Task 1: Calculating the Mean First Passage Matrix for a Regular Markov Chain

Prepare my personal Markov chain probability transition matrix P:

```
[[0.00000 0.00000 0.00000 0.21064 0.00000 0.18934 0.00000 0.00
 [0.00000 0.23340 0.44311 0.00000 0.00000 0.00000 0.00000 0.00
 [0.50398 0.00000 0.00000 0.00000 0.00000 0.43275 0.00000 0.00
 [0.00000 0.00000 0.49275 0.00000 0.50725 0.00000 0.00000 0.00
 [0.30974 0.00000 0.00000 0.00000 0.00000 0.69026 0.00000 0.00
 [0.02998 0.00000 0.00000 0.00000 0.00000 0.39229 0.00000 0.57
 [0.00000 0.00000 0.00000 0.00000 0.43948 0.00000 0.00000 0.00
 [0.47684 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00
 [0.48492 0.00000 0.00000 0.00000 0.29683 0.00000 0.00000 0.00
 [0.00000 0.08205 0.36807 0.00000 0.34591 0.00000 0.00000 0.00
 [0.00000 0.00000 0.12319 0.36212 0.12949 0.00000 0.38520 0.00
```

Solution:

1. It is obvious that all entries of p^5 are positive. So, the chain is regular:

```
[[0.20637 0.01028 0.07725 0.03308 0.10660 0.19397 0.01009 0.13
 [0.22075 0.00919 0.05300 0.04798 0.07790 0.24392 0.01465 0.10
 [0.13866 0.00916 0.06864 0.05973 0.11437 0.20185 0.00573 0.14
 [0.26026 0.00497 0.02169 0.05889 0.06842 0.22417 0.01305 0.07
 [0.15700 0.01121 0.07777 0.05873 0.12763 0.18605 0.00368 0.11
 [0.18608 0.01370 0.08514 0.04938 0.11358 0.23814 0.01283 0.09
 [0.16447 0.00329 0.04372 0.05193 0.09626 0.21108 0.00592 0.16
 [0.16419 0.00763 0.05123 0.05289 0.07343 0.25040 0.01235 0.16
 [0.17783 0.00628 0.04738 0.05524 0.08889 0.20965 0.01131 0.15
 [0.21696 0.00544 0.03747 0.05441 0.08256 0.22218 0.01103 0.11
 [0.23071 0.00657 0.05797 0.03904 0.08751 0.20971 0.02189 0.13
```

2. Rows of P^t for t=40 are equal to an accuracy of 3-4 significance digits:

```
[[0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
 [0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.1
```

So, vector w:

```
[0.18793 0.00935 0.06424 0.04970 0.09950 0.21732 0.01075 0.12
```

3. Calculate the mean first passage matrix E .

The Fundamental matrix Z:

```
[[0.78786 0.00634 0.06166 0.11870 0.02389 -0.04108 -0.00815 -
 [-0.06473 1.28406 0.43689 -0.07346 -0.13310 -0.21784 -0.0215
 [0.18647 -0.01700 0.91224 -0.00366 -0.14056 0.19045 -0.00356
 [-0.06980 -0.02819 0.31210 0.91497 0.23769 0.11288 -0.03270 -
 [0.05176 -0.02063 -0.14424 -0.06611 0.80128 0.46594 -0.03981
 [-0.00628 -0.01919 -0.14357 -0.07704 -0.15446 1.00829 -0.038
 [-0.34188 -0.04089 -0.06092 0.09446 0.37441 -0.08223 1.21919
 [0.27781 -0.00432 -0.04303 -0.00117 0.00852 -0.29201 -0.0213
 [0.17216 0.00384 -0.01566 -0.01545 0.18470 -0.10532 -0.01290
 [-0.17100 0.07443 0.28176 -0.01691 0.15932 -0.02927 0.06244 -
 [-0.31523 -0.04008 0.11902 0.30901 0.21723 -0.12432 0.44144 -
```

The Matrix Z_{dg}:

```
[[0.78786 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.0
 [0.00000 1.28406 0.00000 0.00000 0.00000 0.00000 0.00000 0.0
```

```
[0.00000 0.00000 0.91224 0.00000 0.00000 0.00000 0.00000 0.0(
[0.00000 0.00000 0.00000 0.91497 0.00000 0.00000 0.00000 0.0(
[0.00000 0.00000 0.00000 0.00000 0.80128 0.00000 0.00000 0.0(
[0.00000 0.00000 0.00000 0.00000 0.00000 1.00829 0.00000 0.0(
[0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 1.21919 0.0(
[0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.7(
[0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.0(
[0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.0(
[0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.0(
```

The $I-Z+JZ_{dg}$:

```
[[1.00000 1.27772 0.85058 0.79627 0.77739 1.04937 1.22734 0.85
 [0.85258 1.00000 0.47535 0.98843 0.93439 1.22613 1.24072 0.95
 [0.60139 1.30106 1.00000 0.91863 0.94184 0.81784 1.22275 0.72
 [0.85765 1.31226 0.60014 1.00000 0.56359 0.89541 1.25189 0.76
 [0.73610 1.30469 1.05648 0.98107 1.00000 0.54235 1.25900 0.56
 [0.79413 1.30325 1.05581 0.99201 0.95574 1.00000 1.25762 0.24
 [1.12973 1.32495 0.97316 0.82051 0.42687 1.09052 1.00000 0.87
 [0.51004 1.28838 0.95527 0.91614 0.79277 1.30030 1.24058 1.00
 [0.61570 1.28022 0.92790 0.93042 0.61658 1.11361 1.23209 0.89
 [0.95886 1.20963 0.63048 0.93188 0.64196 1.03756 1.15675 0.84
 [1.10308 1.32414 0.79322 0.60596 0.58405 1.13261 0.77775 0.90
```

The Diagonal matrix D:

```
[[5.32100 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00
 [0.00000 106.91190 0.00000 0.00000 0.00000 0.00000 0.00000 0.
 [0.00000 0.00000 15.56705 0.00000 0.00000 0.00000 0.00000 0.0
 [0.00000 0.00000 0.00000 20.12221 0.00000 0.00000 0.00000 0.0
 [0.00000 0.00000 0.00000 0.00000 10.05008 0.00000 0.00000 0.0
 [0.00000 0.00000 0.00000 0.00000 0.00000 4.60159 0.00000 0.00
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 92.99140 0.0
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 7.90
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00
```

The Mean first passage matrix E:

```
[[5.32100 136.60355 13.24099 16.02262 7.81287 4.82879 114.132
 [4.53659 106.91190 7.39974 19.88938 9.39066 5.64214 115.3761
 [3.20001 139.09856 15.56705 18.48482 9.46557 3.76336 113.705
 [4.56358 140.29576 9.34235 20.12221 5.66417 4.12032 116.4150
 [3.91676 139.48732 16.44626 19.74139 10.05008 2.49566 117.07
 [4.22559 139.33261 16.43582 19.96137 9.60525 4.60159 116.947
 [6.01132 141.65260 15.14917 16.51055 4.29010 5.01811 92.9914
 [2.71394 137.74332 14.87069 18.43484 7.96736 5.98343 115.363
 [3.27612 136.87071 14.44464 18.72202 6.19671 5.12438 114.573
 [5.10209 129.32352 9.81470 18.75148 6.45173 4.77443 107.5679
 [5.86951 141.56625 12.34811 12.19331 5.86974 5.21180 72.3236
```