Cutting the Queue to the Dentist: Waiting Time, Public-Private Interaction and Consumer Surplus

Tuomas Markkula*

November 21, 2024

Click here for the most recent version

Abstract

In industries where private and public production coexist, public production can incentivize private competition and address distributional concerns. Using a structural model that incorporates affordable but capacity-constrained public health care producers into a framework with price-setting private producers and consumer demand, I examine how increasing public production's capacity affects waiting times, private practices' prices, and the distribution of consumer surplus. I estimate the model using consumer-level visit data on the Finnish dental care industry. In the counterfactual simulation, I increase the number of dentists in public practices by 20%. This reduces waiting times at public practices on average by 5%, or 1.5 days, but does not affect private practice prices as consumers do not switch from private to public practices. The increase in capacity most benefits middle-income and younger consumers, as they gain the most from shorter public practice waiting times.

*Aalto University and Helsinki GSE. I am grateful to Otto Toivanen, Tanja Saxell, and Ciprian Dominisoru for their constant guidance and support. I also thank Jaakko Markkanen, Milla Hägg, Mikael Mäkimattila, Cristina Bratu, Markku Siikanen, Alina Ozhegova, Ossi Tahvonen, Julia Salmi, Prottoy Akbar, Eero Mäenpää and Mateusz Mysliwski for their advice, as well as participants of Helsinki GSE IO PhD seminar, Helsinki GSE Urban and Regional Economics PhD seminar, and NHH CBE Seminar for their feedback. I thank Atte Kaljonen for excellent research assistance. Finally, I thank Yrjö Jahnsson Foundation and OP Research Foundation for generous funding. All remaining errors are my own. Contact email: tuomas.markkula@aalto.fi

1 Introduction

In many industries such as education, housing, and health care, goods and services are produced by both private and public producers. What is the rationale for having public production alongside private production? Competition between private producers promotes efficiency and innovation (Bloom et al., 2015; Backus, 2020), and privatization has been found to improve production efficiency (Chen et al., 2021). On the other hand, when competition is lacking private producers can raise prices, and incentives for innovation are dampened (Igami and Uetake, 2020; Asker and Nocke, 2021). Moreover, private producers can price out the most vulnerable consumer groups, which is especially worrisome for basic necessities like education, housing, and health care. Public production is a way to address these shortcomings of private production (Stiglitz, 1988).[1] The existence of public production can increase competitive pressure on private producers in markets where competition might otherwise be lacking. Moreover, direct government provision can ensure universal access to basic necessities. However, how these mechanisms play out in practice is not well understood.

I study a typical health care setting where affordable but congested public producers coexist with expensive private producers that allow consumers to bypass the queue. Using a structural model that incorporates capacity-constrained public producers into a framework with price-setting private producers and consumer demand, I examine how increasing public production's capacity affects market outcomes, and the distribution of consumer surplus. I estimate the model using consumer-level visit data on the Finnish dental care industry. Using the model, I show that increasing the production capacity of public practices reduces waiting times but fails to exert competitive pressure on private producers. Middle-income and younger consumers benefit the most.

To study how increasing public practices' capacity affects public practices' waiting times, private practices' prices, and the distribution of consumer surplus, I need to model consumers' dental practice choices, equilibrium waiting times, and private practices' price-setting. In the model, heterogeneous consumers make discrete choices between public dental practices, private dental practices or not visiting a practice at all. Consumers' choices between public and private practices depend on how consumers weigh waiting against higher prices, and I allow consumers' preferences for waiting times and prices to depend on income. In addition, my model takes into account how consumers' income and age influence their decision not to visit a dental practice.

[1] Alternatively, the government could contract out the production of services to private firms, and negotiate a contract that would address the flaws of private production. However, it might be difficult to specify the characteristics of the service and to monitor that the contracted level of quality is maintained(Stiglitz, 1988). Additionally, in the presence of only a few suitable private producers contracting out might be expensive.

Public and private dental practices are differentiated by their prices, locations, and waiting times. Public practices are nonstrategic and have an exogenous production capacity that depends on the number of dentists at the practice. The waiting times are determined in the equilibrium as they depend on endogenous demand and demand depends on waiting times. High demand leads to longer waiting times, which in turn reduces demand, subsequently shortening waiting times, and so on. I obtain a steady state expression for average waiting times from queuing theory and use it as a reduced form equation. In the reduced form equation equilibrium waiting times depend on the demand for practice and the exogenous number of dentists at the practice. Private practices choose prices to maximize profits and take into account congestion externality: private practices' pricing decisions affect public practices' waiting times through demand. As a private practice increases prices, some consumers switch to public practices. As demand for public practices increases, waiting times increase. An increase in waiting time decreases the substitution to public practices, and as a result, the own-price elasticity of private practices is lower than it would otherwise be. This can result in higher prices for private practices.

The Finnish dental care industry is an ideal setting for studying the questions at hand. Affordable but congested public dental practices and expensive private dental practices share the market with comparable market shares. There is a large number of markets with varying market structures and a natural measure of capacity: the number of dentists working at the practice. Finally, I have individual-level data on dental care visits to both sectors for many years including visit-level prices and waiting times and I can link the visits to consumers' demographics, allowing me to study the distribution of consumer surplus. Finally, dental care is an "important and common type of primary care" (Buchmueller et al., 2016).

In my counterfactual, I increase the number of dentists working at public practices by 20%. This corresponds to an increase of 226 dentists in total in the markets in my sample and the salaries of these additional dentists would amount to €16.8 million in a year.[2] If the policy change would be implemented in the whole Finland a 20% increase would require 450 additional dentists, which would cost €33.6 million in salaries. The counterfactual corresponds to a policy that would lengthen Finnish dental schools' mandatory six-month practical training and increase the dental schools' intake to fill the additional public practice dentist vacancies while keeping the number of private practice dentists constant.

The increase in capacity decreases waiting times at public practices on average by 5% or 1.5 days, which is a small decrease relative to the 20% increase in the number of dentists, and likely a result of the rather high practice-level own-waiting time elasticity of -2.71.

[2] Assuming the dentists are paid €6212 per month, which is the average salary of a public practice dentist in 2017 from a survey by Finnish Dental Association (2019).

The increase in capacity decreases the waiting times at first but as waiting times decrease, demand increases. Because of the substantial elasticity, waiting times decrease only by 5%, even though the market share of a public practice increases on average by 13%.

The increase in capacity and the decrease in waiting times do not affect the prices of private practices. Even though the average market share of public practices increases by 13% the average private practice market share decreases only by 2.8% and most of the increase in public practices' market shares comes from consumers who substitute from not visiting a dentist to a public practice. Demand estimates reveal that high-income consumers are more responsive to waiting times and less responsive to prices, and vice versa for low-income consumers. Thus, it seems that private practices have successfully segmented the market and consumers do not view public and private practices as close substitutes due to the differences in prices and waiting times. This means that it is difficult for public production to apply competitive pressure on private practices.

The increase in capacity leads to the largest increases in consumer surplus for middle-income consumers and for the youngest consumers. As prices stay constant, all of the change in consumer surplus must come from decreased waiting times. First, the decrease in waiting times increases the consumer surplus the most for consumers who dislike waiting, as the reduction in the disutility of waiting is larger the more consumers dislike waiting. Second, the decrease in waiting times benefits most consumers who are likely to choose a public practice. Middle-income consumers strike the balance of disliking waiting, while still having a relatively high probability of visiting public practices, and thus they benefit the most.

A reason why the private practice prices do not change is that the increase in capacity has only a small effect on the equilibrium waiting times. This in turn is because there appear to be relatively many consumers on the extensive margin that entered the market when waiting times decreased by a small amount. Siciliani et al. (2014) mentions how it might be difficult to decrease waiting times purely by increasing supply as the demand will immediately react to decrease in waiting times. Instead, supply increases should be accompanied by demand restrictions, for example allowing consumers to access non-emergency dental care at public practices only every two or three years. This would reduce the substitution from not visiting to public practice and would lead to larger decreases in waiting times and to possible decreases in private practice prices.

The reason why low-income consumers did not benefit the most is that they are the consumer group with the lowest disutility for waiting, even though they have a high probability of visiting a public practice. At the same time, low-income consumers have the highest opportunity cost of money. Thus it might be that a policy change that would most benefit low-income consumers would be to decrease the out-of-pocket price of the public practices even though it would also increase waiting times. This would likely also improve the targeting of public production towards low-income consumers.

Related Literature My work is mainly related to two strands of literature. First, I contribute to the literature on markets with both public and private production, particularly on the effects that an increase in public production has on market outcomes. My main contribution to this literature is studying how an increase in public production affects market outcomes when public production is rationed through waiting in addition to prices. Second, I contribute to the literature that builds and estimates structural models where congestion is determined in the equilibrium as a function of demand and producer capacity. My contribution is the development of a framework that features two types of producers: one that experiences endogenous congestion and cannot set prices, and another that operates without congestion and sets prices strategically. Furthermore, I contribute to the literature estimating waiting time elasticities in health care, where I estimate waiting time elasticities for a common primary care service. Finally, I also contribute to a large theoretical literature on markets with both public and private producers, by constructing a structural model and taking it to the data.

In the literature on markets with both public and private production, to my knowledge closest to my work is Dinerstein and Smith (2021), who show that an increase in public school funding, which increases the public schools' capacity, can result in some private schools exiting the market. A major difference between me and Dinerstein and Smith (2021) is that in the education context, all children must attend school, and thus there is no outside option. Thus the increased capacity in public schools resulted in a substantial substitution away from private schools. While my results show that increased capacity led consumers not using the service to start using public dental care, with minimal switching from private providers. My study highlights how the effects of an increase in public production's capacity on private producers depend on consumers' substitution patterns. This is especially true with waiting times, as substitution from not using a service to using publicly produced services dampens the reduction in waiting times that increased capacity can achieve.

My result that private practices' prices are not affected by the capacity increase is similar to the findings of Atal et al. (2024). They study how introduction of public pharmacies affected the prices of private pharmacies, and find that private pharmacies increased their prices instead of incentivizing competition. Atal et al. (2024) explain the finding by arguing that even though the public pharmacies were more affordable, they were also of lower quality and the entry allowed private pharmacies to segment the market. Price-sensitive consumers switched to public pharmacies and thus, private pharmacies' marginal consumers became less price-elastic. My findings provide more direct evidence of how market segmentation can insulate private producers from competition by public producers.

In addition, the empirical literature on public-private interaction contains studies that analyze how introducing a public production to a market with private producers affects prices, demand, and consumer welfare in markets like milk, and insurance (Jimenez-Hernandez and Seira, 2022; Saltzman, 2023). I contribute to this body of work by highlighting how public-private interaction is different when public production is allocated with waiting times and prices instead of only prices. For example, congestion externality increases private practices' prices when public production is congested.

The second strand of literature my study contributes to is the body of work building structural models with congestion as an equilibrium outcome. Russo (2024) quantifies the trade-off between redistribution and efficiency when rationing access to care with waiting times rather than with prices for physician office visits of the US veterans. She also draws on insights from the queuing literature, but her model, while allowing for capacity constraint and waiting times, does not allow her to change the congested practices' capacity, which is my main interest.

Brancaccio et al. (2024) and Elliott et al. (2024) study how a change in production capacity affects congestion and other outcomes, in addition to treating congestion as an equilibrium outcome. Brancaccio et al. (2024) study congested ports and ask how investments in port capacity would affect welfare through changes in trade volume and speed at which goods move from one country to another. They model congestion with a full dynamic queuing model with a production function for the time a ship spends at a port. However, their framework does not feature strategic price-setting competitors that could react to changes in congested producers' capacity. Elliott et al. (2024) build and estimate a model of mobile network operators where network operators invest in infrastructure and set prices. In their model, the quality of the operator's network depends on how fast the download speed is, which in turn depends on how congested the network is in the equilibrium. However, in their paper operators only differed in their prices and download speeds, there is no uncongested alternative, and firms can influence how congested their network is by varying prices. I contribute to this literature by studying and modeling a setting with two different types of producers: congested public practices with exogenous prices and uncongested price-setting private practices.

My study is also related to studies estimating waiting time elasticities in health care. To my knowledge, the only other study estimating waiting time elasticity for primary care is Russo (2024) who estimates the elasticity for physician office visits of the US veterans. There is a larger number of studies estimating waiting time elasticities for inpatient care such as Martin et al. (2007); Sivey (2012); Riganti et al. (2017) and Bruni et al. (2021). Strobel (2024) estimates waiting time elasticity for emergency care. There is also a large theoretical literature on public-private interaction and waiting times including Hoel and Sæther (2003); Marchand and Schroyen (2005); Brekke and Sørgard (2007) and surveyed by Barros and Siciliani (2011). I contribute to this literature by taking a model with public-private interaction and waiting times to data.

2 Institutional Setting

The Finnish dental care industry is worth €1 billion in expenditures according to the Finnish Institute for Health and Welfare (2019). The industry consists of both public and private dental care providers, with comparable market shares at the national level[3]. All permanent residents of Finland have access to public dental care but can choose to use private dental care. Public dental care's out-of-pocket prices are one-third of the private practices' out-of-pocket prices, but waiting times for the public providers can be long. Anecdotal evidence suggests that the waiting times are negligible at the private practices[4].

2.1 Public and Private Dental Care Providers

Public and private dental practices provide dental care services that aim to preserve and restore the oral health of the patient. In both types of practices, dental care is performed during a treatment episode, which consists of several visits during which the patient's condition is treated. The treatment episode begins with a diagnostic visit, during which a dentist performs a dental examination and X-rays. Based on the findings of the examination, the dentist makes a diagnosis and determines what dental treatments a patient needs. The treatments, most commonly fillings, teeth removals, root canal treatments, and periodontal treatments, are then performed during one or several follow-up visits.

Public dental care is organized by municipalities, which are required by law to provide dental care to their residents. Municipalities can also form a joint municipality authority to provide the services together. The central government regulates public providers' prices

[3]Expenditure on public dental care was €377.6 million and expenditure on private dental care was €542.3 million in 2014. The remaining expenditure classes are university students' and Finnish Defence Forces' dental care with €13.8 million and dental prosthetics with €82.6 million.

[4]There are multiple available times for dental examination for the next day (Saturday) in the three of the four largest dental care chains in Helsinki as checked in the afternoon of Friday 28.6.2024. All the chains offer multiple times for dental examinations for the next Monday. The same holds in Turku for two of the large chains and all offer multiple times for the Monday and in Tampere all chains offer multiple times for the Monday.

and the waiting times to obtain non-urgent care. The public providers' prices are capped at the dental procedure level, and most of the municipalities set prices at the cap. In general, non-urgent dental care should be provided within three months of the assessment of the need for treatment, but the three-month limit can be extended by an additional three months if the patient's health is not compromised. Violating the waiting time limit is a reputational concern and can lead to fines if the issue is not addressed sufficiently. Public dental care also provides urgent dental care. In the case of an urgent visit, a consumer's need is deemed urgent enough so that treatment should begin within a day. Public dental care does not provide aesthetic treatments or dental implants.

Private dental care is provided by firms, either national chains or independent practices owned by dentists. Private practices' pricing is not regulated, and private practices' services are covered by the National Health Insurance. The insurance covers all permanent residents of Finland and reimburses a fixed amount per procedure. According to Blomgren et al. (2017) the reimbursement paid for 31% of the expenses at the private providers in 2014, but the reimbursements were cut in 2015 and 2016, and as a result, the reimbursement covered only 15.7% of consumers' expenses in 2016. Private dental practices might provide higher quality care than public dental practices, as they might use more expensive materials and spend more time to perform the treatments.

2.2 Dentists

Dentists are the most important factor of production in dental care. The law states that they are the only dental care professionals who are allowed to diagnose oral diseases. A dentist treats different inflammations and infections of the mouth area and plans and performs dental treatments like fillings, root canal treatments, and surgical treatments like teeth removal. There are on average 3915 working dentists per year between 2011 and 2019, and approximately 60% of dentists work in the public sector. A dental hygienist is the closest substitute to a dentist, but rather than examining, diagnosing, and treating oral disease like dentists, they focus on promoting oral health, performing dental check-ups without making a diagnosis, and performing assessments of the need for treatments. Dental hygienists often perform periodontal treatments, like tartar removal. Finally, dental nurses mainly assist dentists, perform limited dental check-ups without making a diagnosis and perform assessments of the need for treatments.

3 Data

I use comprehensive consumer-level data on dental care visits for all Finnish residents for 2014–2017 allows me to link visits to public and private dental practices to consumer characteristics. Moreover, I observe if a consumer does not visit a dental practice in a given year. I observe prices at the public and private practices, and individual-level waiting times at the public practices and I can link dentists to the practices where they work. Finally, I observe both practices' locations and consumers' residence locations.

Data on visits to private practices is administrative data from Kela, the Social Insurance Institution of Finland. The data contains all procedures performed at the private dental practices, that were reimbursed by the National Health Insurance[5] for 2014-2017. For each procedure, I observe the patient, the day of the visit, the daily price of the procedure, the treating dentist, and the dental practice. In addition, I observe what is the reimbursement level for the procedure and whether a higher reimbursement level applies due to the use of a specialized dentist's services or due to an unusual visit time[6]. Finally, I observe if a practice belongs to a chain, in which chain the practice belongs to, and the location of the practice in Statistics Finland's 250m$\times$250m grid.

Data on visits to public practices is administrative data from the Finnish Institute for Health and Welfare. The data contains all visits to public dental practices for the years 2014–2017. For each visit, I observe the patient, the date on which the visit was booked and the day of the visit, what procedures were performed during the visit, and the location of the practice in Statistics Finland's 250m$\times$250m grid. Moreover, I observe whether the visit is an urgent visit or not. I calculate waiting time as a difference between the date of the first visit of the treatment episode and the date when the first visit was booked as done by the previous literature (Yee et al., 2022; Chartock, 2023; Russo, 2024). Unfortunately, the data on visits to public practices does not include prices. Therefore, I use data on the government-set procedure-specific price caps on public practice prices collected by Haaga (2019) instead, and assume that all municipalities set the prices at the caps.

Data on consumer characteristics is administrative data from Statistics Finland. These data contain consumer characteristics like age, sex, socioeconomic status, education, and disposable income as well as consumer's location of residence in 250m$\times$250m grid for all permanent residents at the end of the year for years 2014-2017. As I observe the universe

[5] The procedures that fall outside the scope of the insurance are prosthetic and aesthetic procedures, so I do not observe these procedures. Additionally, from 2015 onward dental examination is only reimbursed every other year.

[6] A higher reimbursement level applies if the dentist performing the procedure provided treatment as a specialized dentist or if the treatment was performed outside regular hours.

of permanent residents and can link consumers to their dental care visits, I observe if a consumer does not visit a dental practice in a given year.

Data on dentists comes from The National Supervisory Authority for Health and Welfare Valvira and Statistics Finland. I observe the age and specialization of dentists working at private practices in Valvira data. In addition, as I can link a dentist and a private sector visit using the private practice visit data from Kela, I observe how many patients the dentist treats, what treatments the dentist performs on a given patient, and in which private practice the dentist works at. For dentists working in the public sector, I observe the municipality where the dentist works and various characteristics like earnings, age, and sex in Statistics Finland data.

3.1 Constructing Treatment Episodes and Expected Prices

I model demand for dental care as a discrete choice between public and private practices and not visiting a practice, and thus I need to define a discrete unit of dental care. A natural choice for a unit of dental care is a treatment episode as it contains all the services that the consumer pays for, from diagnostics to treatments. The downside of using treatment episodes is that I need to group the individual dental care visits into treatment episodes myself. I start by defining a dental care visit. The public sector data contains a variable that indicates to which visit the treatments belong, while in the private sector data, I define a single visit as a unique combination of a patient, a practice, the date of the visit, and whether the person providing the treatment is a dentist or a dental hygienist.

In the public sector data, I construct the episodes from individual visits in the following way. If for two visits the interval between the booking date and the visit date overlap, I assume the visits belong to the same episode. Moreover, if the booking for one or more visits was done on a previous visit date, then the booked visits also belong to the same episode as the visit. Finally, if a visit was booked within 90 days of the previous visit, then the booked visit belongs to the same treatment episode as the previous visit. This is in accordance with the legislation stating that the waiting time for non-urgent care should be at most 90 days. In the private practice visit data, I assume that visits belong to the same treatment episodes if they are within 90 days of each other.

An additional issue is that consumers do not know what treatments their treatment episode contains before a diagnostic visit to a dentist. As the price of the treatment episode depends on how many and which treatments the dentist prescribes to the consumer, I need to take a stand on the consumers' expectations about their treatment bundle. As the focus of this paper is not on imperfect information, I make a simplifying assumption that all consumers expect to get the same sector-year-specific average bundle of treatments. I allow the average bundle to vary across sectors and years to allow for different treatment practices between private and public practices, as well as across years.

I construct the average bundles as follows. I take the 50 most common procedures performed on the study population. I then calculate weights for the procedures by dividing the number of procedures by the number of treatment episodes separately for each sector-year combination. The average bundle is then the weighted sum of the 50 most common procedures. I construct practice-specific prices by summing over the practice's prices for the 50 most common procedures while weighting the procedure-specific prices according to the procedure weights. Thus, the price that consumers consider when choosing between public and private practices and not visiting is the price of the average sector-year-specific bundle in a given practice. Even though the treatments do not vary across practices, private practices' prices for treatments vary and thus the price of the average bundle also varies across private practices. Public practice prices are the same within a year, but the price caps were increased in 2016, and thus also prices increased.

The average bundle assumption simplifies the analysis and allows me to model private practices as setting only one price. The one price can be thought of as a menu of prices for different procedures. For example with ten different bundle types, each private practice would need to set ten different prices that would correspond to the menu of prices as well as a bundle-type specific set of services. The downside of the assumption is that it can lead to biased price elasticity estimates, if consumers have rational expectations about their bundle of treatments and consumers' choices depend on the bundle they expect to get. However, this seems unlikely to me.

3.2 Market, Practice and Consumer Samples

I define a market as a combination of a geographic area and a year. To simplify the analysis I focus on isolated local markets, where the consumers residing in the market almost exclusively visit the dental practices located within the market. I assume that the geographical area is a municipality[7], and if many municipalities produce public dental care together, I assume that the municipality cooperation area is the geographical market. This definition produces sensible local markets for more rural areas but does not work for larger urban areas. Thus, I exclude the ten most populous municipalities from the sample, as well as their neighboring municipalities.

[7]An alternative to this would be to use the locations of consumers and practices to construct catchment area-based markets. This would likely produce overlapping markets, which would complicate empirical analysis.

Table 1 presents summary statistics for 322 markets in my sample. I have an unbalanced panel of markets as some of the 101 geographic areas are in the sample for only a subset of 2014–2017. In a market, there are on average 2.46 public practices and 3.24 private practices. There is also significant variation in the number of practices across markets with a standard deviation of 1.87 for the number of public practices and a standard deviation of 2.75 for the private practices. Chains' role in these markets is limited. A 5% of the practices belong to a chain and only 2.5% of the practices have two practices by the same chain present. An important feature of the markets is that only 73.9% of the markets have a private practice present. This allows me to compare markets with and without public-private interaction. 90.1% of the markets have a public practice present. Finally, 46.9% of the markets are rural, 31.4% are semi-urban 31.4% and 21.7% are urban.

Table 2 presents the summary statistics for the practices in the included markets that sum up to 1484 practice-year observations for 2014–2017. My four-year panel of dental practices is unbalanced. There are 218 unique public practices, 713 public practice-year observations, 256 unique private practices, and 771 private practice-year observations. The average treatment episode costs out-of-pocket €119 in an average public practice and €180 in an average private practice, a difference of €61s. The price of the treatment episode varies a lot in the private sector as the standard deviation is €47. The variation in public practice prices comes only from the across-year variation in the price caps. Affordable public practices are congested, and the average first-visit waiting time is approximately 32 days, with a substantial standard deviation of 16 days. Within-episode waiting times are longer, on average 40 days. Moreover, public practices are on average larger than private practices. A public practice has on average a market share of 30%, while a private practice has on average a market share of 14%. In addition, an average public practice has 50% more visits per year, treats more unique patients per year, performs more dental procedures per year and completes more treatment episodes per year than an average private practice. Finally, 12.5% of private practice-year observations are by practices that belong to chains.

I restrict my consumer sample to 25–80 years old consumers residing in the included markets. The restriction is necessary as university students in Finland have their own dental care, which I do not have data on[8]. I drop all consumers younger than 25 years, as dropping only university students would result in a selected sample. The upper bound restricts the sample to consumers who are in the market for dental care. I take a 40% random sample of my consumer sample that I use in the empirical analysis.

Table 3 presents summary statistics for my unbalanced panel of two million consumer-

[8]University studies in Finland usually take at least five years, three years for the undergraduate degree and two years for the master's degree, and the typical age to begin studies is between 19 and 21 years.)

year observations by a half million unique consumers. The table's last three columns divide the unique consumers into three categories based on which dental practices they have visited between 2014–2017. Half of the consumers have visited a public practice, 35% have visited a private practice, and 27% have not visited a dental practice between 2014–2017. If a consumer has visited both a public and a private practice, then they are included in both public and private practice visit columns. The share of not visiting is slightly inflated due to the unbalancedness of the consumer panel. For example, some consumers are in the data only in 2017 because they turned 25 in 2017, and they are included in the "No visit" column if they did not visit a dental practice in 2017. The table shows that consumers who have visited a private practice are on average oldest and have the highest income, while consumers who have visited a public practice are younger and have lower income. Consumers who have not visited have the lowest income on average and are slightly younger on average than consumers with a private visit. The average number of episodes per year is 0.5, meaning that on average a consumer goes through a treatment episode every other year. Consumers with a public visit a dental practice more often, on average once every 1.7 years, and consumers with a private visit visit a dental practice on average almost every year.

Figure 1 displays how consumers' choices between public and private practice, and not visiting vary by yearly disposable income. The figure shows that all income groups use public dental care, but private dental care is more common for high-income consumers. As income increases from €27 thousand the share of consumers choosing a private practice increases, while the share of consumers visiting a public practice decreases. Additionally, the share not visiting a dental practice decreases as we move from lowest yearly incomes to incomes over €27 000 and levels off afterward.

4 Model

I am interested in what would happen to public practice waiting times, private practice prices, and distribution of consumer surplus if public practices would employ more dentists. To this end, I build a structural model of the industry that endogenizes consumer demand, prices, and waiting times. Heterogenous consumers choose whether to purchase dental care services and from which practice. Private practices choose their prices to maximize profits and public practice waiting times are determined in the equilibrium by consumers' choices and an exogenous number of dentists. Private and public dental practices are differentiated by their location. All choices are simultaneous. I model the demand for non-urgent dental care, and I am taking as given the number of dentists in the local market, as well as the number of dental practices and their locations. In addition, I am not modeling public sector wages.

4.1 Consumer Demand

A consumer can choose to visit one of the private dental practices or one of the public dental practices in the market or decide not to visit a dental practice. A market is a municipality or a municipality cooperation area-year combination. Outside good is not visiting a dental practice in a given year and using income on other things. Market size, M_t, is the number of individuals in the consumer sample residing in the market. The indirect utility for consumer $i \in I_t$ from purchasing a dental care treatment episode from practice $j \in J^{pri} \cup J^{pub}$, where $j = 0$ denotes outside good, in market $t \in T$, is

$$u_{ijt} = \delta_{jt} + \mu_{ijt} + \epsilon_{ijt} \tag{1}$$

$$\delta_{jt} = x_{jt}^{\intercal}\beta + \lambda_y + \xi_{jt} \tag{2}$$

$$\mu_{ijt} = (x_{jt} \otimes z_{it})^{\intercal}\theta_2 + \theta_3 h_{ijt} \tag{3}$$

$$u_{i0t} = z_{it}^{\intercal}\theta_1 + \epsilon_{i0t}, \tag{4}$$

where $\otimes$ denotes a Kronecker product. ϵ_{ijt} is i.i.d. and follows the type-I extreme value distribution and is consumer i's idiosyncratic preference shock for practice j. δ_{jt} captures consumers' mean utility from choosing the practice j. It is a linear function of practice characteristics, x_{jt}, including the price of the average treatment bundle and a yearly average of individual consumers' waiting times for practice j. Waiting times are zero for private practices. λ_y is a year fixed effect for year y. ξ_{jt} is the unobservable component of the mean utility and contains the unobservable practice-specific quality as well as the average waiting time for private practice j if the private practice has positive waiting times. Thus, the demand model captures these important unobservable practice characteristics and they are kept constant in the counterfactuals. ξ_{jt} is correlated with prices and waiting times and hence, private practice prices and public practice waiting times are endogenous. Public practice prices are set at the national price cap and are thus exogenous.

Access to consumer-level data with consumer characteristics allows me to model how observable consumer characteristics affect consumers' choices. μ_{ijt} captures flexibly how consumers' preferences for the practice j's characteristics depend on the consumers' observable characteristics, z_{it}. z_{it} contains consumer i's age, and disposable income, and their 2nd powers.[9] Income and its 2nd power are interacted with price and waiting times, while age

[9] I choose the quadratic function instead of income bins, as Miravete et al. (2024) recommends against

and its 2nd power are not. For example, the model allows consumers with higher incomes to have a lower price elasticity and a higher waiting time elasticity. The interactions allow me to study how the distribution of consumer surplus changes when public practices' capacity is increased (Griffith et al., 2018). This is because the interactions between prices and waiting times and consumer characteristics allow observationally different consumers to have different preferences for prices and waiting times as well as different elasticities and substitution patterns. Absent interactions, increasing the public practice capacity would affect all consumers the same. Additionally, interacting product characteristics with a nonlinear function of consumer's income ensures that the demand model does not restrict the range of pass-through the model can produce (Griffith et al., 2018; Miravete et al., 2024).[10]

The availability of consumer-level data with consumer and practice locations allows me to include consumer i's distance to practice j, h_{ijt} into the model. Thus, the consumer's indirect utility for practice j depends on the distance from the consumer's residence of locations to the practice in a granular 250m×250m grid. I also allow consumers' preferences for the outside good of not visiting a practice to depend on consumers' characteristics. For example, I expect older and higher-income consumers to visit a dental practice more often. Formally, consumer is' indirect utility for not visiting a practice, u_{i0t}, depends on consumer's characteristics z_{it}.

Consumer i chooses a practice to maximize her indirect utility. Together with the assumptions on ϵ_{ijt}, this leads to the following expression for the probability that consumer i chooses practice j in market t

$$\mathcal{P}_{ijt} = \frac{\exp(\delta_{jt} + \mu_{ijt})}{1 + \sum_{k=1}^{J} \exp(\delta_{kt} + \mu_{hkt})}. \tag{5}$$

Integrating over consumer characteristics z_{it} gives the market share of practice j in market t

$$s_{jt} = \frac{1}{I_t} \sum_{i \in I_t} \mathcal{P}_{ijt}(z_{it}). \tag{6}$$

4.2 Public Practice Waiting Times

I model public practices' waiting times as equilibrium objects that depend on how many consumers book an appointment at the practice, how many dentists work at the practice, and how productive the practice is. An expression for the average waiting time within a year

using bins as they result in discrete consumer types which strongly restricts the possible combinations of elasticity and pass-through that the model can accommodate.

[10]As shown by Miravete et al. (2024), see also Griffith et al. (2018), a logit demand without preference heterogeneity restricts pass-through to be below one.

that depends on these variables can be derived from a simple queuing theory model,[11] see Bhat (2008). Using the steady-state average waiting times that arise from the model as a reduced-form equation, I get the following parameterized expression for the average waiting time in days for a consumer booking an appointment at a public practice j in market t

$$b_{jt} = \psi_1 \frac{q_{jt}}{d_{jt}} + \tau_y + \omega_{jt}, \quad j \in J^{pub}. \tag{7}$$

The right-hand side of the equation consists of how many consumers seek care at the practice during a year, per the number of dentists working at the practice, q_{jt}/d_{jt}. The number of consumers is divided by the number of dentists working at the practice, which corresponds to how many consumers can be treated simultaneously. Dividing the length of the queue by the number of dentists gives the number of patients a single dentist must treat before it is consumer i's turn. ψ_1 is service time: how large share of a day does it take to treat a consumer with an average treatment bundle. ω_{jt} allows some practices to be more productive, in the sense that with the same consumers per dentists ratio, practices can have different waiting times. Finally, I allow for different productivity levels across years shared by all public practices with year-fixed effects, τ_y.

Waiting times depend on the private practices' prices as well as consumers' choices in the equilibrium, as these choices affect the demand for the public practice's services q_{jt}. Thus, demand and waiting times are determined simultaneously in the equilibrium, and thus demand is endogenous in equation 7. I assume that the waiting time cannot be negative $b_{jt} \geq 0$. Waiting times for the private practices are included in the demand model and are not allowed to change in the counterfactuals. Instead, private practices can change prices.

4.3 Private Practices

Private practices choose the price of the average treatment bundle to maximize profits. I model private practices as single-practice firms as there are only a few markets where the same chain has two practices in the market.[12] Moreover, chains do not have uniform prices across markets. Private practice j's pricing problem in market t is

$$\arg\max_{p_{jt} \in J_t^{pri}} \sum_{j \in J_t^{pri}} \Big(p_{jt} - mc_{jt}\Big) q_{jt}(p_t, b_t(q_t^{pub})). \tag{8}$$

[11] The expression arises as a steady state of the M/M/K queuing model, where consumers' arrivals to a dental practice follow a Poisson distribution (M), how long does it take to serve a consumer follows an exponential distribution (M) and there are (K) dentists treating the consumers. Brancaccio et al. (2024) use this kind of model to study congestion at ports.

[12] I will use the multiproduct formula for these chain practices in future versions of this paper.

p_{jt} is the price of the average service bundle for practice j, $q_{jt}(p_t, b_t(q_t^{pub}))$ is the number of patients treated, $q_{jt}(p_t, b_t(q_t^{pub})) = s_{jt}(p_t, b_t(q_t^{pub}))M_t$, M_t is the number of consumers in the local market and $b_t(q_t^{pub})$ contains public practices' average waiting times. mc_{jt} is practice j's marginal cost function, which I assume to be constant.

The standard first-order condition of the private practice's problem is

$$p_{jt} = mc_{jt} + q_{jt}(p_t, b_t(p_t))\left[-\frac{\partial q_{jt}(p_t, b_t(p_t))}{\partial p_{jt}}\right]^{-1} \tag{9}$$

We can see that a less elastic demand for practice j implies higher prices for practice j. The price-setting problem accounts for the fact that private practices' prices affect public practices' waiting times, and the waiting times affect private practices' prices. Private practices' prices impose a congestion externality on public practices, as an increase in the private practices' prices increases demand for the public practice. Thus, waiting times at the public practice increase. The congestion externality results in higher private practice prices, as the congestion externality shifts consumers' price elasticity of demand towards zero. The intuition is following. If all private practices set high prices, then many consumers prefer to visit a public practice. However, because of the limited capacity of the public practice, the increase in demand will increase waiting times at the public practice. As consumers dislike waiting, with large enough waiting times some consumers will prefer to visit one of the private practices even with high prices. This will result in a lower price elasticity of demand for private practices.

The congestion externality enters the private practice's pricing problem through price derivatives of demand, which differ from the usual price derivative of demand obtained from a logit demand. This is because in the equilibrium the public practices' waiting times depend on the private practices' prices and the private practices' prices depend on the waiting times. The derivative of the private practice j's quantity with respect to the p_{jt} is

$$\begin{aligned} \frac{\partial q_{jt}}{\partial p_{jt}} &= M_t \frac{1}{I_t} \sum_{i \in I_t} \frac{\partial \mathcal{P}_{ijt}}{\partial p_{jt}} && (10) \\ &= M_t \frac{1}{I_t} \sum_{i \in I_t} \left(\frac{\partial u_{ijt}}{\partial p_{jt}} \mathcal{P}_{ijt}(1 - \mathcal{P}_{ijt}) - \sum_{k \in J_t^{pub}} \left[\frac{\partial u_{ikt}}{\partial b_{kt}} \frac{\partial b_{kt}}{\partial s_{kt}} \frac{\partial s_{kt}}{\partial p_{jt}} \mathcal{P}_{ijt} \mathcal{P}_{ikt} \right] \right) && (11) \end{aligned}$$

The first term of equation 11 is negative as it is a standard logit own-price derivative telling how the demand for practice j changes when practice j changes its price. The second term expresses how the waiting times of the public practices change when practice j changes its price. The second term is positive under sensible conditions: consumers dislike waiting, an increase in public practice demand increases waiting time, and public practices' demand increases in private practices' prices. Thus, a larger increase in the waiting times of public practices after a change in p_{jt} results in a less price elastic demand for practice j.

The extent to which the congestion externality reduces consumers' price sensitivity to private practices' prices depends on how much consumers dislike waiting, how much waiting times change when public practices' demand changes, and how much public practice demand changes when private practice changes their price, $\partial q_{ijt}^{pub}/\partial p_{jt}$. The first two components I get straightforwardly from the demand model and from the waiting time model, but the third component requires more work.

The expression for the market share of public practice is a fixed point relation, where on the left-hand side I have the demand for public practice j, and on the right-hand side, I have a function that depends on the demand for public practice through the waiting times.

$$q_{jt} = M_t \frac{1}{I_t} \sum_{i \in I_t} \mathcal{P}_{it}(b_{jt}(q_{jt})), \text{ for } j \in J_t^{pub} \implies s_{jt} = \frac{1}{I_t} \sum_{i \in I_t} \mathcal{P}_{it}(b_{jt}(s_{jt} M_t)), \text{ for } j \in J_t^{pub} \quad (12)$$

I differentiate this equation using implicit differentiation. I get for the public practice j's cross-price derivative with respect to private practice k's price in a market with only one public practice

$$\frac{\partial s_{jt}}{\partial p_{kt}} = \frac{-\sum_{i \in I_t} \left[\frac{\partial u_{ikt}}{\partial p_{kt}} \mathcal{P}_{ikt} \mathcal{P}_{ijt} \right]}{\sum_{i \in I_t} \left[1 - \left(\frac{\partial u_{ijt}}{\partial b_{jt}} \frac{\partial b_{jt}}{\partial s_{jt}} \right) \mathcal{P}_{ijt}(1 - \mathcal{P}_{ij}) \right]}, \text{ for } j \in J_t^{pub}, k \in J_t^{pri} \quad (13)$$

which is positive assuming consumers dislike higher prices and longer waiting times. This expression for the cross-price derivative of a public practice is intuitive. The numerator is the standard logit cross-price derivative, which is then scaled up or down by the denominator. The denominator is one minus the own-waiting time derivative of a public practice. If consumers dislike waiting, then the denominator is larger than one and the cross-price derivative is scaled down. An increase in a private practice price increases demand for a public practice less than without waiting times. Moreover, the congestion externality is larger, the larger the own-waiting time derivative of the public practice. If there is more than one public practice in a market the $\frac{\partial s_{jt}}{\partial p_{kt}}$ depends also on other public practices' cross-price derivative. In that case, I turn each public practice-specific expression of equation 13 into a linear system of equations and solve it numerically. See appendix A.1 for the derivations of cross-price derivatives of public practices and own-price derivatives of private practices.

5 Estimation and Results

I estimate the parameters of consumer demand using maximum likelihood and two-stage least squares, marginal costs using the first-order conditions of the private practices problem, and parameters of the waiting time model using two-stage least squares.

5.1 Estimating Demand

My consumer-level visit data allows me to estimate demand using a simple two-step procedure.[13] In the first-step I estimate the mean utilities δ_{jt}s as fixed effects and the preference heterogeneity parameters θ in

$$\mathcal{P}_{ijt} = \frac{\exp(\delta_{jt} + \mu_{ijt}(x_{jt}, z_{it}, h_{ijt}; \theta))}{1 + \sum_{k=1}^{J} \exp(\delta_{kt} + \mu_{ikt}(x_{kt}, z_{it}, h_{ikt}; \theta))}, \tag{14}$$

using maximum likelihood.[14] As prices and waiting times as well as the unobservable quality and unobservable private practice waiting times are kept constant in this step as they are in δ_{jt}, there is no endogeneity problem even though the $\mu_{ijt}(x_{jt}, z_{it}, h_{ijt}; \theta)$ contains interactions between prices and waiting times and consumer characteristics.

In the second step, I estimate the mean effects of the product characteristics, prices and waiting times, using 2SLS and the estimates for the practice-market-specific mean utility, $\widehat{\delta_{jt}}$, obtained in the first step.

$$\widehat{\delta}_{jt} = x_{jt}^{\intercal}\beta + \xi_{jt} \tag{15}$$

where x_{jt} contains prices and waiting times. As almost all variation in prices comes from private practices and all variation in waiting times comes from public practices, I estimate the price parameter using private practices and the waiting time parameter using public practices.

For private practices prices are endogenous. In addition to prices being determined in the equilibrium, any correlation between unobservable quality and prices leads to an endogeneity problem. I instrument for prices with the number of dentists working at the public practices in the same municipality where the practice is located. This instrument is a combination of a BLP instrument and a cost-shifter. The number of public practice dentists is a measure of

[13] Compared to a Berry et al. (1995) style demand estimation with market-level data, I do not need to recover δ_{jt} using BLP inversion that requires a standard error correction (Grieco et al., 2023). Moreover, I do not need to use nonlinear IV estimators which are vulnerable to a weak instrument problem (Gandhi and Houde, 2023).

[14] This is a conditional logit model with around 1500 δ_{jt}s that I estimate with Stata's asclogit command.

the capacity in the public sector, and the larger the capacity larger the competitive pressure on the private practices in the market is. Additionally, the more there are public practice dentists, the easier it is for the private practice to hire dentists, and thus private practices' marginal costs should be lower when there are many public practice dentists.

BLP instruments are widely used in the industrial organization literature to instrument for prices and for demand (Berry and Haile, 2021; Gandhi and Houde, 2023). They affect prices, as practices can set higher prices for equally desirable products that are more isolated in the product characteristics space. BLP instruments satisfy the exclusion restriction because the number of competitors and other practices' characteristics do not enter the consumer's indirect utility for practice j. Finally, other product characteristics than prices are commonly assumed to be exogenous, in the sense that practice is not choosing them in the short term. Cost shifters are instrumental variables that affect prices through practices' marginal costs and need to be mean independent of unobservable quality, ξ_{jt}. I can also introduce fixed effects to control for unobserved quality and time-varying demand shocks. However, I need to consider the trade-off between the fixed effects and variation left in the instrumental variables after controlling for the fixed effects. Introducing too many fixed effects can lead to a weak instrument problem.

For public practices waiting times are endogenous as they are determined in the equilibrium together with demand and prices. Moreover, endogeneity could also arise from any unobserved changes in quality, like improvements in premises that would affect demand. I instrument for the waiting times with the number of private practice dentists working in the same municipality where the practice is located. Again, the instrument is a combination of a BLP instrument and a cost-shifter. I assume that public practice prices are exogenous, as I am using the price caps set by the government. Instead of including the public practice prices in the model, I include year fixed-effects that absorb the prices as there is only across-year variation in public practice prices.

5.1.1 Demand Estimation Results

Table 4 presents the estimates on how prices and waiting times affect the mean utility. The estimates are in utilities, so magnitudes are not interpretable, but the estimates' signs are. All estimates are statistically significant at the conventional levels. The first two columns of the table show OLS and 2SLS estimates for price. The OLS estimate is negative but small, and the 2SLS estimate is negative and larger in absolute value. This is what we would expect if the endogeneity problem arises from a positive correlation between unobservable quality and prices. The last two columns show the estimates for waiting times. OLS estimate of the waiting time is negative and the 2SLS estimate of the waiting time is more negative than the OLS estimate. Again, this is as expected because we should expect a positive correlation between waiting times and unobservable quality.

Table 5 displays preference heterogeneity estimates. The estimates are of the expected sign: consumers with higher income are less sensitive to price, but at a decreasing rate, while higher income consumers dislike waiting more and at an increasing rate. Moreover, higher income and older consumers are more likely to visit a dental practice but at a decreasing rate. All consumers dislike practices that are far, but the effect decreases in distance.

With demand estimates at hand, I can calculate elasticities with respect to prices and waiting times. Table 6 shows average aggregate and practice-level own-price and own-waiting time elasticities calculated by sector. The elasticities in the table do not account for the congestion externality and thus should be interpreted as lower bounds, as the congestion externality reduces the absolute value of the elasticity. The average aggregate price elasticity measures how consumers' decision to use any dental care depends on prices. I find an estimate of -0.84, so demand for dental care is inelastic and a one percent increase in dental care price decreases demand for dental care by 0.84%. This is a large estimate compared to the previous studies on health care markets, for example, a common benchmark for the price elasticity of the demand for medical care is -0.2 from the Rand Health Insurance Experiment (Aron-Dine et al., 2013). A major reason for the difference in the estimates is that in my setting there is no coinsurance[15] and no cap for the out-of-pocket expenditures, so consumers pay the price increase in full, which is reflected in the price elasticity. For example, Keeler and Rolph (1988)'s analysis of Rand Health Insurance Experiment data results in elasticity estimates of -0.12 and -0.39 for medical spending on dental care with 0%–25% and 25%–95% coinsurance, respectively.[16] I find sector-specific aggregate price elasticities of -1.64 for the private sector and -1.30 for the public sector.

The average practice-level own-price elasticity is -4.68, meaning that a one percentage increase in price decreases the practice's demand by 4.68%. Private practices' elasticities are on average higher, with a mean of -5.83, while public practices' elasticities are lower, with a mean of -3.47. The private practices' own-price elasticity is comparable to Gowrisankaran et al. (2015)'s own-price elasticity of hospitals when consumers do not have health insurance. The elasticity is lower for public practices, as they have substantially lower out-of-pocket prices. Average cross-price elasticities are small and vary between 0.14 and 0.19 across within and cross-sector elasticities.

I can only calculate aggregate waiting time elasticity for public practices as private prac-

[15] With coinsurance consumer pays a percentage share of their expenses, for example with coinsurance of 20% consumer pays 20% of expenses themselves.

[16] More on the difficulties of obtaining price elasticities from the Rand Health Insurance Experiment see (Aron-Dine et al., 2013).

tices waiting time is zero. I find an aggregate waiting time elasticity of -1.01 for the public sector. There is only a small amount of work estimating aggregate waiting time elasticities. The literature on hospitals typically finds an elasticity in the range of -0.1 and -0.2, see for example (Martin et al., 2007; Sivey, 2012; Riganti et al., 2017). The difference in the estimates likely stems from a difference in the costs of waiting between care given in a hospital and dental care. Practice-level own-waiting time elasticity is -2.71, which is three times larger than a recent estimate for the US veterans choosing a physician's practice by Russo (2024). A reason for the difference is that Russo (2024)'s sample's consumers are on average 61 years old while my sample's consumers are on average 53 years old. My sample is thus likely to have fewer pensioners who have a lower opportunity cost of time. Moreover, it could be that waiting for a physician's visit is more costly than waiting for a dental care visit. Average cross-waiting time elasticities are small. The average within-sector cross-waiting time elasticity is 0.11 for public practices and across-sector cross-waiting time elasticity is 0.13.

The elasticities also vary by income. Figure 2 shows average own-price elasticity by five income bins. We can see from the figure that the own-price elasticity decreases from -5.35 to -4.98 as we move from the lowest bin to the highest. Interestingly own-waiting time elasticity increases from -2.75 to -3.32 between the lowest and highest bins. Thus, the heterogeneity in own-elasticity is more pronounced for the waiting times than for the prices, a likely outcome given the difference between how disutility for price and waiting times change as income increases as can be seen from the preference heterogeneity estimates in table 5. Disutility for waiting times increases at an increasing rate as a function of income, while the disutility for price decreases at a decreasing rate as a function of income. Finally, we should expect this pattern of heterogeneity as we might expect high-income consumers to have a low opportunity cost of money and high opportunity cost of time, while low-income consumers are more likely to have a high opportunity cost of money and low opportunity cost of time.

5.2 Backing Out Marginal Costs

I follow a common practice and use private practices' first-order conditions to back out the marginal costs.

$$mc_{jt} = p_{jt} - q_{jt}(p_t, b_t(p_t)) \left[- \frac{\partial q_{jt}(p_t, b_t(p_t))}{\partial p_{jt}} \right]^{-1} \tag{16}$$

As everything on the right-hand side of the equation is either data or estimated, I know the left-hand side of the equation. I assume that marginal costs are constant, which is a common simplifying assumption in the industrial organization literature. Table 8 shows summary statistics on backed-out marginal costs and compares them to prices. The average marginal cost is €203.27 and as the average price is €233.73, the average markup is €30.5 and the average markup percentage is 13%. The markups are on average quite low, indicating that the competition is relatively fierce. For example, my estimates are lower than markups for for-profit (26%) and not-for-profit (20%) hospitals by Gaynor and Vogt (2003). The competitiveness of the industry is at least partly explained by the lack of coinsurance and out-of-pocket cost maximum.

5.3 Estimating Public Practice Waiting Time Equation

I estimate the waiting time equation using 2SLS.

$$b_{jt} = \psi_1 \frac{q_{jt}}{d_{jt}} + \tau_y + \omega_{jt}, \quad j \in J^{pub}. \tag{17}$$

The challenges in estimating this equation are that consumer demand, the numerator of demand per dentists $\frac{q_{jt}}{d_{jt}}$, is endogenous and determined simultaneously with demand in the equilibrium. Moreover, the measure for the number of dentists, the number of public sector dentists working in the municipality, is a noisy measure of dentist capacity at the public practices. I solve the endogeneity of demand by using a BLP instrument: the difference to the nearest practice from practice j. Instrumenting for the $\frac{q_{jt}}{d_{jt}}$ also solves the measurement error problem.

I expect that the average waiting time should increase when the demand for the practice relative to the number of dentists at the practice increases. The number of patients a dentist can treat is assumed to be constant, and thus an increase in demand should result in higher waiting times. Table 9 presents the estimates of the queuing model. Using OLS, I find that increasing treatment episodes per dentist at a public practice reduces waiting times. This is the likely direction of the bias as a positive waiting time shock increases waiting times, and as consumers dislike waiting, consumer demand and positive waiting time shock are negatively correlated. Thus, the OLS estimate is biased downwards. When I turn to the instrumental variable specification I obtain a statistically significant positive estimate of 0.0692. Now increasing the treatment episodes per dentist by one increases a practice's average waiting time by 0.07 days. As I add year-fixed effects to the specification, standard errors increase, but the estimate is unchanged. I the instrumental variables specification with year-fixed effects in the counterfactuals.

6 Counterfactual Analysis

I am interested in how capacity increase in public practice affects public practices' waiting times, private practices' prices, and the distribution of consumer surplus. Moreover, I ask what the marginal cost of the public practices needs to be so that the increase in capacity would increase total welfare.[17] In the counterfactuals, I increase the capacity of the public practices by increasing the number of dentists at the public practices by 20%.[18] This is equal to an increase of 226 dentists in total in the markets in my sample. If the policy was implemented in the whole of Finland, it would mean an increase of approximately 500 dentists. To put these numbers in context, 182 dentists graduated from Finnish dental schools in 2020. The counterfactual corresponds to a policy that would lengthen the mandatory six-month practical training that is a part of the dental school curriculum in Finland. Counterfactuals are calculated for the year 2015.[19]

When calculating the counterfactual equilibrium I need to calculate three objects: the counterfactual market shares, the counterfactual waiting times, and the counterfactual prices. A complication is that public practice market shares are an implicit function as they depend on waiting times and waiting times depend on market shares. So for every new set of equilibrium prices, I need to solve for the public practice market shares as a fixed point. This raises the issue of the existence and uniqueness of equilibria of the public practice market shares. Fortunately, Bayer and Timmins (2005) prove the existence and uniqueness of equilibrium for logit models in the presence of congestion effects, which in my context requires that consumers dislike waiting. Thus, there exists a unique equilibrium in public practice market shares.

6.1 Market Outcomes, Consumer Surplus, and Profits

Figure 3a shows the percentage change in the mean waiting time and mean private practice prices when public practice capacity is increased by 20%. The average waiting time decreases by 5% or by 1.5 days against the baseline mean of 29.45 days, while the average price stays practically constant. Figure 3b explains the lack of effect on private practice prices. The figure shows how the average percentage point changes in the market share of public and private practice as well as not visiting after the capacity increase. The market share of the public practices increases on average by 2.6 percentage points while the private practice

[17] I am ignoring the cost of obtaining the additional dentists in these calculations, and I assume that public and private sectors' dentists' welfare does not change for the sake of simplicity.

[18] I multiply the number of dentists in the public practices by 1.2.

[19] The public practice treatment bundle costs €105 and the National Health Insurance reimbursement is €60 for the private sector treatment bundle.

market share decreases by 0.4% percentage points and not visiting share decreases by 2.2 percentage points. Most of the increase in the public practices' market shares comes from the outside good. As private practices' market share is decreases only by 2.7%, private practices do not need to defend their market share by lowering their prices.

Figure 5a shows that the total consumer surplus increases by 0.2% and total profits go down by 3%. Profits decrease largely because private practices lose market share to public practices. Figure 5b shows that the euro increase in total consumer surplus is similar to the euro decrease in private practices' profits. Total consumer surplus increases by €172 thousand while private practices' profits decrease by €155 thousand. The increase in public practice capacity moves surplus from private practices to consumers. Figure 6a shows that the capacity increase has a surprisingly even effect on the consumer surplus of consumers with different incomes. It seems that consumers with middle incomes benefit the most, but the percentage difference between the highest increase in the consumer surplus, the middle-income bin, and the lowest-income bin is 7.2%. Surprisingly, the consumer surplus for the lowest-income consumers increases the least. Figure 6b shows the percentage change of consumer surplus across different age bins. The youngest consumers' consumer surplus increases the most: 0.05 percentage points and 19% more than the worst-off third-age bin.

6.2 Is the Capacity Increase Likely to be Welfare Improving?

Even though I am not able to calculate the change in the total welfare as I do not have an estimate of public practices' marginal costs, I can evaluate whether the increase in the public practice dentists is likely to be welfare increasing. The expression for the change in total welfare after the capacity increase is, ignoring dentists,

$$\Delta\pi + \Delta\mathrm{CS} - \Delta Q^{Pri} r_y + \Delta Q^{pub} \times (p^{pub} - mc^{pub}) > 0. \tag{18}$$

The first term, $\Delta\pi$, is the change in the private practice profits, and the second term, $\Delta\mathrm{CS}$ is the change in consumer surplus. $\Delta Q^{Pri} r_y$ represents how the payments of National Health Insurance reimbursements change as the demand for private practices changes. Finally, there is the increase in the number of consumers treated by public practices, times the out-of-pocket price minus the unknown public practice marginal costs. Using this equation, I get an expression for the upper bound of the public practice marginal costs such that the change in total welfare is positive

$$\frac{\Delta\pi + \Delta\mathrm{CS} - \Delta Q^{Pri} r_y}{\Delta Q^{pub}} + p^{pub} > mc^{pub}. \tag{19}$$

When I plug in the numbers, I find that the public practice's marginal cost must be less than €118.5, €118.5 $> mc^{pub}$. The first term of the expression states that increase in total consumer surplus and reimbursement savings outweigh the private practices' loss of profits by €10.51 per a new consumer treated at the public practices.

$$€10.51 + €108 > mc^{pub}. \tag{20}$$

I can compare this condition to my estimates of the marginal costs of private practices. The 25th percentile of private practices' marginal costs is €174.88, which is €56.4 more than the upper bond for the public practice marginal cost. Usually, we assume that private practices are more efficient and have lower marginal costs, so it seems unlikely that public practice marginal costs would be 38.4% lower than the 25th percentile of the private practice marginal costs. Thus, I conclude that the increase in public practice capacity is unlikely to improve total welfare.

6.3 Discussion

The aim of the counterfactual is to evaluate how the capacity increase in public practice affects public practices' waiting times, private practices' prices, and the distribution of consumer surplus. I find that a sizable 20% increase in the public practices' dentists reduces waiting times by only 5% or 1.5 days. Most, of the capacity increase seems to be reflected in the market share of the public practices, which increases by 13%. The root source of the modest decrease in waiting times is that the demand that an individual public practice faces is quite elastic to waiting times, with an average of -2.71. Thus, as waiting times initially decrease due to the capacity increase, the demand increases a lot, which brings the waiting times back up with only a modest reduction in the average waiting times. Thus, it is difficult to reduce waiting times with capacity increases when the demand is elastic to waiting times.[20]

The small decrease in waiting times is one reason why private practices do not reduce prices. As high-income consumers are more likely to visit private practices and are very sensitive to waiting times, a small reduction in waiting times is not enough to affect their choices. It appears that private practices have managed to segment the market so that private practices and public practices are relatively distant substitutes, which is similar to what Atal et al. (2024) find for the pharmaceutical industry in Chile where entry of affordable lower quality pharmacies increased the private pharmacies prices. Thus, it seems that the public

[20]Siciliani et al. (2014) mentions that waiting time reductions with supply-side policies might fail if the policy is not accompanied by a way to keep demand under control

practices' capacity increase fails to increase competition among private dental practices.

Finally, my results indicate that the increase in capacity did not improve the position of the lowest-income consumers, but increased the distance between them and middle-income consumers. A likely explanation for why middle-income consumers benefitted more than low-income or high-income consumers is that middle-income consumers are not extremely sensitive to either prices or waiting times, so they are moderately likely to use public practices services but are still sensitive enough to waiting times to benefit from the decrease in the waiting times. Consumers of the lowest age bin likely benefit the most across all age bins, because they are less likely to visit a dentist and when they visit, they visit a public practice. I find that the capacity increase did not shape the distribution of consumer surplus to the benefit of low-income consumers, but it did increase the consumer surplus most for the youngest consumers.

7 Conclusion

Public and private production coexist in many industries, like education, housing, and health care. The rationale for introducing public production alongside private production is that public production can increase competition if the incentives at the market are otherwise lacking and public production can address distributional concerns. As typical industries with both public and private producers produce important services it is important to know how they work. I build a structural model of the Finnish dental care industry that represents a typical health care setting with public and private producers: public production is affordable but congested, and private production is expensive and allows consumers to cut the queue. Consumer demand, waiting times, and private practices' prices are endogenous in the model. To gain insight on how public practices' capacity affects waiting times, and how capacity increases and the change in waiting times affects private practices prices and the distribution of consumer surplus, I perform a counterfactual exercise where I increase the public practices' capacity by 20%. I find that waiting times decrease on average by 5% or 1.5 days and private practices' prices do not change. I find that consumer surplus increases the most for consumers with average incomes and for the youngest consumers. My findings highlight the complex interactions between public and private production that can produce unexpected outcomes, especially in markets where public production is congested. This is particularly important to keep in mind when considering government policies in these markets.

References

Aron-Dine, A., L. Einav, and A. Finkelstein (2013). The rand health insurance experiment, three decades later. *Journal of Economic Perspectives 27*(1), 197–222.

Asker, J. and V. Nocke (2021). Collusion, Mergers, and Related Antitrust Issues. In *Handbook of industrial organization*, Volume 5, pp. 177–279.

Atal, J. P., J. I. Cuesta, F. González, and C. Otero (2024). The economics of the public option: Evidence from local pharmaceutical markets. *American Economic Review 114*(3), 615–644.

Backus, M. (2020). Why is Productivity Correlated with Competition? *Econometrica 88*(6), 2415–2444.

Barros, P. P. and L. Siciliani (2011). Public and private sector interface. In *Handbook of health economics*, Volume 2, pp. 927–1001. Elsevier.

Bayer, P. and C. Timmins (2005). On the Equilibrium Properties of Locational Sorting Models. *Journal of Urban Economics 57*(3), 462–477.

Berry, S., J. Levinsohn, and A. Pakes (1995). Automobile prices in market equilibrium. *Econometrica 63*(4), 841–890.

Berry, S. T. and P. A. Haile (2021). Foundations of Demand Estimation. In *Handbook of industrial organization*, Volume 4, pp. 1–62.

Bhat, U. N. (2008). *An introduction to queueing theory: modeling and analysis in applications*, Volume 36. Springer.

Blomgren, J., T. Maljanen, and L. Virta (2017). Yksityisten sairaanhoitopalveluiden käyttö, kustannukset ja Kela-korvaukset vuosina 2013-2016 – Kuinka paljon Kela-korvausten alentamiset ovat vaikuttaneet palveluiden käyttöön? (in Finnish). Kela Working Papers 121.

Bloom, N., C. Propper, S. Seiler, and J. Van Reenen (2015). The Impact of Competition on Management Quality: Evidence from Public Hospitals. *The Review of Economic Studies 82*(2), 457–489.

Brancaccio, G., M. Kalouptsidi, and T. Papageorgiou (2024). Investment in Iinfrastructure and Trade: The Case of Porst. Technical Report Working Paper 32503, NBER.

Brekke, K. R. and L. Sørgard (2007). Public Versus Private Health Care in a National Health Service. *Health economics 16*(6), 579–601.

Bruni, M. L., C. Ugolini, and R. Verzulli (2021). Should i wait or should i go? travelling versus waiting for better healthcare. *Regional Science and Urban Economics 89*, 103697.

Buchmueller, T., S. Miller, and M. Vujicic (2016). How Do Providers Respond to Changes in Public Health Insurance Coverage? Evidence from Adult Medicaid Dental Benefits. *American Economic Journal: Economic Policy 8*(4), 70–102.

Chartock, B. L. (2023). Quality disclosure, demand, and congestion: Evidence from physician ratings.

Chen, Y., M. Igami, M. Sawada, and M. Xiao (2021). Privatization and Productivity in China. *The RAND Journal of Economics 52*(4), 884–916.

Dinerstein, M. and T. Smith (2021). Quantifying the supply response of private schools to public policies. *American Economic Review 111*(10), 3376–3417.

Elliott, J., G. V. Houngbonon, M. Ivaldi, and P. Scott (2024). Market Structure, Investment and Technical Efficiencies in Mobile Telecommunications.

Finnish Dental Association (2019). Työmarkkinatutkimus 2017. https://www.hammaslaakariliitto.fi/sites/default/files/mediafiles/kuvat/liiton_toiminta/tmt2017_nettiin.pdf.

Finnish Institute for Health and Welfare (2019). Health expenditure and financing 2017. Statistical Report 15/2019, The Finnish Institute for Health and Welfare.

Gandhi, A. and J.-F. Houde (2023). Measuring Substitution Patterns in Differentiated-products Industries. NBER working paper w26375.

Gaynor, M. and W. B. Vogt (2003). Competition among Hospitals. *RAND Journal of Economics 34*(4), 764–785.

Gowrisankaran, G., A. Nevo, and R. Town (2015). Mergers When Prices Are Negotiated: Evidence from the Hospital Industry. *American Economic Review 105*(1), 172–203.

Grieco, P. L., C. Murry, J. Pinkse, and S. Sagl (2023). Conformant and efficient estimation of discrete choice demand models.

Griffith, R., L. Nesheim, and M. O'Connell (2018). Income Effects and the Welfare Consequences of Tax in Differentiated Product Oligopoly. *Quantitative Economics 9*(1), 305–341.

Haaga, T. (2019). Terveyskeskusten asiakasmaksut joulukuussa 2018: Aineisto lääkäri-ja hoitajavastaanottojen asiakasmaksuista 2013-2018 (in Finnish).

Hoel, M. and E. M. Sæther (2003). Public Health Care with Waiting Time: the Role of Supplementary Private Health Care. *Journal of health economics 22*(4), 599–616.

Igami, M. and K. Uetake (2020). Mergers, Innovation, and Entry-Exit Dynamics: Consolidation of the Hard Disk Drive Industry, 1996–2016. *The Review of Economic Studies 87*(6), 2672–2702.

Jimenez-Hernandez, D. and E. Seira (2022). Should the government sell you goods? evidence from the milk market in mexico. Technical report, Federal Reserve Bank of Chicago.

Keeler, E. B. and J. E. Rolph (1988). The demand for episodes of treatment in the health insurance experiment. *Journal of health economics 7*(4), 337–367.

Marchand, M. and F. Schroyen (2005). Can a Mixed Health Care System be Desirable on Equity Grounds? *Scandinavian Journal of Economics 107*(1), 1–23.

Martin, S., N. Rice, R. Jacobs, and P. Smith (2007). The market for elective surgery: Joint estimation of supply and demand. *Journal of health economics 26*(2), 263–285.

Miravete, E. J., K. Seim, and J. Thurk (2024). Elasticity and Curvature of Discrete Choice Demand Models.

Riganti, A., L. Siciliani, and C. V. Fiorio (2017). The Effect of Waiting Times on Demand and Supply for Elective Surgery: Evidence from Italy. *Health economics 26*, 92–105.

Russo, A. (2024). Waiting or Paying for Healthcare: Evidence from the Veterans Health Administration.

Saarela, H.-L., T. Taskinen, and M. Härkönen (2024). Suun terveydenhuollon toimenpideluokitus 2024 (in Finnish).

Saltzman, E. (2023). What Does a Public Option Do? Evidence from California. Working paper.

Siciliani, L., V. Moran, and M. Borowitz (2014). Measuring and Comparing Health Care Waiting Times in OECD Countries. *Health policy 118*(3), 292–303.

Sivey, P. (2012). The effect of waiting time and distance on hospital choice for english cataract patients. *Health economics 21*(4), 444–456.

Stiglitz, J. (1988). *Economics of the Public Sector*. New York: WW Norton & Company.

Strobel, S. (2024). Who responds to longer wait times? the effects of predicted emergency wait times on the health and volume of patients who present for care. *Journal of Health Economics 96*, 102898.

Yee, C., K. Barr, T. Minegishi, A. Frakt, and S. Pizer (2022). Provider Supply and Access to Primary Care. *Health Economics 31*(7), 1296–1316.

Tables and Figures

Table 1: Summary Statistics: Markets

Characteristic	Mean (SD) N = 322
Population (thousands)	21.00 (18.75)
Number of public practices	2.46 (1.87)
Number of private practices	3.24 (2.75)
Share practices belonging to a chain	0.05 (0.11)
Has a private practice	73.9%
Has a public practice	90.1%
Has two practices by the same chain	2.5%
Urban municipalities	21.7%
Semi-urban municipalities	31.4%
Rural municipalities	46.9%
Unique geographic areas	101

Note: Calculated for the included markets and for years 2014–2017. An observation is a market and a market is a combination of municipality or municipality cooperation area and a year. A unique geographic area is the geographic part of the market definition, so either a municipality or municipality cooperation area.

Table 2: Summary Statistics: Practices

Characteristic	Public practices N = 713	Private practices N = 771
Market share excluding outside good	0.30 (0.31)	0.14 (0.15)
Total visits	4,349.84 (3,944.04)	2,857.16 (1,964.11)
Mean out-of-pocket price (€)	119.37 (13.57)	180.11 (47.21)
Mean first visit waiting time (days)	31.71 (15.77)	0.00 (0.00)
Mean within-episode waiting time (days)	39.67 (16.06)	0.00 (0.00)
Total yearly consumers	1,297.44 (1,081.71)	1,073.91 (727.09)
Total yearly episodes	1,433.61 (1,202.43)	1,304.45 (901.14)
Total yearly procedures	9,289.84 (8,887.41)	4,589.74 (3,199.63)
Mean episode complexity	14.35 (2.98)	10.48 (2.28)
Belongs to a chain	0.0%	12.5%
Unique practices	218	256

Note: Standard deviation in parenthesis. Calculated for the included markets and for years 2014–2017. One observation is a dental practice in a market, where the market is a combination of a municipality or a municipality cooperation area and a year. Episode complexity refers to how time-consuming a given treatment episode is. It takes into account the number of procedures and how long performing each procedure takes. Procedure-specific complexity (Vaativuus in Finnish) is defined in Saarela et al. (2024).

Table 3: Summary Statistics: Consumers

Characteristic	All	Has a public visit	Has a private visit	No visits
Age	52.46 (15.74)	50.63 (15.491)	54.28 (14.425)	53.71 (17.14)
Yearly net income (€1000s)	28.48 (15.64)	27.75 (13.718)	32.50 (18.321)	25.02 (13.72)
Episodes per year	0.51 (0.68)	0.63 (0.695)	0.88 (0.746)	0.00 (0.00)
More than one episode per year	8.7%	10.0%	18.2%	0.0%
Unique consumers	524,769	253,667	185,143	141,906

Note: Standard deviation in parenthesis. Calculated for a 40% random sample of 25–80 years old consumers residing in included markets and for years 2014–2017. One observation is a consumer-year combination. "Has a public visit" means that a consumer has visited a public practice between 2014–2017. Similarly "Has a private visit" means that a consumer has visited a private practice between 2014–2017. If a consumer has visited both public and private practice between 2014–2017, then they are included in both columns. "No visits" means that a consumer has not visited either a public or a private practice between 2014–2017.

Table 4: Results: Mean Utility

Dependent Variable:	$\widehat{\delta}_{jt}$					
Sample:	Private Practices			Public Practices		
Estimator:	OLS	OLS	IV	OLS	OLS	IV
Constant	-4.613***			-3.501***		
	(0.1266)			(0.0734)		
Price	-0.0041***	-0.0047***	-0.0378***			
	(0.0007)	(0.0006)	(0.0006)			
Waiting Time (days)				-0.0138***	-0.0141**	-0.0876***
				(0.0021)	(0.0037)	(0.0066)
N	771	771	771	780	780	780
Year FE	No	Yes	Yes	No	Yes	Yes
F-test (1st stage)			97.099			124.39

Notes: Significance thresholds for stars are: *** for 0.01, ** for 0.05 and * for 0.1.

Table 5: Results: Preference Heterogeneity

Variable	Estimate	Standard Errors
Price × Income (Thousands)	1.160e-04	5.690e-06
Price × Income^2	-4.140e-07	5.140e-08
Waiting Time × Income	-1.749e-04	1.520e-05
Waiting Time × Income^2	-1.260e-06	1.640e-07
Outside Good × Income	-6.628e-03	1.095e-03
Outside Good × Income^2	8.030e-05	1.030e-05
Outside Good × Age	-8.272e-02	7.880e-04
Outside Good × Age^2	6.948e-04	7.490e-06
Distance	-9.641e-02	3.615e-04
Distance^2	3.756e-04	8.170e-06

Note: 16.4 million consumer-alternative observations. The estimates are from a conditional logit model with 1,484 alternative specific constants or δ_{jt}s.

Table 6: Results: Aggregate Elasticities and Practice-level Own-elasticities

Elasticity	Variable	Sector	Mean elasticity	SD
Aggregate elasticity	Price	Both	-0.84	
Aggregate elasticity	Price	Private	-1.64	
Aggregate elasticity	Price	Public	-1.30	
Aggregate elasticity	Waiting time	Public	-1.01	
Practice-level elasticity	Price	Both	-4.68	1.60
Practice-level elasticity	Price	Private	-5.83	1.48
Practice-level elasticity	Price	Public	-3.47	0.31
Practice-level elasticity	Waiting time	Public	-2.74	1.31

Note: Aggregate elasticity corresponds to how the extensive margin of visiting a dental practice varies as price or waiting time changes, and is calculated as $\sum_j^{J_t} \frac{\partial s_j}{\partial p_j} \frac{p_j}{s_j} \frac{s_j}{s^{pri}+s^{pub}}$, where s^{pri} is the total market share of private practices in a given market.

Table 7: Results: Practice-level Cross-elasticities

Variable	Sector	Within sector	SD	Across sector	SD
Price	Private	0.17	0.15		
Price	Public	0.14	0.14		
Price	Public			0.16	0.18
Price	Private			0.19	0.16
Waiting time	Public	0.11	0.13		
Waiting time	Public			0.13	0.17

Note: Cross-elasticities correspond to how the demand for practice changes when a characteristic of another practice changes.

Table 8: MCs and Prices

Variable	Mean	SE	25th percentile	Median	75th percentile
MC	203.27	3.36	174.88	200.82	234.16
Price	233.73	3.37	204.70	231.61	265.32

Note: Marginal costs are obtained from the private practices' first-order conditions. Price includes the National Health Insurance reimbursement, and is thus higher than the out-of-pocket price.

Table 9: Results: Queuing Model

Dependent Variables:	Yearly Average Waiting Time		
Estimator:	OLS	IV	IV
Constant	32.92***	0.4706	
	(0.9824)	(8.693)	
Demand Per Dentists	-0.0023	0.0692***	0.0692*
	(0.0018)	(0.0191)	(0.0268)
N	713	713	713
Year FE	No	No	Yes
IV: Distance to the Nearest Competitor			
F-test (1st stage)		20.634	21.117

Notes: Significance thresholds for stars are: *** for 0.01, ** for 0.05 and * for 0.1.

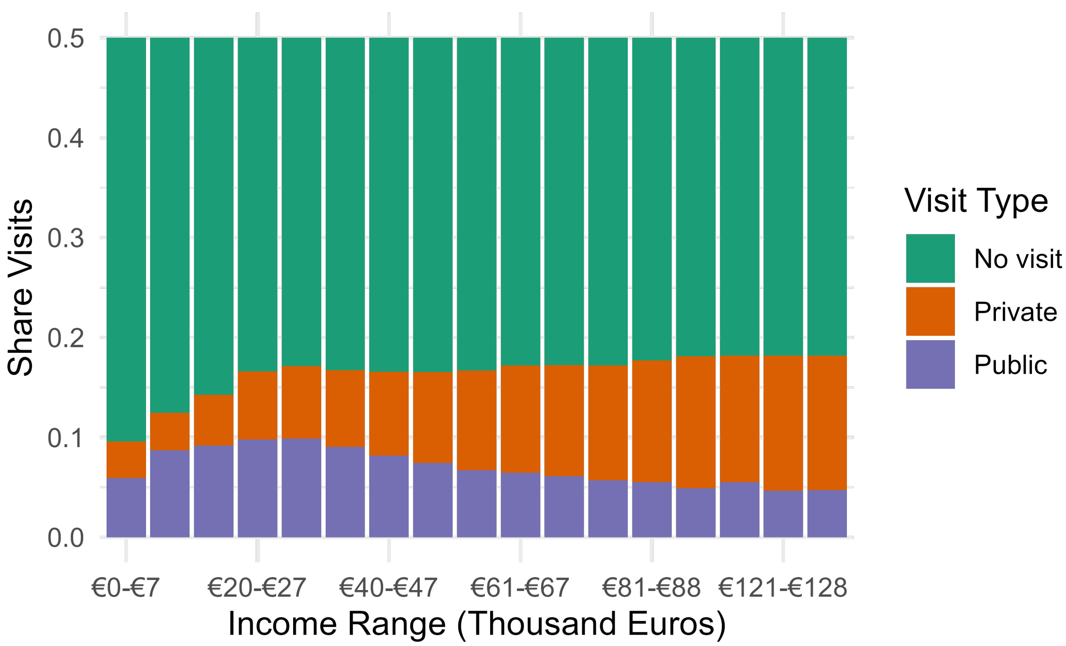


Figure 1: Consumers' Practice Choices by Income Range

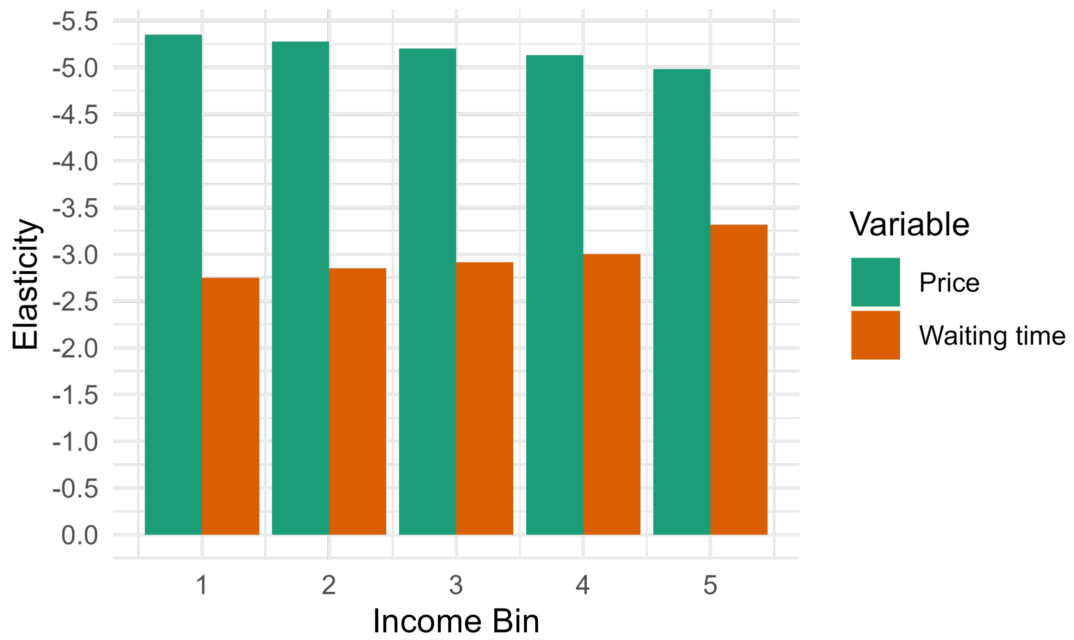
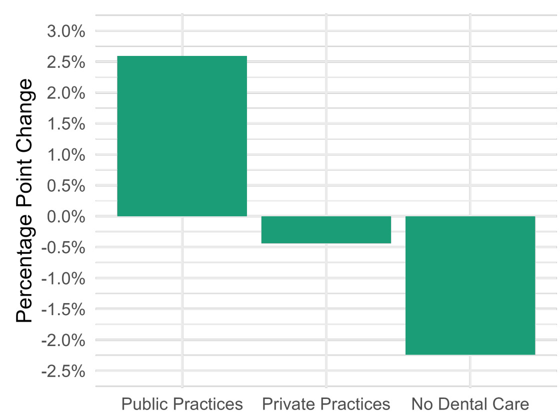


Figure 2: Consumers' Average Own-elasticities by Income Bins

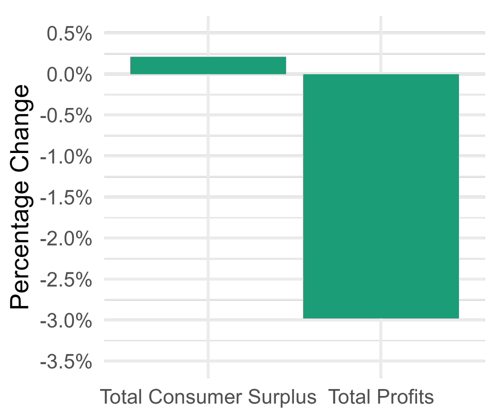
(a) Changes in Prices and Waiting Times



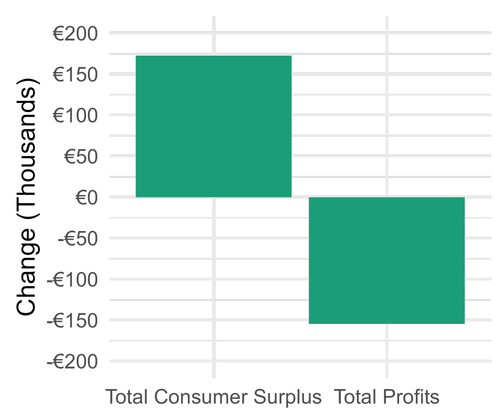
(b) Changes in Market Shares

Figure 3: Counterfactuals: Waiting Times, Prices and Market Shares

Figure 4: Counterfactuals: Percentage Change in Average Market Share

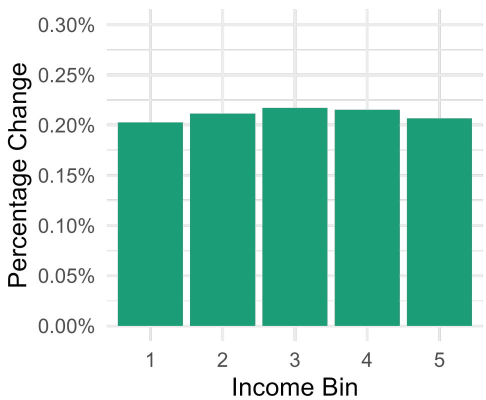


(a) Percentage Changes

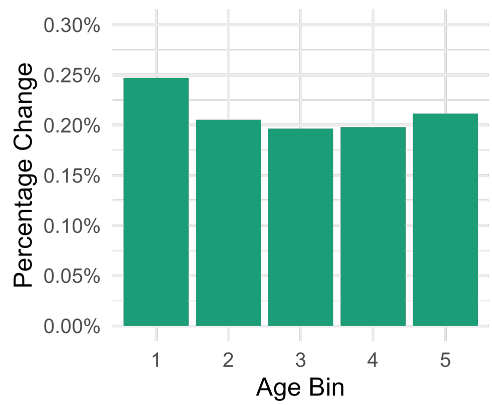


(b) Absolute Changes

Figure 5: Counterfactuals: Total Consumer Surplus and Profits



(a) Income Bin



(b) Age Bin

Figure 6: Counterfactuals: Change in Consumer Surplus by Income and Age

Appendices

A.1 Derivatives

Derivatives of the logit model with respect to prices and waiting times are more complicated than usually, because the expression for the market share of a public practice is an implicit function that depends on the public practice market share through the waiting time equation. To solve for the own-price derivatives of private practices that I need for private practice's first-order condition, I first solve for cross-price derivatives of public practices and use those to solve for the own-price derivatives of private practices. I also solve for own-price derivatives of public practices.

A.1.1 Cross-Price Derivatives for Public Practices

In a given market the market share for the public practice k is an implicit function.

$$s_{kt} = \frac{1}{I_t}\sum_i^I \mathcal{P}_{ik} = \frac{1}{I_t}\sum_i^I \frac{\exp(\delta_{kt}(s_{kt}) + \mu_{ijt})}{1 + \sum_{h=1}^{J^{pri}} \exp(\delta_{ht} + \mu_{iht}) + \sum_{c=1}^{J^{pub}} \exp(\delta_{ct}(s_{ct}) + \mu_{ict})}$$

The first step of implicitly differentiating these equations, taking a derivative with respect to the p_{jt} and solving for the derivative, which gives the following.

$$\frac{\partial s_{kt}^{pub}}{\partial p_{jt}} = \frac{-\sum_i^I [P_{ik}(\frac{\partial u_{ijt}}{\partial p_{jt}} P_{ij} + \sum_{o \neq k}^O \frac{\partial u_{iot}}{\partial b_{ot}} \frac{\partial b_{ot}}{\partial s_{ot}} \frac{\partial s_o}{\partial p_j} P_{io})]}{\sum_i^I \left[-\frac{\partial u_{ikt}}{\partial b_{kt}} \frac{\partial b_{kt}}{\partial s_{kt}} P_{ik}(1 - P_{ik}) + 1 \right]}$$

This can be manipulated into a linear equation in the derivative of market share with respect to the price. First, multiply the equation by the denominator on the right-hand side.

$$\frac{\partial s_{kt}^{pub}}{\partial p_{jt}} \left(\sum_i^I \left[-\frac{\partial u_{ikt}}{\partial b_{kt}} \frac{\partial b_{kt}}{\partial s_{kt}} P_{ik}(k - P_{ik}) + 1 \right] \right) = -\sum_i^I \left[P_{ik} \left(\frac{\partial u_{ijt}}{\partial p_{jt}} P_{ij} + \sum_{o \in J^{pub}, o \neq k} \frac{\partial u_{iot}}{\partial b_{ot}} \frac{\partial b_{ot}}{\partial s_{ot}} \frac{\partial s_o}{\partial p_j} P_{io} \right) \right]$$

Rearrange

$$\begin{aligned} \sum_i^I P_{ik} \left(\frac{\partial u_{ijt}}{\partial p_{jt}} P_{ij} \right) = & -\frac{\partial s_{kt}^{pub}}{\partial p_{jt}} \left(\sum_i^I \left[-\frac{\partial u_{ikt}}{\partial b_{kt}} \frac{\partial b_{kt}}{\partial s_{kt}} P_{ik}(1 - P_{ik}) + 1 \right] \right) \\ & - \sum_i^I \left[P_{ik} \left(\sum_{o \in J^{pub}, o \neq k} \frac{\partial u_{iot}}{\partial b_{ot}} \frac{\partial b_{ot}}{\partial s_{ot}} \frac{\partial s_o}{\partial p_j} P_{io} \right) \right] \end{aligned}$$

Notice that we know the left-hand side, as it is just data and a function of demand estimates. On the right-hand side, the equation is linear in all derivatives of all public practice market shares in market t with respect to price p_j. I can stack these equations for all public practices and solve the resulting system of equations numerically to obtain the cross-price derivatives for the public practices.

A.1.2 Own-Price Derivatives for Public Practices

Own-price derivatives can be obtained using implicit differentiation and the expression for public practices market shares. First, differentiate with respect to p_{jt}.

$$\frac{\partial s_{jt}}{\partial p_{jt}} = M_t \frac{1}{I_t} \sum_{i \in I_t} \left(\frac{\partial u_{ijt}(b_{jt}(s_{jt}))}{\partial p_{jt}} \mathcal{P}_{ijt}(1 - \mathcal{P}_{ijt}) - \sum_{k \in J_t^{pub} \setminus j} \left[\frac{\partial u_{ikt}}{\partial s_{kt}} \frac{\partial s_{kt}}{\partial p_{jt}} \mathcal{P}_{ijt} \mathcal{P}_{ikt} \right] \right)$$

Then solve for $\frac{\partial s_{jt}}{\partial p_{jt}}$. Note that $\frac{\partial u_{ijt}(b_{jt}(s_{jt}))}{\partial p_{jt}}$ is a function of $\frac{\partial s_{jt}}{\partial p_{jt}}$ because of the waiting times.

$$\frac{\partial s_{jt}}{\partial p_{jt}} = -\frac{\sum_i^I \left[\frac{\partial u_{ijt}(p_{jt})}{\partial p_{jt}} (1 - P_{ijt}) P_{ijt} - \sum_{k \in J_t^{pub} \setminus j} \frac{\partial u_{ikt}}{\partial b_{jt}} \frac{\partial b_{kt}}{\partial s_{kt}} \frac{\partial s_{kt}}{\partial p_{jt}} P_{ikt} P_{ijt} \right]}{\sum_i^I \left[\frac{\partial u_{ijt}}{\partial b_{jt}} \frac{\partial b_{jt}}{\partial s_{jt}} (1 - P_{ijt}) P_{ijt} + 1 \right]}$$

Rearrange.

$$\begin{aligned} \sum_i^I \left[\frac{\partial u_{ijt}(p_{jt})}{\partial p_{jt}} (1 - P_{ijt}) P_{ijt} \right] &= -\frac{\partial s_{jt}}{\partial p_{jt}} \sum_i^I \left[\frac{\partial u_{ijt}}{\partial b_{jt}} \frac{\partial b_{jt}}{\partial s_{jt}} (1 - P_{ijt}) P_{ijt} + 1 \right] \\ &\quad + \sum_{k \in J_t^{pub} \setminus j} \left[\sum_i^I \frac{\partial u_{ikt}}{\partial b_{jt}} \frac{\partial b_{kt}}{\partial s_{kt}} \frac{\partial s_{kt}}{\partial p_{jt}} P_{ikt} P_{ijt} \right] \end{aligned}$$

Then we can form a linear system of equations consisting of the following for the public practices $j \in J^{pub}$.

$$\begin{aligned} \sum_i^I \left[\frac{\partial u_{ijt}(p_{jt})}{\partial p_{jt}} (1 - P_{ijt}) P_{ijt} \right] &= -\frac{\partial s_{jt}}{\partial p_{jt}} \sum_i^I \left[\frac{\partial u_{ijt}}{\partial b_{jt}} \frac{\partial b_{jt}}{\partial s_{jt}} (1 - P_{ijt}) P_{ijt} + 1 \right] \\ &\quad + \sum_{k \in J_t^{pub} \setminus j} \left[\sum_i^I \frac{\partial u_{ikt}}{\partial b_{jt}} \frac{\partial b_{kt}}{\partial s_{kt}} \frac{\partial s_{kt}}{\partial p_{jt}} P_{ikt} P_{ijt} \right] \end{aligned}$$

As we know the cross-price derivatives of public practices, the only unknowns are the own-price derivatives and we can solve them numerically.

A.1.3 Own-Price Derivatives for Private Practices

Similarly, as for the public practices, I can derive the expression for the own-price derivative of private practice j.

$$\sum_{i}^{I}\left[\frac{\partial u_{ijt}(p_{jt})}{\partial p_{jt}}(1-P_{ijt})P_{ijt}\right] = -\frac{\partial s_{jt}}{\partial p_{jt}} + \sum_{k\in J_t^{pub}\setminus j}\left[\sum_{i}^{I}\frac{\partial u_{ikt}}{\partial b_{jt}}\frac{\partial b_{kt}}{\partial s_{kt}}\frac{\partial s_{kt}}{\partial p_{jt}}P_{ikt}P_{ijt}\right]$$

By stacking these equations for all private practices in a market, I can solve for the own-price derivatives from the resulting linear system of equations numerically, as I know the cross-price derivatives of public practices.