Task 2: Analysis of an ergodic Markov chain. Calculation of the steady-state vector of Markov chain by Cesàro summation

Prepare my personal Markov chain probability transition matrix P:

```
[[0.00000 0.00000 0.12819 0.36008 0.00000 0.00000 0.00000 0.0(
 [0.34321 0.00000 0.00000 0.00000 0.00000 0.65679 0.00000 0.0(
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.35969 0.4(
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.53366 0.4(
 [0.35442 0.00000 0.00000 0.00000 0.00000 0.64558 0.00000 0.0(
 [0.00000 0.00000 0.55971 0.05475 0.00000 0.00000 0.00000 0.0(
 [0.00000 0.29669 0.00000 0.00000 0.49574 0.00000 0.00000 0.0(
 [0.00000 0.60745 0.00000 0.00000 0.36487 0.00000 0.00000 0.0(
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.31611 0.2(
 [0.39972 0.00000 0.00000 0.00000 0.00000 0.60028 0.00000 0.0(
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00130 0.5(
 [0.00000 0.12827 0.00000 0.00000 0.52640 0.00000 0.00000 0.0(
```

Solution

Find cyclic classes in which the chain is divided:

For finding the chain period d and cyclic classes we begin moving from any state of the chain.

Let the state 3 is starting. From the state 7, 8, 12 can be reached; from the states 7, 8, 12, states [2, 5, 10] are reachable. This moving can be listed in the following:

- step.1 -> 3
- step.2 -> 7, 8, 12
- step.3 -> 2, 5, 10
- step.4 -> 1, 6
- step.5 -> 3, 4, 9, 11
- step.6 -> 7, 8, 12
- step.7 -> 2, 5, 10

- step.9 -> 1, 6
- step.10 -> .....

It is obvious that d=4 and cyclic classes are d=4 and cyclic classes are $C_0 = \{7, 8, 12\}$, $C_1 = \{2, 5, 10\}$, $C_2 = \{1, 6\}$, $C_3 = \{3, 4, 9, 11\}$

Transitions between classes: C_0 -> C_1 -> C_2 -> C_3 -> C_0 -> C_1 -> ...

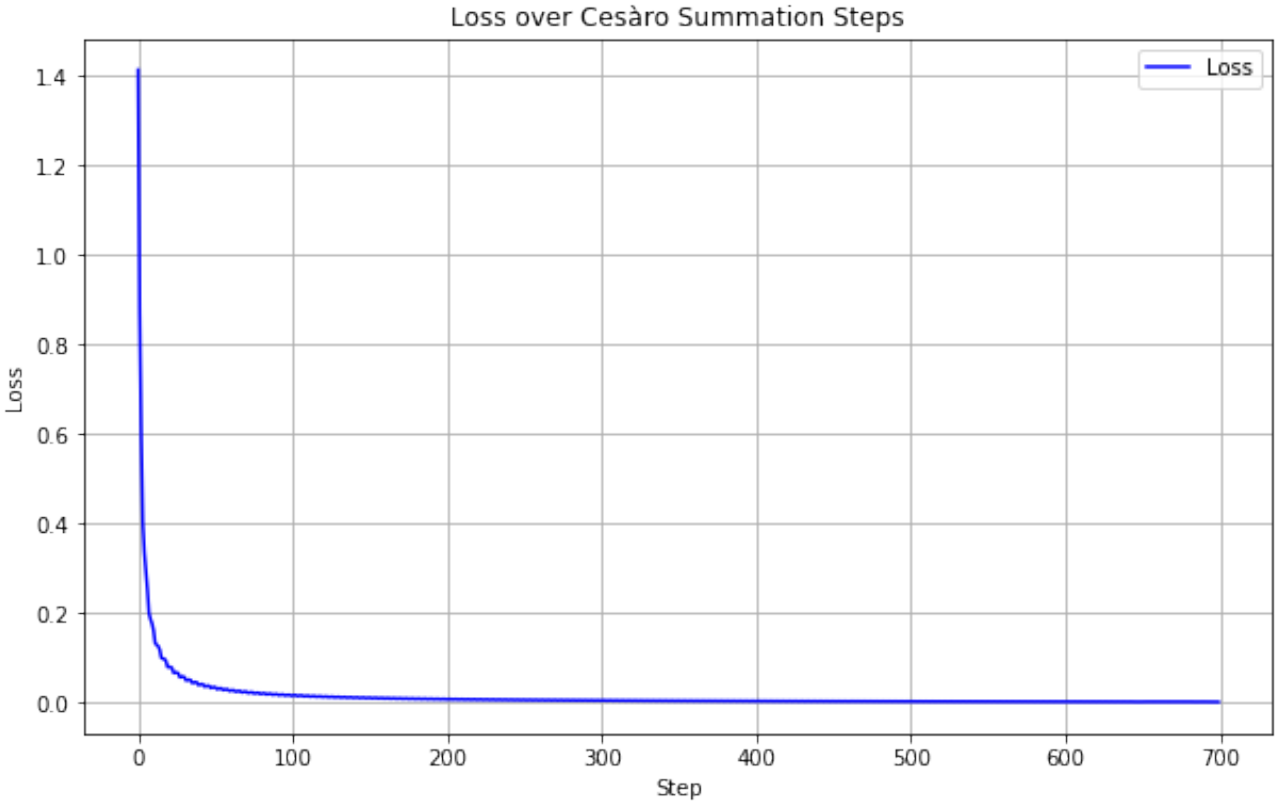
The limiting matrix A can be found as the limit $A(t) = \frac{1}{t}\sum_{i=0}^{t-1} P^i$ when $t \to \infty$ (summation by Cesàro). The number t should be enough large.

The limiting matrix A for t = 700:

```
[[0.09038 0.09531 0.10093 0.04128 0.11219 0.15962 0.07316 0.10
 [0.08944 0.09619 0.10133 0.04100 0.11155 0.16056 0.07317 0.10
 [0.08946 0.09534 0.10217 0.04077 0.11218 0.16054 0.07327 0.10
 [0.08946 0.09539 0.10075 0.04220 0.11218 0.16054 0.07351 0.10
 [0.08946 0.09476 0.10133 0.04100 0.11298 0.16054 0.07317 0.10
 [0.08895 0.09530 0.10154 0.04085 0.11220 0.16105 0.07317 0.10
 [0.08947 0.09518 0.10132 0.04100 0.11226 0.16053 0.07418 0.10
 [0.08945 0.09563 0.10133 0.04100 0.11207 0.16055 0.07275 0.10
 [0.08947 0.09516 0.10074 0.04077 0.11224 0.16053 0.07320 0.10
 [0.08952 0.09476 0.10130 0.04102 0.11155 0.16048 0.07317 0.10
 [0.08946 0.09530 0.10074 0.04077 0.11218 0.16054 0.07275 0.10
 [0.08948 0.09494 0.10132 0.04101 0.11231 0.16052 0.07275 0.10
```

The rows of the matrix A are approximately equal, but not identical.

The L2 error between each row is given by 0.002298541379180601.


Loss over Cesàro Summation Steps
Loss
Step
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500
600
700