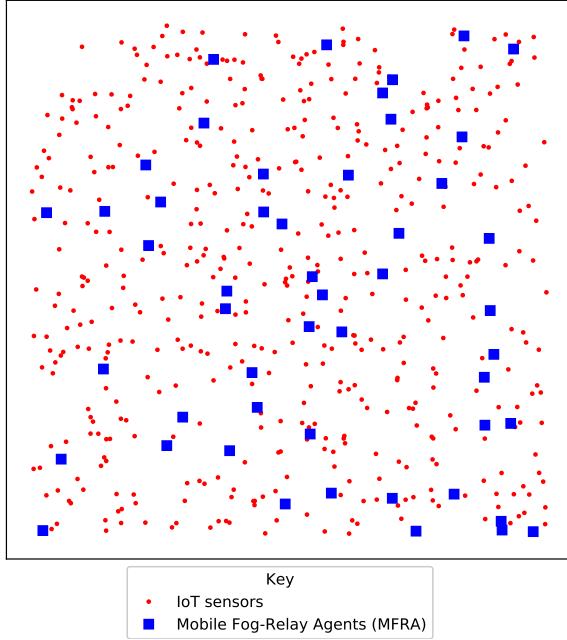
Fig. 7. A pictorial view of how MFRA and IoT sensors are randomly deployed.

outage.

However, the dynamic IoT environment increases the complexity in system design, as such, many classical approaches will fail when domain knowledge of the environment mismatches the actual environment [11].

In Fig. 1, we present micro-view of randomly deployed IoT sensors trying to send/receive data/service request to a remote fog device via a potential mobile fog-relay agent (MFRA). Intuitively, if a fog device does not have the required computational/processing capability, it may decide to forward the data/service request to other available fog devices.

V. IDEA 2

A. Problem Formulation

Devices/agents in an IoT network are resource-constrained, as such, it will be proper to deploy lightweight RL-based techniques that will improve the performance of the network. Furthermore, we may have to experiment on a very dynamic environment considering factors that depict a realistic IoT scenario to meet strict quality-of-service requirements.

In this work, a finite-horizon MDP is considered with continuous state and action spaces defined by the tuple $\langle \mathcal{S}, \mathcal{A}, p, p_0, \mathcal{P}_{out}, \gamma \rangle$, where $\mathcal{S}$ is the set of states, $\mathcal{A}$ is the set of actions, $p : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{G}^+$ is the conditional probability density over successor states given the current state and action, $p_0 : \mathcal{S} \to \mathbb{G}^+$ is the probability density over initial states, $\mathcal{P}_{out}$ is a function that maps state to cost, and the discount factor is $\gamma \in (0, 1]$. In the RL techniques, the agent has a choice to take certain actions in each time step, causing the environment to respond with new conditions, and

consequently, the agent receives reward for that action as a form of feedback. The reward could be positive, negative or even zero, and the main objective of the agent is to maximize the positive reward or minimize the negative reward (often the cost) over the entire time step N.

Our objective is to learn a stochastic policy $\pi^* : \mathcal{S} \times \mathcal{A} \to \mathbb{G}^+$, which is a conditional probability density over the present state, in such a way as to minimize the expected cumulative cost.

$$\pi^* = \arg\min_{\pi} \mathbb{E}_{s_0, a_0, s_1, a_1, \dots, s_N} \Big[\sum_{i=0}^{N} \gamma^i \mathcal{P}_{out}(s_i) \Big], \quad (4)$$

We take the expectation over the joint distribution of all state-action pairs, with the density give as,

$$q(s_0, a_0, s_1, a_1, \dots, s_N) = p_0(s_0) \prod_{i=0}^{N-1} \pi(a_i|s_i) p(s_{i+1}|s_i, a_i). \quad (5)$$

Fig. ?? shows a dynamic IoT environment with an IoT sensor attempting to request some services from a remote target fog agent/node through randomly deployed fog devices. The devices act as relays to forward traffic from the source to the destination. However, based on their position, line-of-sight(LoS) obstruction, which affect the conditions of the wireless channel, some degree of communication outage may occur. Our aim is to ensure that the agents are able to learn the optimal route to take through their experience with the environment.

VI. IDEAS2

VII. IDEAS3

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