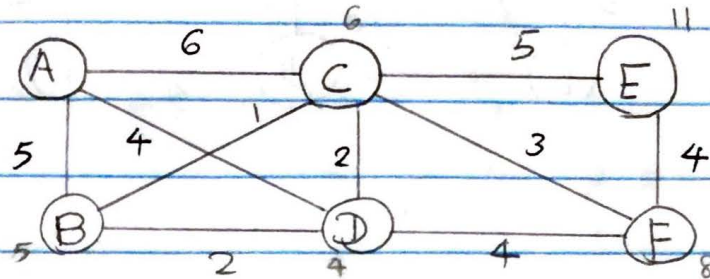


A. Graphs

1.



Vertex	Shortest distance from A	Previous Vertex
A	0	
B	5	A
C	6	A
D	4	A
E	11	C
F	8	D

visited vertex = [A, D, C, D, E, F]

$$A \rightarrow B = 5$$

$$A \rightarrow C = 6$$

$$A \rightarrow D = 4$$

Dijkstra's algorithm

$$A \rightarrow C \rightarrow E = 11$$

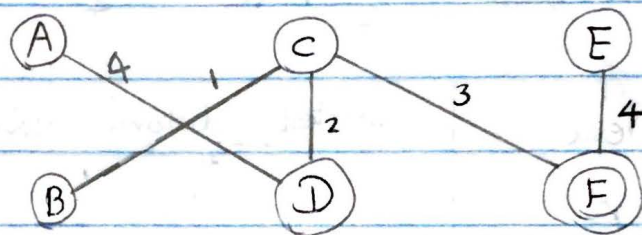
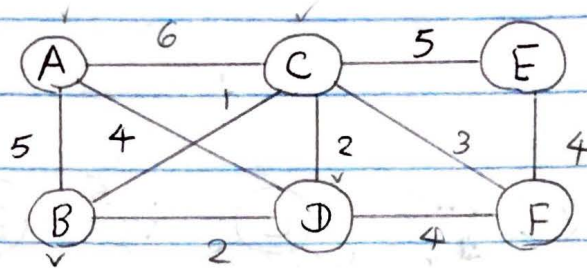
$$A \rightarrow D \rightarrow F = 8$$

We always choose the shortest path to reach the vertex.

For example: $A \rightarrow C \rightarrow F = 9 > A \rightarrow D \rightarrow F = 8$

So, we choose $A \rightarrow D \rightarrow F = 8$

2.



~~A → B = 5~~

B → C = 1

C → D = 2

C → F = 3

~~A → D = 4~~

F → E = 4

D → A = 4

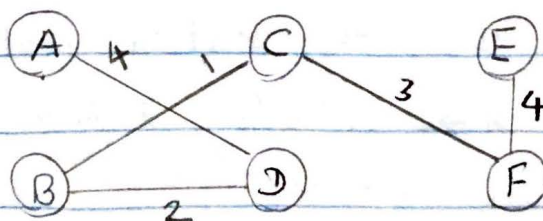
Prims doesn't specify the starting node. It connects all the node with possible minimum cost. Then, it gives the minimum ~~spanning~~ spanning tree.

Minimum spanning tree = ~~5 + 1 + 2 + 3 + 5~~ = 16

= ~~4~~ + 1 + 2 + 3 + 4 = 14

3.

The minimum spanning tree is not unique. We can build another different tree with same cost.



A → D = 4

D → B = 2

B → C = 1

C → F = 3

F → E = 4

MST = 4 + 2 + 1 + 3 + 4 = 14

A. 3.(a)		Dijkstra's	Prim's	Another
		SPT	MST	MST
	A → B	5	7	6
	A → C	6	6	7
	A → D	4	4	4
	A → E	8	13	14
	A → F	11	9	10
	Total	34	39	41

$$d_{\text{SPT}}^{\text{avg}} = 34/5 = 6.8$$

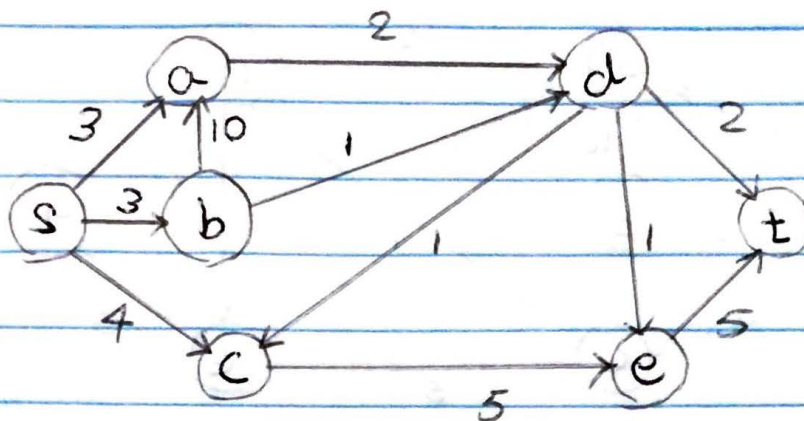
$$(b) \quad d_{\text{MST1}}^{\text{avg}} = 39/5 = 7.8$$

$$d_{\text{MST2}}^{\text{avg}} = 41/5 = 8.2$$

$d_{\text{SPT}}^{\text{avg}} < d_{\text{MST}}^{\text{avg}}$ because SPT is defined to find the shortest path from the root to other nodes.

For graph G , the average distance using SPT is smaller than using any possible MSTs.

B. 1.



s a b c d e t
 0 ∞ ∞ ∞ ∞ ∞ ∞
 3 3 4
 4 5 5

Node	Next hop	Distance to t
s	b	6
a	d	4
b	d	3
c	e	10
d	t	2
e	t	5

$$s \rightarrow b \rightarrow d \rightarrow t = 6 < s \rightarrow a \rightarrow d \rightarrow t = 7$$

2.

$\therefore s \rightarrow b \rightarrow d \rightarrow t = 6$ is chosen.

$$a \rightarrow d \rightarrow t = 4$$

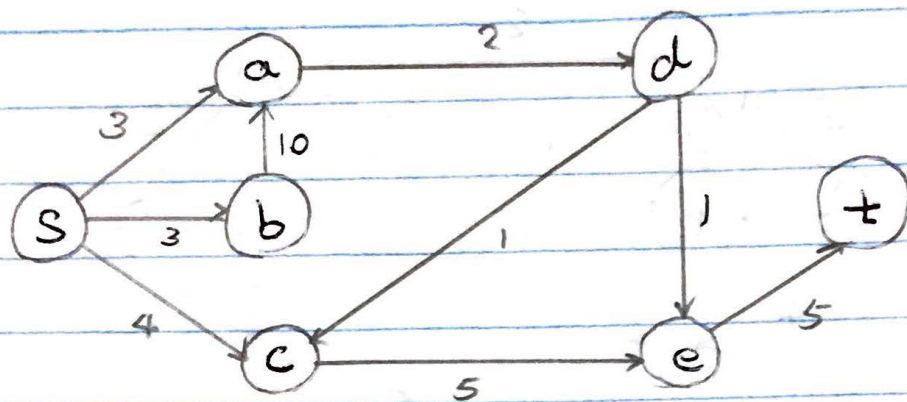
$$b \rightarrow d \rightarrow t = 3$$

$$c \rightarrow e \rightarrow t = 10$$

$$d \rightarrow t = 2$$

$$e \rightarrow t = 5$$

2.



Node	Next hop	distance t
s	a	11
a	d	8
b	a	18
c	e	10
d	e	6
e	t	5

$$s \rightarrow a \rightarrow d \rightarrow e \rightarrow t = 11$$

$$a \rightarrow d \rightarrow e \rightarrow t = 8$$

$$b \rightarrow a \rightarrow d \rightarrow e \rightarrow t = 18$$

$$c \rightarrow e \rightarrow t = 10$$

$$d \rightarrow e \rightarrow t = 6$$

$$e \rightarrow t = 5$$