

Def: Vertex betweenness <sup>of a vertex, v</sup> is a weighted sum of the number of shortest paths in which a vertex, v, appears. If there is ~~are~~ more than one path between 2 vertices we divide by the number of shortest paths.

$$b_v = \sum_i \sum_j \frac{|\{S_{ij}(v)\}|}{|\{S_{ij}\}|}$$

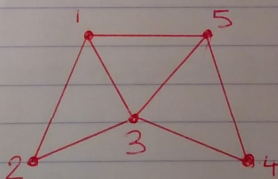
where: a) the sums are over ~~all~~ ~~connected~~ connected pairs of vertices (i, j) in the network.

b)  $\{S_{ij}\}$  is the set of all shortest paths between i & j

c)  $\{S_{ij}(v)\}$  is the subset of  $S_{ij}$  ~~is the~~ which include vertex v

d)  $b_v$  is the betweenness of vertex v.

e.g.



← Our network.  
Fig. 1

Shortest path		Via node *					Path(s)
From node	To node	1	2	3	4	5	
1	2						(1,2)
1	3						(1,3)
1	4			0.5		0.5	(1,3,4), (1,5,4)
1	5						(1,5)
2	3						(2,3)
2	4			1			(2,3,4)
2	5	0.5		0.5			(2,1,5), (2,3,5)
3	4						(3,4)
3	5						(3,5)
4	5						(4,5)
Vertex betweenness		0.5	0	2	0	0.5	

\* Values are 1 if the node is part of a unique shortest path between i & j and  $\frac{1}{n_{sp}(i,j)}$  if there are multiple shortest paths

Def: Edge betweenness <sup>from to</sup> of an edge (i, j) is a weighted sum of the number of shortest paths in which the edge appears. If there is more than one path between two vertices we divide by the number of shortest paths between those vertices.

e.g. For the network in fig. 1

Edge	$\Sigma(\text{\# of appearances} / \text{\# of shortest paths})$
(1,2)	<del>1</del> 2
(1,3)	1.5
(1,4)	N/A
(1,5)	2
(2,3)	2.5
(2,4)	N/A
(2,5)	N/A
(3,4)	2.5
(3,5)	1.5
(4,5)	1