

An Annotated Timeline of Sensitivity Analysis

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Abstract

The last half a century has seen spectacular progresses in computing and modelling in a variety of fields, applications, and methodologies. Over the same period, a cross disciplinary field known as sensitivity analysis has been making its first steps, evolving from the design of experiments for laboratory or field studies, also called ‘in-vivo’, to the so-called experiments ‘in-silico’. Some disciplines were quick to realize the importance of sensitivity analysis, whereas others are still lagging behind.

Major tensions within the evolution of this discipline arise from the interplay between local vs global perspectives in the analysis as well as the juxtaposition of the mathematical complexification and the desire for practical applicability. In this work, we retrace these main steps with some attention to the methods and through a bibliometric survey to assess the accomplishments of sensitivity analysis and to identify the potential for its future advancement.

1 - Introduction

Models simulate the real-world by synthesizing a multitude of input configurations in their output, mapping potential present and future system’s states of interest. Their primary objective is to extract valuable insights regarding the relationship between inputs and outputs. Defining the nature of mathematical models is not easy, due to the variety of contexts and applications (Page 2018). Various authors identified modelling as an art (Morrison 1991) or a craft (Rosen 1991), with models being performative (Espeland and Stevens 2008), and acting as mediators between theories and the world (Morgan and Morrison 1999). What remains undisputed is the remarkable development realised in computing and modelling in recent decades. Computer models are so widely used in a variety of fields, applications, and methodologies that they are seemingly affecting any aspect of our lives (Morgan and Morrison 1999).

Together with modelling, a new field of research called sensitivity analysis has come to life, moving from the design of experiments for laboratory or field studies to experimental techniques performed by computers, namely the experiments ‘in-silico’. While uncertainty analysis studies the uncertainty in the output, sensitivity analysis studies how the uncertainty in the output can be allocated to the different sources of uncertainty in the input (Saltelli 2002). In other words, sensitivity analysis elucidates the intimate relationship between the system output and its influential factors. It is easy to recognize the strong bond between sensitivity analysis and modelling. For some, sensitivity analysis, namely the drawing of the connection

between model output and relevant input, is the very *raison d'être* of models (Hall and Adams 2020).

Sensitivity analysis can effectively tackle a multitude of issues, serving a dual role in the model development phase as well as during its utilization by users to enhance decision-making processes. Sensitivity analysis serves various purposes, including model validation, dimensionality reduction, prioritization of research efforts, pinpointing critical regions within the space of uncertainties under investigation, and aiding decision-making by quantifying how input variations impact outcome uncertainty (Saltelli et al., 2008; Tarantola et al., 2002).

At present, sensitivity analysis is evolving toward an independent discipline recognised also by institutional guidelines (European Commission 2021; Azzini et al., 2020). However, whilst some disciplines promptly embraced sensitivity analysis, its potential has not yet been exploited in other fields, or its full adoption proceeds with hesitation.

The present concise historical account of sensitivity analysis attempts to chart the evolution of the field and gain insights into its contemporary challenges. In this study, we will prioritize interpretations of sensitivity analysis that emphasize the global exploration of uncertain inputs. This “global” understanding started in the 70s with the pioneering work of Holling et al., 1978 who recognised that “... simultaneous variation of the parameters... ” over a wide range of uncertainty “... is necessary to give reliable results”.

As global sensitivity analysis techniques have advanced in recent decades, becoming capable of handling complex models alongside the growing computational power of computers, user-friendly tools and software have been developed to broaden accessibility for a wide spectrum of researchers and practitioners and contribute to the wider dissemination of the discipline. Nevertheless, the sensitivity analysis panorama is still dominated in practically all disciplines by the so-called “local” approaches. To make an example, in operation research, where the objective is the optimal allocation of tasks and resources, sensitivity analysis is mostly pursued looking at factors one at a time (Hillier, 2014), ignoring possible crucial interactions of factors that may change the optimal solution only when changed jointly, not one at a time.

The outline of the paper is as follows: section 2 traces the significant steps in sensitivity analysis development: milestones which made its advances are underlined starting from the early developments (2.1) to the modern communities (2.3) through the crucial transition to computer experiments (2.2). The following section illustrates the results of the bibliometric analysis conducted to assess the impact of global sensitivity analysis. In section 4, the authors focus on the tension still characterizing different schools of sensitivity analysis. Finally, section 5 summarizes and concludes our investigation.

2 – Evolution of Sensitivity Analysis

Sensitivity analysis has undergone remarkable development over time, achieving several historical milestones that have significantly shaped its evolution. These crucial advancements (see Figure A1), show the progressive journey of sensitivity analysis, and highlight its growing importance as a fundamental tool within various scientific disciplines.

2.1 – Early developments

Sensitivity analysis is, after all, finding things that have effect on a certain phenomenon out of many things that could potentially be causes. So, if we look at sensitivity analysis as to a science of the causes, all the scientific revolution can be taken as anticipating sensitivity analysis. So, one sensitivity author compares Leonardo' experiment leading to laws of sliding friction to an early sensitivity experiment (Razavi et al., 2021). If we remain instead to the realm of the causes that can be discovered in mathematical constructs – rather than in the real, then perhaps a good date to set the start of sensitivity analysis is 1905, when Karl Pearson, the founder of modern statistics, proposed the idea of correlation ratio (known as the η^2 index), to link two variables associated by a non-linear relation.

A further milestone in the development of sensitivity analysis is the formalization of the experimental design in the 1920s and 1930s by the statistician Ronald Fisher. Experimental design is the process of planning and conducting experiments to test a hypothesis, answer a research question, or optimize the use of resources, including measuring or manipulating variables, hence the link with sensitivity analysis. The process whereby statistics managed to adjudicate the authority to assess the 'realism of causes' is well described in the classic book of Desrosières (1998, pp 103-146).

World War 2 provided a significant impetus for the expansion and application of sensitivity analysis within the field of operational research (Gass and Assad 2005; Hillier 2014). During this global conflict, nations were faced with unprecedented challenges in terms of strategic planning, resource allocation, and decision-making. The complexities of managing large-scale military operations, logistics, and supply chains required innovative approaches to optimize resource utilization and maximize efficiency.

Experimental design continued to develop with several important advancements in this field in the 1950s, including the widespread adoption of factorial designs. These designs allowed researchers to investigate the effects of multiple factors or variables on an outcome of interest. In a factorial design, each factor is varied at multiple levels, and the effects of each factor and their interactions are examined, thus allowing to identify the unique effects of each independent variable and to test complex hypotheses.

Another important development in the 1950s was the introduction of the response surface methodology, which provided a way to optimize a response variable influenced by several input or process variables. The response surface methodology involves the use of mathematical models to describe the relationships between the input variables and the response variable, allowing researchers to identify the optimal values for each input variable to achieve the desired response. Notably, the Polynomial Chaos Expansions (PCE) (Wiener, 1938) and Kriging, also called Gaussian Processes (GP) (Matheron, 1963) got some traction.

In the following decades, Cukier and co-workers (Cukier et al., 1978) developed the Fourier amplitude sensitivity test (FAST) in the early 1970s, one of the most elegant methods of sensitivity analysis. In FAST, the sensitivity of model output is computed by using spectral analysis through Fourier transformation of the input parameters. The method was primarily used in chemistry, but its applications extended to engineering (Lee and Cho, 2013), finance (Huang and Tao, 2011) and environmental modelling.

2.2 – Transition to computer experiments

In the 1980s, the advancement of computing resources revolutionized sensitivity analysis and significantly expanded its capabilities. Prior to this period, sensitivity analysis was often limited to manual and analytical techniques, which were practical only for simple models with a few input parameters. However, with the increased computing power, researchers could now conduct sensitivity analyses on complex models that involved numerous input parameters and interactions.

One of the key breakthroughs during this time was the adoption of random sampling techniques. Instead of relying solely on analytical methods, researchers began to generate random samples of input parameters within specified ranges and then execute the model for each combination of these sampled inputs. This process allowed them to explore a vast range of possible input combinations, covering a wide spectrum of parameter values and assumptions. As a result, sensitivity analysis became more comprehensive, enabling the identification of critical factors that significantly influenced the model's behaviour. For a detailed overview of the progress made over these decades, see Myers et al., 1989.

As computing resources continued to advance throughout the 1990s, sensitivity analysis reached another milestone with the pioneering work of the Russian mathematician Ilya M. Sobol'. In 1993, Sobol' introduced an innovative approach to sensitivity analysis based on the decomposition of the output variance (Sobol' 1993). This method, known as Sobol' indices, allowed researchers to quantify the contribution of each input parameter to the variance of the model's output accurately.

The Sobol' indices provided a deeper level of insight into the model's behaviour by quantifying the individual and combined effects of input parameters on the output variance. This method not only allowed researchers to rank the importance of different inputs but also enabled them to identify interactions and nonlinearities between parameters, which were crucial for understanding complex systems.

Over the years, Sobol' sensitivity indices have become a widely used and well-established tool in various scientific domains, including engineering and environmental sciences (Saltelli et al., 2000b). The method's versatility and reliability have contributed significantly to the robustness of sensitivity analysis, making it an essential component in the study of models.

2.3 – The modern communities

Towards the end of the 1990s, a brand-new community of sensitivity-analysis practitioners emerged, reflecting on the concept of “global sensitivity analysis”. This approach involves simultaneously varying model inputs across a wide range of values to uncover interactions between parameters (Saltelli et al., 2000a). Appendix 1 synthesises this concept.

Concomitantly, the concept of ‘uncertainty quantification’ gained traction in various scientific fields like climate modelling (Annan and Hargreaves, 2005), computational physics (Knio and Karniadakis, 2012) and materials science (Hennig et al., 2018), focusing on propagating uncertainties through models to estimate prediction uncertainty.

The global approach to uncertainty quantification and sensitivity analysis garnered interest from institutions and communities worldwide. Efforts from U.S. national laboratories such as Sandia and Los Alamos (Helton 1993; McKay 1997), along with the European Commission's Joint Research Centre (JRC) in Italy through the SAMO community

(<https://www.sensitivityanalysis.org/>), played a crucial role in advancing such techniques. The first software started to emerge, such as the PREP (Preprocessor) and SPOP (Statistical Post-processor) codes for uncertainty and sensitivity analysis developed in the context of nuclear waste management where modellers engaged in a model intercomparison program, which included a benchmark on sensitivity analysis (Probabilistic System Assessment Group (PSAG), 1987, 1989, 1993 and 1994).

The growing enthusiasm led to the formation of new communities of practitioners, such as the UK's MUCM (Managing Uncertainty in Complex Models) community, which developed Bayesian techniques for computer experiments (Oakley and O'Hagan 2004).

The Society for Industrial and Applied Mathematics (SIAM) in the United States and the CNRS (Centre national de la recherche scientifique) research group MASCOT-NUM (Methodes d'Analyse Stochastique pour les Codes et Traitements NUMériques) in France further contributed to spread uncertainty quantification and sensitivity analysis in their respective regions.

To support these efforts, software packages emerged from the early 1990s to 2010s, with a notable boom in the 2010s (see Figure 1). The inclusion of SA in notable software like SciPy has proven to be challenging as SA is only starting to be seen as a fundamental scientific tool by the scientific software community. Considering the reach of some of these software, it is expected that SA will get lot of attention in these communities.

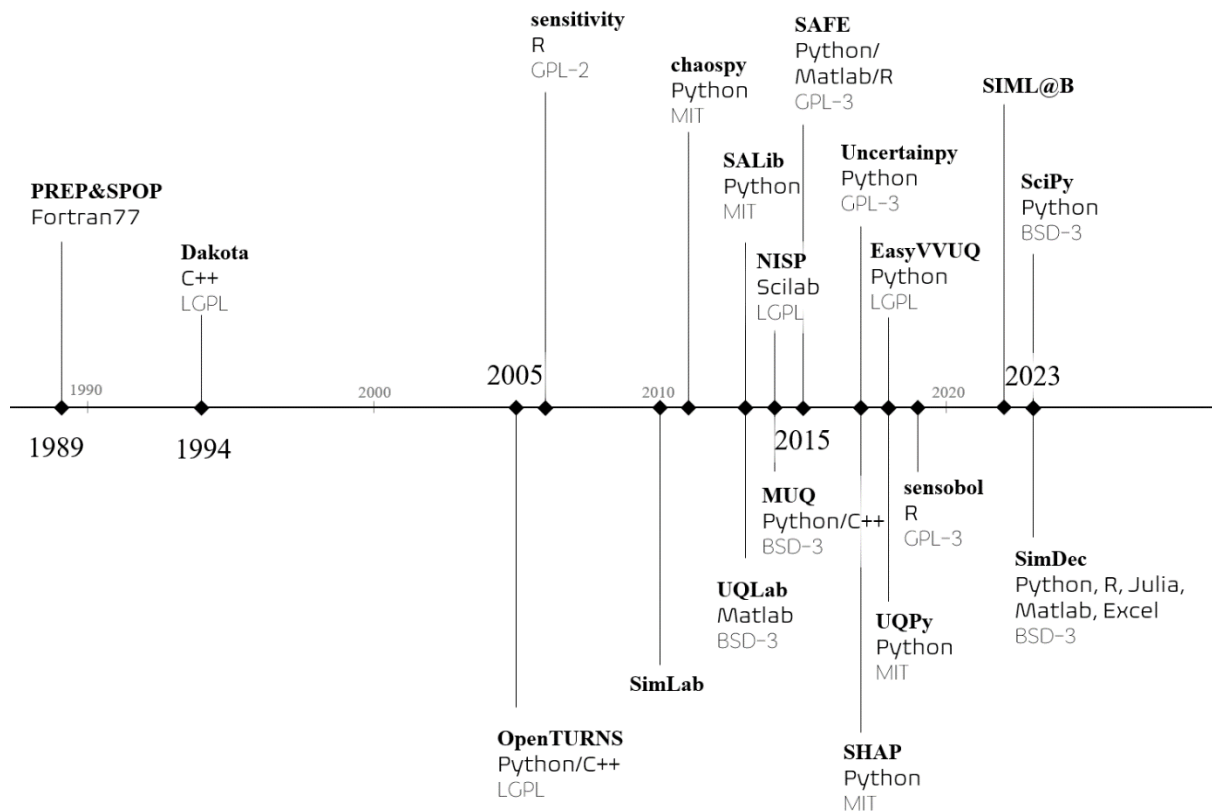


Figure 1: Introduction of software packages for sensitivity analysis

Several handbooks on sensitivity analysis (Helton 1993; Saltelli and Funtowicz 2004; Saltelli et al., 2004, 2008), also contributed to the mainstreaming of sensitivity analysis by emphasizing its application in various settings like factor prioritization, factor fixing, and variance reduction

(Saltelli and Tarantola 2002). These authors advocated for global methods other than local approaches for analysing non-linear and non-additive models.

To foster collaboration and knowledge exchange, the JRC school began the organization of international conferences on global sensitivity analysis in 1995 (<https://www.sensitivityanalysis.org/conferences/>). Despite these advancements, local sensitivity analysis methods remained prevalent across disciplines. Researchers investigated the underpinning reasons and the practical implications of these trends in several contributions (Saltelli and Annoni 2010; Ferretti et al., 2016; Saltelli et al., 2019; Lo Piano and Benini 2022).

Recent key developments in sensitivity analysis include the introduction of moment-independent methods (Borgonovo 2007), which do not rely on statistical distribution assumptions of input parameters. Other important developments include a variogram-based method to determine sensitivities at different spatial scales (Razavi and Gupta 2016a, 2016b), PAWN (Pianosi and Wagener 2015) and others.

Faithful to their own precepts, SA practitioners also started to compare the performance of sensitivity analysis methods using SA itself (Puy and others 2020, 2021).

Furthermore, due to the increasing complexity of models, researchers focused on developing methods for sensitivity analysis of computationally expensive models. These methods involved constructing simpler surrogate models that could replace the original complex model for sensitivity analysis, with adaptive sampling strategies selecting the most informative input parameter combinations (Sudret 2008).

Moreover, a growing interest in integrating sensitivity analysis with machine learning methods cuts across scientific communities. Both approaches are grounded on the exploration of the parameter space to achieve both interpretable and highly predictive solutions, which is promising towards a fruitful synergy (see, i.e., Zhang, 2019, Bénard et al., 2022, Iooss et al., 2022).

The trend toward model complexification emphasized the importance of using sensitivity analysis to ensure accurate and reliable model outputs. This trend tied back to the programmatic introduction of sensitivity analysis as a tool for model transparency within the framework of post-normal science (Funtowicz & Ravetz, 1993). In this context, sensitivity auditing (Saltelli et al., 2013) and the "modelling of the modelling process" (Lo Piano and others 2022) were introduced, urging modellers to retrace their assumptions and enhance transparency in the modelling process.

In the "modelling of the modelling process", it is often advisable to consider multiple candidate models that differ in their assumptions or specifications. The process involves subjecting the various stages of the model-building process to coordinated and simultaneous variation in the modelling assumptions. This exploration can be carried out within a Monte Carlo framework, as discussed by Kroese and others 2014, by introducing random "triggers" that determine the model to be followed in each simulation. By combining the predictions from these models, we can account for the uncertainty associated with each model's parameter, assumptions and structural components.

2.4 The politics of sensitivity analysis

Recent years have seen an extra impetus to sensitivity analysis coming from policy studies. The COVID-19 pandemic was partly instrumental with this development, leading several

authors to question the political use of models (Caduff, 2020; Rhodes & Lancaster, 2020; Saltelli et al., 2020; Winsberg, 2022), with sensitivity analysis being advocate as a tool to make models less opaque (Saltelli et al., 2020). Impact assessment is also a field where increasingly sensitivity analysis is seen as a useful lens to peers at models (Saltelli et al., 2023), also in conjunction with sensitivity auditing just mentioned (Di Fiore et al., 2023; Lo Piano et al., 2023). A recent volume devoted to the politics of modelling (Saltelli & Di Fiore, 2023) also includes a relevant discussion of sensitivity analysis (Borgonovo, 2023). Sensitivity analysis and auditing have recently been proposed as tools to jointly match the double demand for technical and normative quality in modelling (Saltelli & Puy, 2023), echoing a parallel discussion in the field of social statistics (Salais, 2022; Sen, 1990).

3 - Bibliometric Survey

Bibliometric tools have recently emerged as valuable instruments for studying the evolutionary dynamics within specific scientific domains (see e.g. Neff & Corley, 2009). These tools have previously been employed to investigate the trajectories of sensitivity analysis and global sensitivity analysis (Ferretti et al., 2016), as well as the patterns of adoption and utilization of software for uncertainty management in the field of environmental sciences (Lo Piano and Benini, 2022; Douglas-Smith et al., 2020).

In this article, a bibliometric analysis¹ was conducted with the explicit aim of exploring the development of sensitivity analysis as a discipline in scholarly literature. The analysis leverages a Scopus dataset containing 16,513 documents including books, book chapters, articles, conference papers, and reviews. These documents were selected based on the presence of the term "sensitivity analysis" (respectively "global sensitivity analysis"), within their abstracts or keywords, coupled with "model" and "uncertainty" as control fields anywhere in the text body. After data cleansing and processing, leading to the creation of infographics and charts, several observations emerged.

Above all, consolidating the findings of Ferretti et al. (2016), the corpus of literature has continued its consistent growth since 2016, as shown in Figure 2. As mentioned in the previous section, the penetration of global sensitivity analysis methods into the broader modelling community has not reached its full potential and, to the present, still represents roughly a fifth of the total number of published documents.

As noted in Saltelli et al. (2019) the slower uptake of GSA methods might be partly due to their intrinsic complexity. These often involve algorithms and computational processes which can be daunting for researchers who are not well-versed in sensitivity analysis methodologies. Additionally, there might be a reluctance among some practitioners to deviate from familiar and established local sensitivity analysis approaches, even when global methods could offer more comprehensive insights into complex models.

¹ Query specification: (ABS ("sensitivity analysis") OR KEY ("sensitivity analysis") AND ALL (model AND uncertainty) AND REF ("sensitivity analysis") AND PUBYEAR > 1900 AND PUBYEAR < 2023) AND (LIMIT-TO (DOCTYPE , "bk") OR LIMIT-TO (DOCTYPE , "ch") OR LIMIT-TO (DOCTYPE , "re") OR LIMIT-TO (DOCTYPE , "cp") OR LIMIT-TO (DOCTYPE , "ar")) Retrieved on Scopus.com through API calls. June 2023

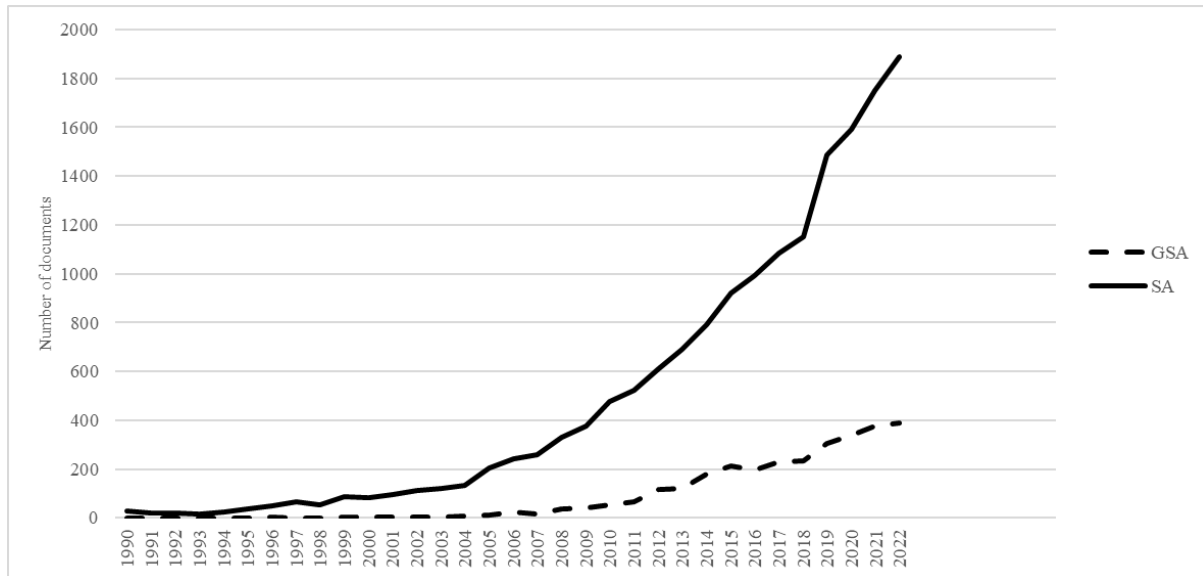


Fig.2 – Publications per year that adopt any kind of sensitivity analysis (SA, solid line) vs those that employ more sophisticated global methods (GSA, dashed line)

Figure 3 provides insight into the distribution of documents across distinct subject areas, highlighting a concentration of documents within engineering and environmental science.

Minimal disparity emerges between sensitivity analysis and global sensitivity analysis and their fields of application: the distribution of the various disciplines' shares is essentially mirrored in global sensitivity analysis, albeit on a more modest scale.

This trend is further reinforced by Figure 4, showing the distribution based on publishing sources. Notably absent fields include finance and economics, and, to a lesser extent, medicine and related fields such as psychology and neuroscience. Considering the relevance of risk within these disciplines, it is rather surprising to observe their substantial absence in the body of literature on sensitivity analysis, a fundamental component in comprehending and managing risk. A tentative of explanation could come from the fact that these fields have traditionally been using other statistical tools such as hypothesis testing (e.g Dunnett test). In some ways, these methods can be used to answer similar questions.

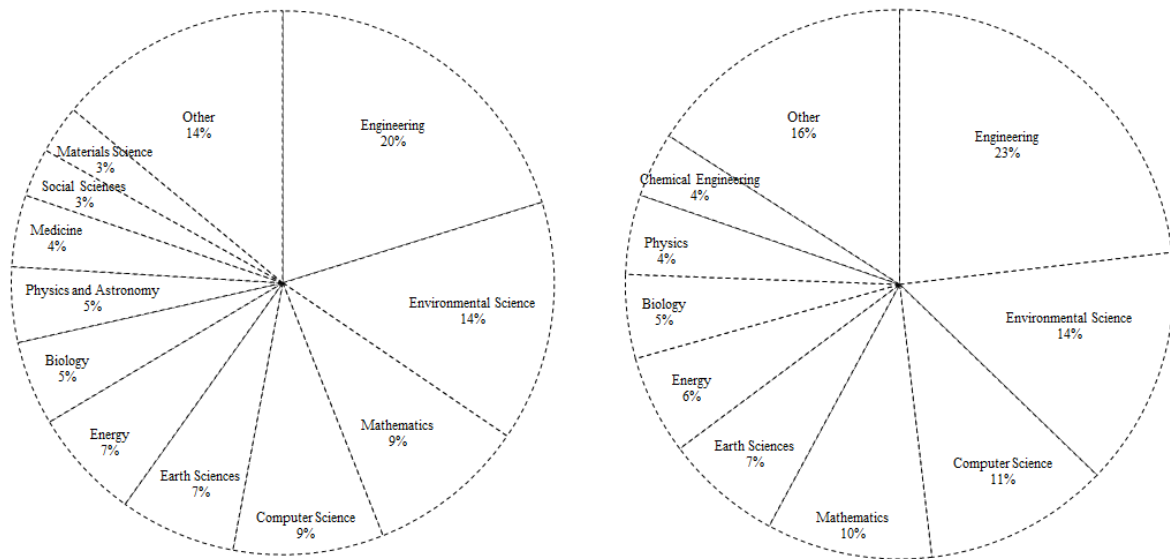


Fig. 3 - Subject area segmentation, on the left SA, on the right GSA

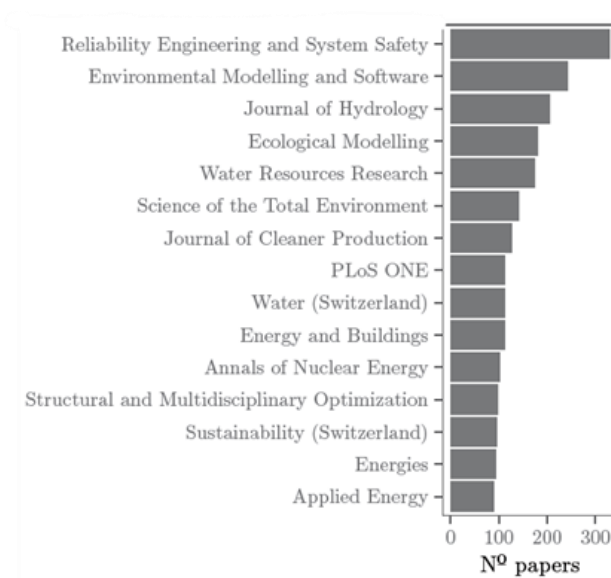


Fig. 4 – Outlets that publish sensitivity analysis studies

The geographical distribution of publishing countries is displayed in Figure 5, wherein the United States and China jointly account for a significant proportion of all published material.

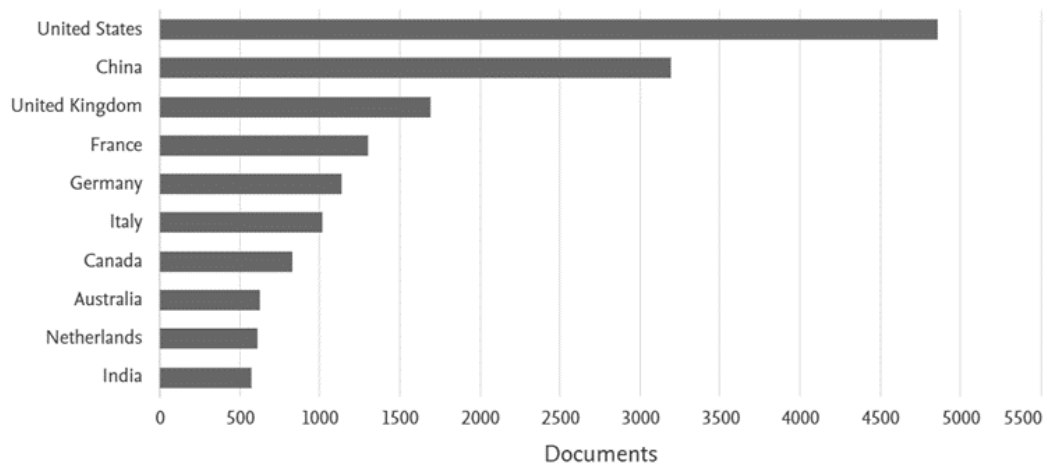


Fig. 5 – Geographical profile of sensitivity analysis publications

Figure 6 attempts to reconstruct the methodological landmarks within the field. Specifically, it focuses on documents that account for more than 500 citations, underscoring the pivotal role played by these works in shaping the methodological landscape of Sensitivity Analysis.

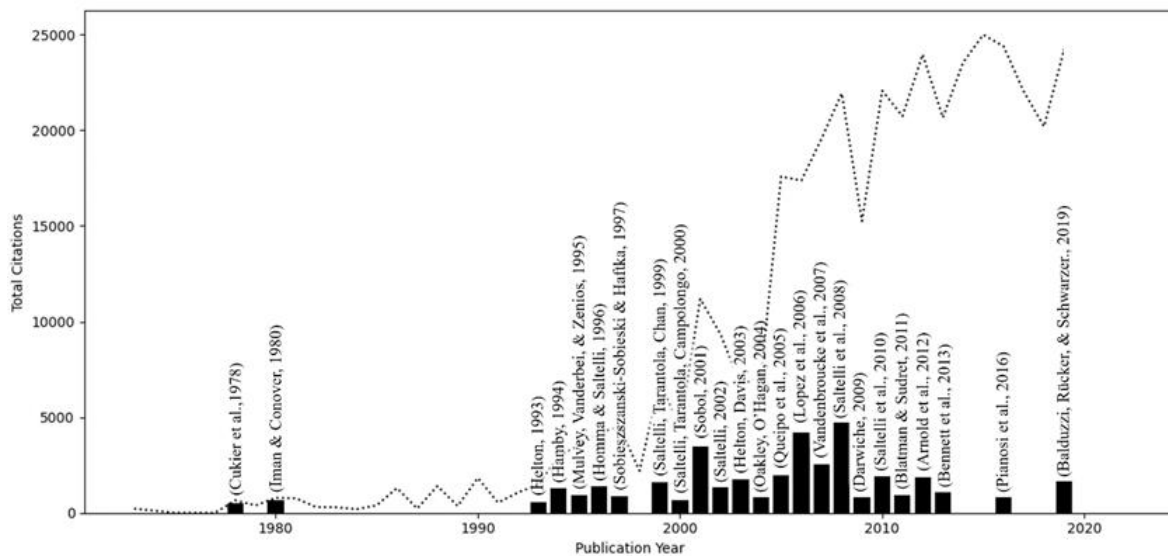


Fig. 6 – Most cited documents and total citations (dotted line)

4 - A fragmented adoption of sensitivity analysis

The historical development of sensitivity analysis is driven by the need to understand the effects of changes in model parameters on model outputs. However, the adoption of sensitivity analysis across various disciplines remains fragmented, with no guarantee that approaches effective in one field will be equally applicable in another. Factors that contribute to this divergence include:

- **Research Culture and Tradition:** Different scientific disciplines may have distinct research cultures and traditions that influence the preferred methods and practices. If sensitivity analysis has not been widely adopted or promoted within a particular discipline, researchers might be less inclined to explore its potential benefits.
- **Computational Resources:** Some sensitivity analysis techniques require significant computational resources, making them less feasible for fields with limited access to high-performance computing facilities or where model evaluations are computationally expensive.
- **Expertise and Awareness:** The level of expertise and awareness of sensitivity analysis methods among researchers in different disciplines can affect their willingness to adopt these techniques. Disciplines with a strong background in statistics might be more likely to embrace sensitivity analysis compared to those less familiar with the concepts.

5 - Conclusions

Sensitivity analysis has progressed from its origins in laboratory and field experiments to in-silico research. Along its journey, it has grappled with crucial challenges, notably the balance between local and global analysis approaches. With the increasing sophistication of sensitivity analysis methods, emerged in order to address the intricacies of increasingly complex models, there is a growing demand for user-friendly tools, aiming to broaden accessibility for researchers and practitioners.

To assess the present landscape of sensitivity analysis, we conducted a bibliometric survey spanning diverse academic disciplines. This analysis offered valuable insights into the spread of documents among different subject areas, underscoring a notable concentration within the fields of engineering and environmental science, whereas absent fields include finance and economics, and, to a lesser extent, medicine and related fields, such as psychology and neuroscience. Interestingly, there is minimal disparity between sensitivity analysis and global sensitivity analysis in terms of their respective applications. The distribution of document shares across various disciplines essentially mirrors that of global sensitivity analysis, albeit on a somewhat smaller scale. This assessment pinpointed areas where further integration and adoption of sensitivity analysis methods is required.

In our perspective, the fragmentation in the adoption of sensitivity analysis across diverse fields can be attributed to a few factors, including i) divergent research cultures and traditions in the different scientific disciplines, ii) heterogeneous computational resources in the various research fields and iii) varying degrees of proficiency and familiarity with sensitivity analysis methods among researchers in the different disciplines.

In the future, sensitivity analysis is expected to play a pivotal role in guiding model development and decision-making processes, especially as simulation models become increasingly bigger and more complex. The ongoing innovation and collaboration among

researchers and practitioners are key to addressing the adoption challenges, fully harnessing the potential of sensitivity analysis and enhancing our grasp of complex systems and their uncertainties. This will eventually lead to more dependable and informed decision-making amid the increasing complexity and uncertainty in our world.

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Appendix 1

A.1 Global Sensitivity Analysis

While sensitivity analysis theory and techniques rapidly developed, a new community of practitioners also embarked on an epistemological journey, delving into the concept of "global sensitivity analysis". Traditionally, sensitivity analysis in various disciplines had been dominated by "local" approaches, involving small perturbations around a reference value while keeping other input parameters fixed. Although this method provided computationally efficient results, it proved inadequate for non-linear and non-additive models or when interactions between inputs played a significant role (Saltelli and Annoni 2010).

In contrast, global sensitivity analysis offered a more comprehensive perspective by simultaneously varying model inputs across a wide range of uncertainty, capturing possible nonlinearities and interactions among parameters. This approach resulted in a broader understanding of input-output dependencies and mitigated the risk of type II errors (nonidentification of influential factors) associated with the traditional one-at-a-time or derivative-based SA.

From the early 1990s, the focus shifted towards exploring and applying new GSA methods across diverse domains, quickly gaining prominence in modelling. Among the early global approaches were variance-based methods (Sobol' 1993), screening methods (Morris 1991), non-parametric or regression-based approaches (Helton 1993; Saltelli and Marivoet 1990), and density-based analysis (Park and Ahn 1994). Over time, global sensitivity analysis continued to evolve and expand, with numerous avenues explored by researchers and practitioners alike (Borgonovo 2007; Saltelli et al., 2010; Mara and Tarantola 2012; Kucherenko et al., 2012; Plischke et al., 2013).

Despite the extensive activity in the field, global sensitivity analysis remains relatively underrecognized as a crucial component of the modelling process, and its applications are not widespread across many fields. The journey of sensitivity-analysis practitioners and the development of global sensitivity analysis techniques have been lengthy, aiming to achieve more comprehensive insights and enhanced reliability in scientific research and decision-making processes. There is still work to be done to fully integrate global sensitivity analysis into various domains and realize its potential as a powerful tool for improving model evaluation and understanding complex systems.

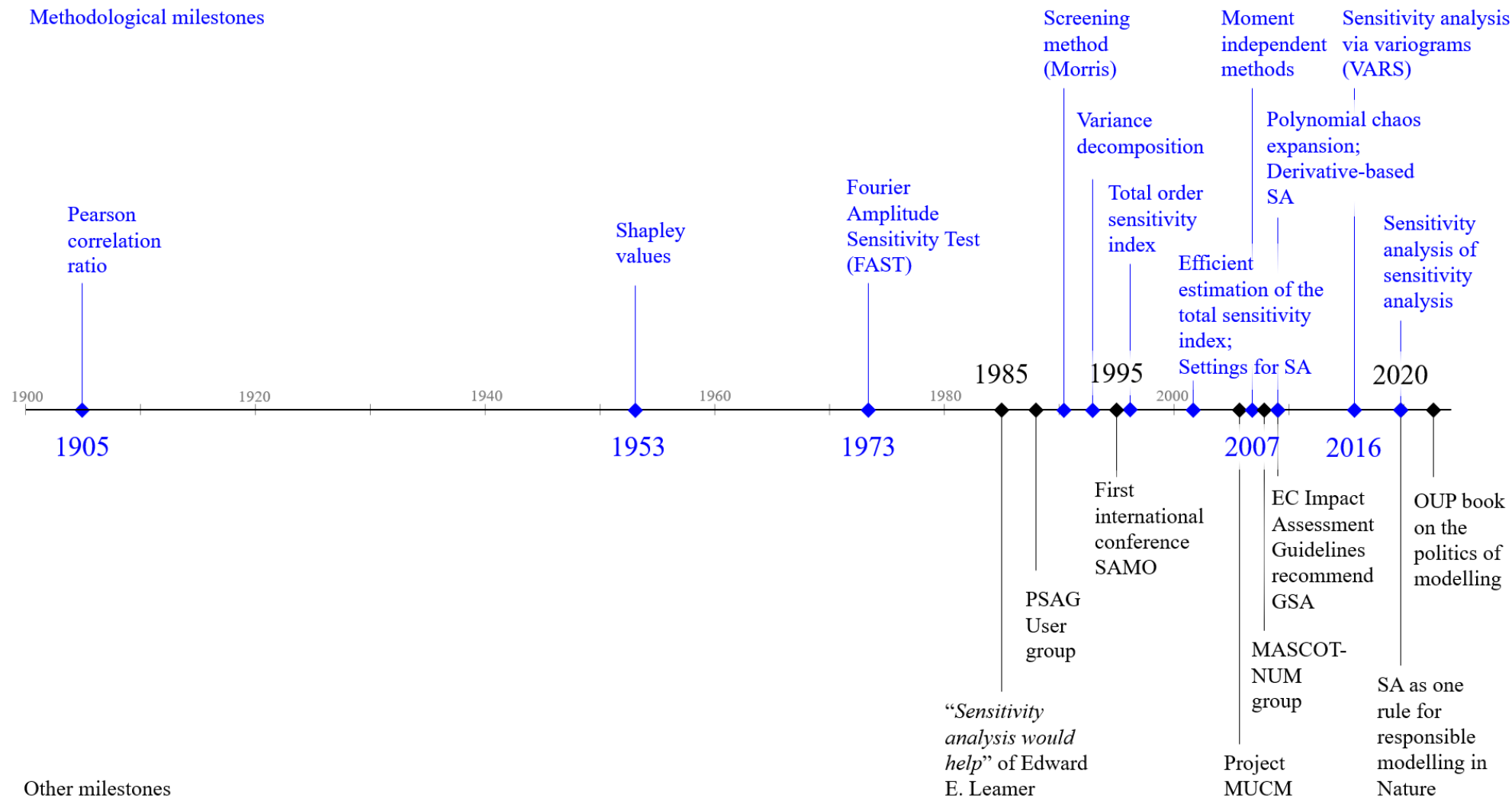


Figure A.1: Milestones of sensitivity analysis: publications, projects and scientific meetings