Anyons

Quantum particles with fractional statistics

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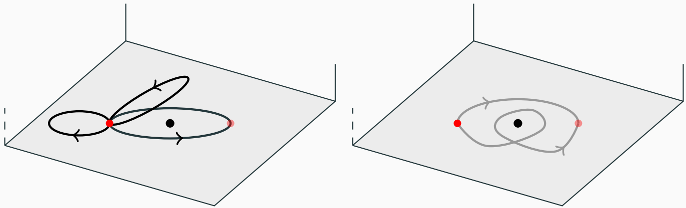
Bosons and Fermions

$M_N^{(d)}$: configuration space of N particles in d spatial dimensions.

$$M_2^{(d)} = \left(\mathbb{R}^d \setminus \{0\}\right)/S_2$$

In $d = 3$ its **fundamental group** is S_2. Be θ the **statistical angle**:

$$e^{i\theta} = \pm 1 \implies \theta = 0 \textbf{ (boson)},\ \pi \textbf{ (fermion)}$$

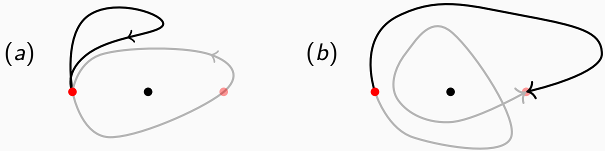


A particle remains fixed at $\bullet$, whereas the other moves. Antipodal points with respect to $\bullet$, like $\bullet$ and $\bullet$, are identified. There are two homotopy classes.

Anyons and fractional statistics

In $d = 2$ the fundamental group is the **braid group** B_2. **Anyons** appear, with **fractional statistical angles**:

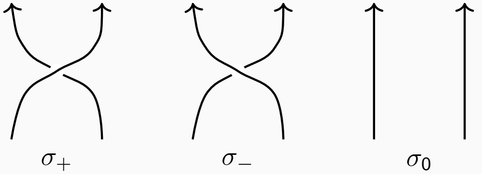
$$\theta \in [0, 2\pi)$$



For $d = 2$, **winding** is well defined, thus homotopy classes are infinite in number.

(a) σ_0 , σ_+^2

(b) σ_-, σ_+^3



Topological phases and flux tube model

If the configurational space is not simply connected, ψ is **polydromous**.

$$\psi \xrightarrow{\gamma(t)\in g} a_g(t)\psi \quad \text{with} \quad a_g(t) = \exp\left(\frac{i}{\pi}\sum_{i<j}\theta_{ij}\ {}_{\gamma}\!\int_0^t dt'\,\dot{\varphi}_{ij}(t')\right)$$

θ_{ij}, φ_{ij} being the (i,j)-couple's and **statistical** and **azimuthal angles**.

$$K(\mathbf{x},t;\mathbf{x}_0,0) = \sum_{g\in\pi_1} a_g(t)\,\overbrace{K_g(\mathbf{x},t;\mathbf{x}_0,0)}^{\text{partial propagators}} = \int_{(\mathbf{x}_0,0)}^{(\mathbf{x},t)} \mathcal{D}[\gamma(t)]\,e^{\frac{i}{\hbar}S}$$

$$L(\mathbf{x},\dot{\mathbf{x}},t) = \overbrace{L_0(\mathbf{x},\dot{\mathbf{x}},t)}^{\text{proper term}} + \overbrace{\frac{\hbar}{\pi}\sum_{i<j}\theta_{ij}\dot{\varphi}_{ij}(t)}^{L_{int}\ \text{(topological term)}} \qquad \textbf{effective lagrangian}$$

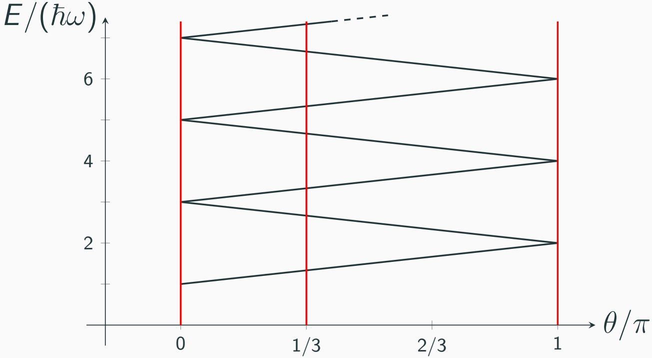
Flux tube model: free anyons are treated like bosons with lagrangian L, carrying **fictitious charge** and **magnetic flux**. L_{int} emerges as flux-charge interaction.

Two harmonically interacting anyons

- **Ansatz** $\psi(r,\phi) = R(r)e^{il\phi}$
- **Anyon statistics** with angle θ: $\psi(r,\phi+\pi) = e^{i\theta}\psi(r,\phi)$

$$\left[\frac{d}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{l^2}{r^2} - \frac{2\mu}{\hbar^2}\left(U(r) - E\right)\right] R(r) = 0, \qquad \begin{cases} l = \theta/\pi + 2k \\ k \in \mathbb{Z} \end{cases}$$

Hence the spectrum is $E_{nk} = \hbar\omega\left(2n + 1 + |\theta/\pi + 2k|\right)$



There and back again

Practical applications of these phenomena seem remote . . .

Wilczek, 1982

- **Anyons exist as quasi-particles** in systems that show **fractional quantum Hall effect** (Laughlin states).
- Significant research in **Topological Quantum Computing**. Anyonic statistics could indeed make available a **fault-tolerant** quantum computation:
  information is stored globally $\longrightarrow$ no local perturbation can alter it

Thanks for your attention

References

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