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Class: CSC 421
Assignment 3
Fall 2022

1)
Convert to clausal form
(p | q | -r) & ((-r | q | p) -> ((r | q) & -q & -p))

Step 1: Remove implication
(p | q | -r) & ((-r | q | p) -> ((r | q) & -q & -p))

Step 2: (a & (b->c)) => a & (-b V c)
(p V q V -r) & ((-(-r V q V p))V((r V q) & -q & -p))

Step 3: Solve for the 2nd part
(p V q V -r) &
((-(-r V q V p))V((r V q) & -q & -p))

Step 4: Remove negation
(p V q V -r) &
(r & -q & -p)V((r V q) & -q & -p))

Step 5: Distributive law
(p V q V -r) &
((r & -q & -p) V (r V q))&
((r & -q & -p) V -q)&
((r & -q & -p) V -p)

Step 6: Distributive Law
(p V q V -r)&
(r V q V r)&
(r V q V -q)&
(r V q V -p)&
(-q V r)&
(-q V -q)&
(-q V -p)&
(-p V r)&
(-p V -q)&
(-p V -p)&

Resolution
Step 7: Number the clause

1(p V q V -r)&
2(r V q V r)&
3(r V q V -q)&
4(r V q V -p)&
5(-q V r)&
6(-q V -q)&
7(-q V -p)&
8(-p V r)&
9(-p V -q)&
10(-p V -p)&
Step 8
1)(p | q | -r) & ((-r | q | p) -> ((r | q) & -q & -p))
2)(p V q V -r) (1)
3)((-r | q | p) -> ((r | q) & -q & -p))(1)
4)((-(-r V q V p))V((r V q) & -q & -p)) (3)
5)(r V q) & -q & -p) (2),(3)
6)(r V q)(5)
7)(-q V -q)(5)
8)(-p V -p)(5)
9)-r (2),(7),(8)
10)r (6),(7)
11){} (9),(10)
Step 9:
From (11), the following propositional sentence is unsatisfiable.

2)
a)
∀h∃d outrun(h,d).
∃g∀r outrun(g,r).
¬∀h∀r outrun(h,r).

all x all y (Horse(x) & Dog(y) -> Faster(x, y)).
exists y (Greyhound(y) & all z (Rabbit(z)->Faster(y, z))).
all y (Greyhound(y)->Dog(y)).
all x all y all z (Faster(x, y) & Faster(y, z) -> Faster(x, z)).

Conclusion
all x all y (Horse(x) & Rabbit(y) -> Faster(x, y)).

b)
∀x hummingbird(x) => richlyColored(x).
¬∃a Bird(x)∧ large(x)=> liveOnHoney(x).
∀x bird(x)∧ ¬liveOnHoney(x) => dullColored(x).
¬∀x hummingbird(x) => small(x).

all x (Hummingbird(x) -> Bird(x)).
-exists x (Bird(x) & Large(x)) -> Liveonhoney(x)
all x Bird(x) & not Liveonhoney(x) -> Coloured(x,dull)

Conclusion
all x Hummingbird(x) -> Small(x).

c)
worthListeningto(talkabout(myGardener,militarySubject)).
¬∃x remember(x,WaterlooBattle) ∨ old(x).
¬∃x worthListeningto(talkabout(x,militarySubject))∨
remember(x,WaterlooBattle).
¬old(myGardener).

Worthlisteningto(talkabout(myGardener,militarySubject))
-exists x Remember(x,WaterlooBattle)| Old(x)
-exists x Worthlisteningto(Talkabout(x,militarySubject))|
Remember(x,WaterlooBattle)

Conclusion
Old(myGardener).

3)

3.1) [13]
3.1.1) P(p13|b12, b21)
P(p13|b12, b21)
= α Σp22 Σ31 P(b12|p13, p22) · P(b21|p31, p22) · P(p13) · P(p22) · P(p31)
= α[P(b12|p13, p22) · P(b21|p31, p22) · P(p13) · P(p22) · P(p31) +
P(b12|p13, ¬p22) · P(b21|p31, ¬p22) · P(p13) · P(¬p22) · P(p31) +
P(b12|p13, p22) · P(b21|¬p31, p22) · P(p13) · P(p22) · P(¬p31) +
P(b12|p13, ¬p22) · P(b21|¬p31, ¬p22) · P(p13) · P(¬p22) · P(¬p31)]
= α × (1 × 1 × 0.01 × 0.01 × 0.01 + 1 × 1 × 0.01 × 0.99 × 0.01 + 1 × 1 ×
0.01 × 0.01 × 0.99 + 0)
= α × (0.000199)

3.1.2) P(¬p13|b12, b21)

= α Σp22 Σp31 P(b12|¬p13, p22) · P(b21|p31, p22) · P(¬p13) · P(p22) · P(p31)
= α[P(b12|¬p13, p22) · P(b21|p31, p22) · P(¬p13) · P(p22) · P(p31) +
P(b12|¬p13, ¬p22) · P(b21|p31, ¬p22) · P(¬p13) · P(¬p22) · P(p31) +
P(b12|¬p13, p22) · P(b21|¬p31, p22) · P(¬p13) · P(p22) · P(¬p31) +
P(b12|¬p13, ¬p22) · P(b21|¬p31, ¬p22) · P(¬p13) · P(¬p22) · P(¬p31)]
= α×(1×1×0.99×0.01×0.01 + 0 + 1×1×0.99×0.01×0.99 + 0)
= α × (0.0099)

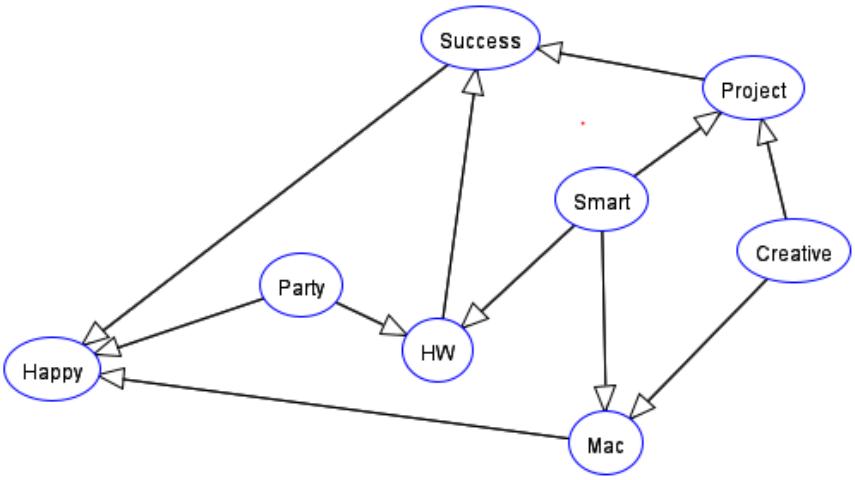
3.1.3) α
P(p13|b12, b21) = 0.000199/(0.000199+0.0099) = 1.97%
P(¬p13|b12, b21) = 0.0099/(0.000199+0.0099) = 98.03%

3.2) [22]
3.2.1) P(p22|b12, b21)
P(p22|b12, b21)
= α Σ p13 Σ p31 P(b12|p13, p22) · P(b21|p31, p22) · P(p13) · P(p22) · P(p31) = α[P(b12|p13, p22) · P(b21|p31, p22) · P(p13) · P(p22) · P(p31) +
P(b12|¬p13, p22) · P(b21|p31, p22) · P(¬p13) · P(p22) · P(p31) +
P(b12|p13, p22) · P(b21|¬p31, p22) · P(p13) · P(p22) · P(¬p31) +
P(b12|¬p13, p22) · P(b21|¬p31, p22) · P(¬p13) · P(p22) · P(¬p31)]
= α × (1 × 1 × 0.01 × 0.01 × 0.01 + 1 × 1 × 0.99 × 0.01 × 0.01 + 1 × 1 × 0.01 × 0.01 × 0.99 + 1 × 1 × 0.99 × 0.01 × 0.99)
= α × (0.01)

3.2.2) P(¬p22|b12, b21)
P(¬p22|b12, b21)
= α Σp13 Σp31 P(b12|p13, ¬p22) · P(b21|p31, ¬p22) · P(p13) · P(¬p22) · P(p31)
= α[P(b12|p13, ¬p22) · P(b21|p31, ¬p22) · P(p13) · P(¬p22) · P(p31) +
P(b12|¬p13, ¬p22) · P(b21|p31, ¬p22) · P(¬p13) · P(¬p22) · P(p31) +
P(b12|p13, ¬p22) · P(b21|¬p31, ¬p22) · P(p13) · P(¬p22) · P(¬p31) +
P(b12|¬p13, ¬p22) · P(b21|¬p31, ¬p22) · P(¬p13) · P(¬p22) · P(¬p31)]
= α × (1 × 1 × 0.01 × 0.99 × 0.01 + 0 + 0 + 0
= α × (0.000099)

3.2.3) α
P(p22|b12, b21) = 0.01/(0.01+0.000099) = 99.019%
P(¬p22|b12, b21) = 0.000099/(0.01+0.000099) = 0.980%

4)
4.1)

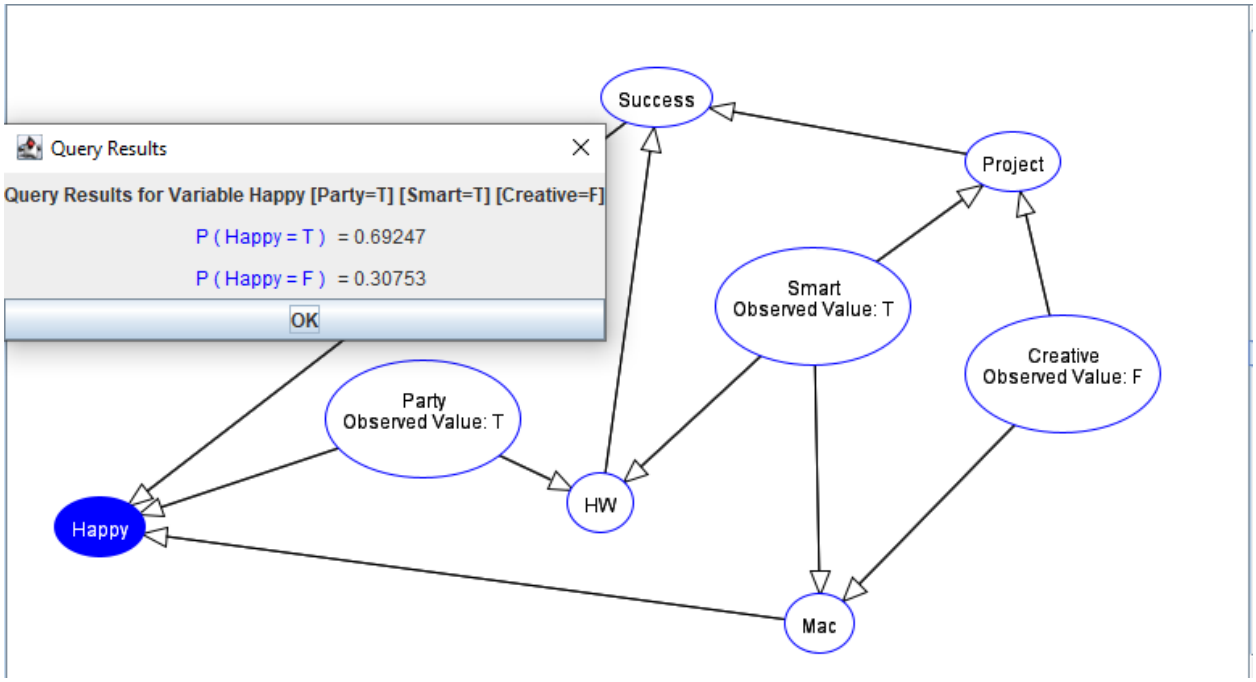


4.2)

		1	0
party	3011	0.6022	0.3978
smart	3524	0.7048	0.2952
creative	3497	0.6994	0.3006
hw	3228	0.6456	0.3544
mac	3112	0.6224	0.3776
project	3536	0.7072	0.2928
success	2789	0.5578	0.4422
happy	2573	0.5146	0.4854

4.3)

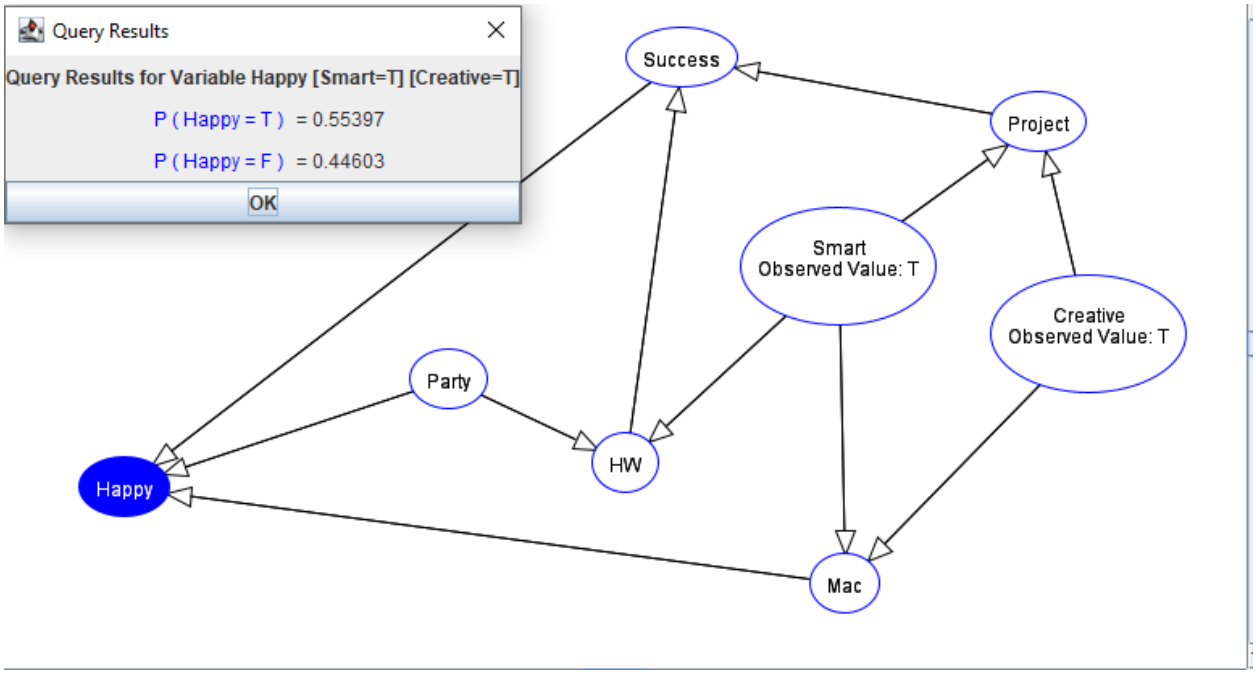
Party	Smart	Creative	cnt1	cnt2(happy=1)	p(Happy|Party,Smart,Creative)
0	0	0	184	22	0.1195652174
0	0	1	426	116	0.2723004695
0	1	0	424	111	0.2617924528
0	1	1	955	285	0.2984293194
1	0	0	252	122	0.4841269841
1	0	1	614	334	0.5439739414
1	1	0	643	453	0.7045101089
1	1	1	1502	1130	0.7523302264



The probability of being happy given that you party often, are wicked smart, but not very creative is about 0.6.

4.4)

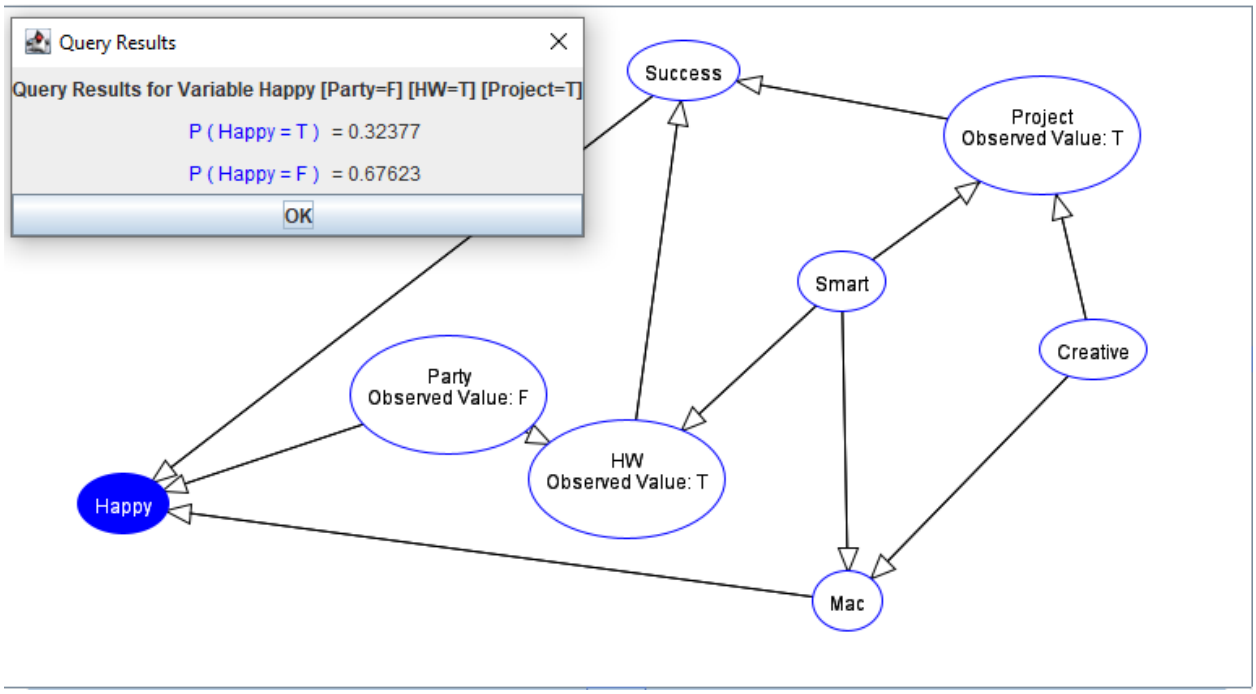
A	B	C	D	E
Smart	Creative	cnt1	cnt2	p(Happy|Smar)
0	0	436	144	0.3302752294
0	1	1040	450	0.4326923077
1	0	1067	564	0.5285848172
1	1	2457	1415	0.5759055759



The probability of being happy given that you are wicked smart and very creative is about 0.55397.

4.5)

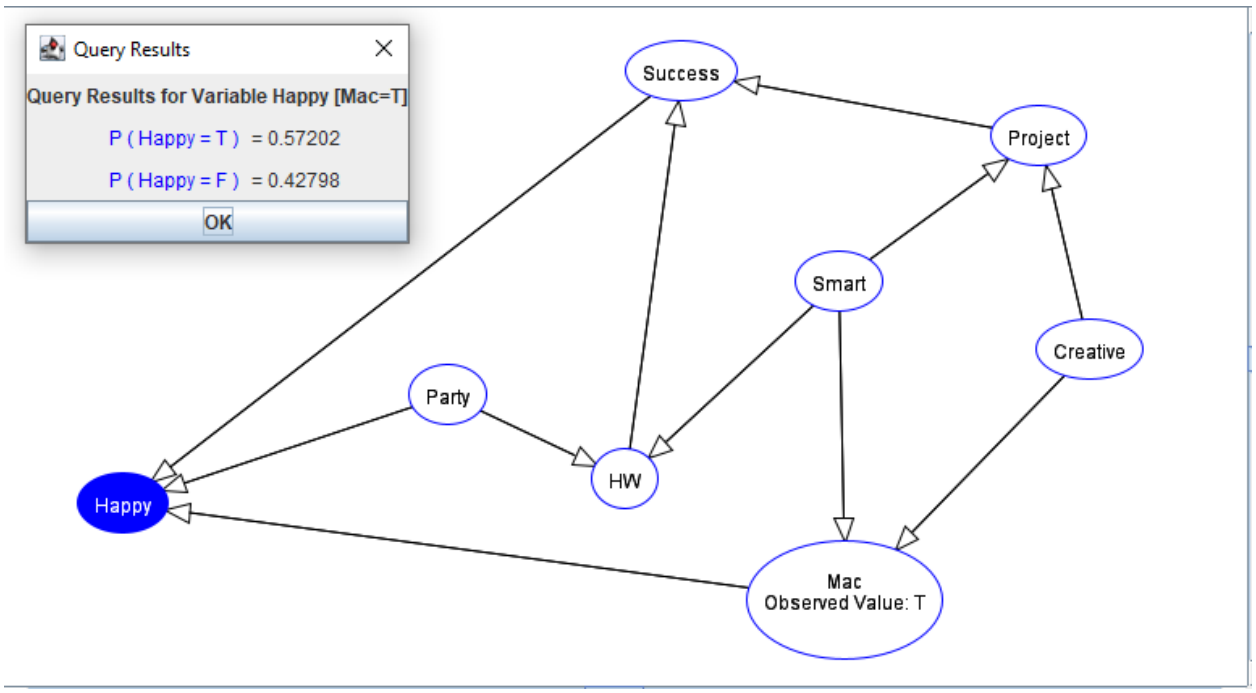
Party	HW	Project	cnt1	cnt2	P(Happy|Party,Hw,Project)
0	0	0	315	60	0.1904761905
0	0	1	249	48	0.1927710843
0	1	0	287	67	0.2334494774
0	1	1	1138	359	0.3154657293
1	0	0	591	286	0.4839255499
1	0	1	617	334	0.5413290113
1	1	0	271	160	0.5904059041
1	1	1	1532	1259	0.8218015666



The probability of being happy given you do not party, and do well on all your homework and class project is about 0.32377.

4.6)

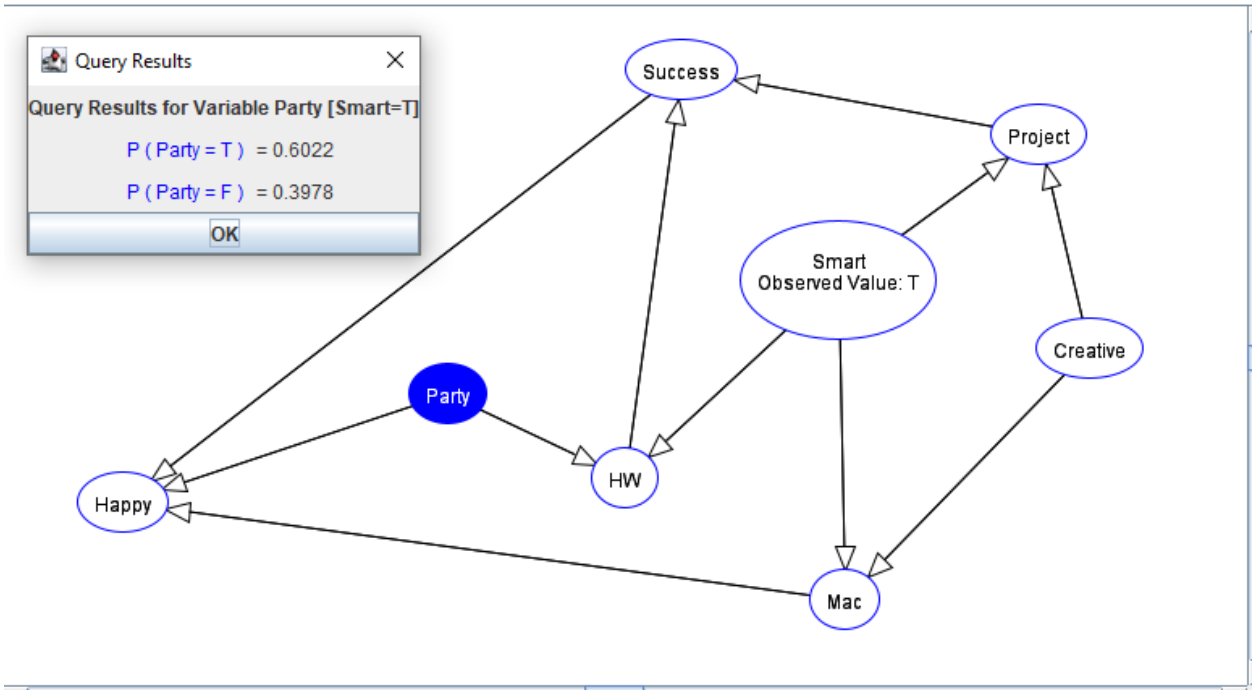
Mac	cnt1	cnt2	P(Happy|Mac)
0	1888	833	0.4412076271
1	3112	1740	0.559125964



The probability of being happy given you own a mac is about 0.57202.

4.7)

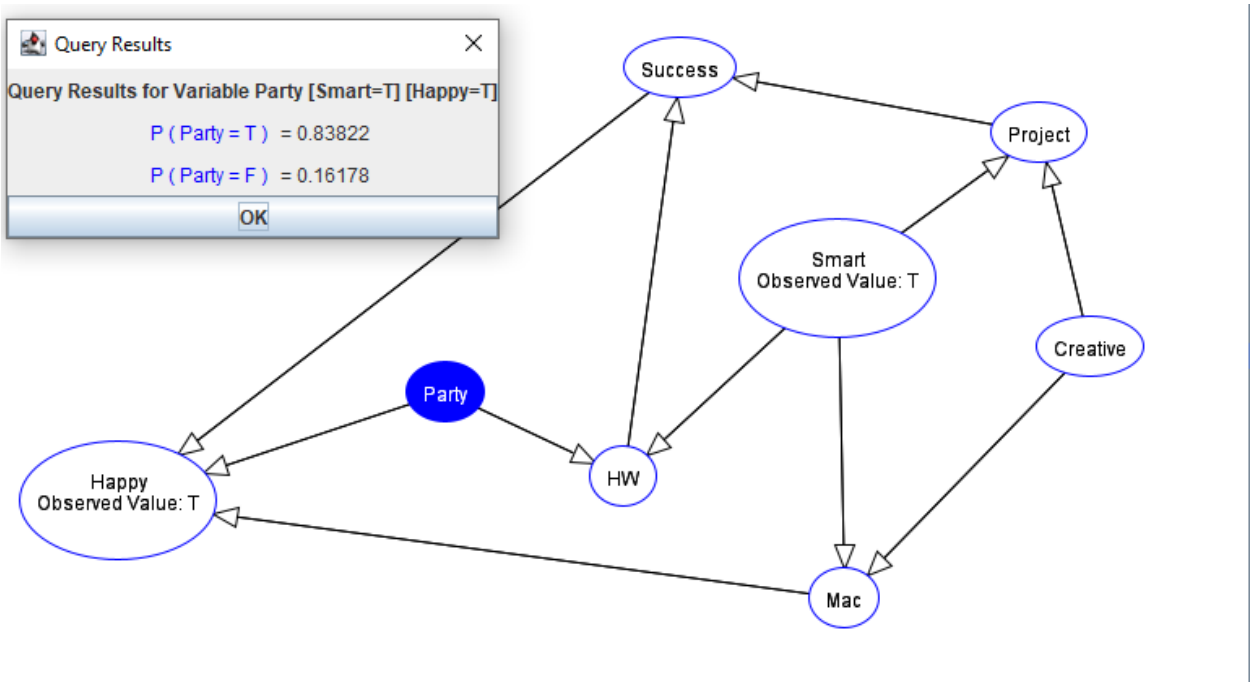
Smart	cnt1	cnt2	P(Party|Smart)
0	1476	866	0.5867208672
1	3524	2145	0.6086833144



The probability that you party often given you are wicked smart is about 0.6022.

4.8)

Smart	Happy	cnt1	cnt1	P(Party|Smart,Happy)
0	0	882	410	0.4648526077
0	1	594	456	0.7676767677
1	0	1545	562	0.3637540453
1	1	1979	1583	0.7998989389



The probability that you party often given you are wicked smart and happy is about 0.83822.