Dynamical systems in neurosciences
Hands-on session

1 Introduction

During this session, you will have to choose from two projects that will be presented here. As a starter, we suggest you to read them carefully, familiarize yourself with the related codes, and ask us questions whenever something is unclear to you. The general goal here is to eventually come up with your own interrogations and propose a way to address them. If the main objective is to familiarize you with "dynamical systems for neuroscience" approaches, it is also the opportunity to do project that is a little less "scholar/academic" and closer to scientific research.

But before let us start with a few reminders on the model and theoretical framework we are using here: the Adaptative Exponential integrate and fire model (AdEx) and the network build from it.

1.1 The AdEx model

In the historical development of neuroscience, the time evolution of the membrane potential observed in single biological neurons has been considered as a relevant observable to explain their functions in neural tissues. The main goal for neuron models is thus to reproduce the shape of these time evolutions. Such models are not "neurons", despite the abusive use of this term in many research areas. To avoid confusion, neuron models will be called "neudel" from now on.

The AdEx neudel is built from the integrate-and-fire model:

$$C\frac{dV}{dt} = I_{inputs} \tag{1}$$

where C is the membrane capacitance, V the membrane potential, and I_{input} is the input current. A "leak current" is then added to reproduce the value of the resting membrane potential:

$$C\frac{dV}{dt} = g_L(E_L - V) + I_{inputs} \tag{2}$$

g_L is the leak capacitance and E_L the leak reversal potential. An exponential term permits to create a dynamics similar to the opening of fast sodium channels:

$$C\frac{dV}{dt} = g_L(E_L - V) + g_L\Delta_T \exp\left(\frac{V - V_T}{\Delta_T}\right) + I_{inputs} \tag{3}$$

where V_T is a threshold that defines when the exponential kicks in, and Δ_T is the slope, which translates the speed at which it "explodes". Finally a second variable, w, permits frequency adaptation and larger repertoire of patterns [2]

$$\begin{aligned} C\frac{dV}{dt} &= g_L(E_L - V) + g_L\Delta_T \exp\left(\frac{V - V_T}{\Delta_T}\right) - w + I_{inputs} \\ \tau_w\frac{dw}{dt} &= a(V - E_L) - w, \end{aligned}$$

where a stand for the conductance of the adaptive channels, and τ_w is the adaptation time constant. After a spike is detected at a fixed threshold V_D, the system is reset as follows:

$$\text{if } V \geq V_D \quad \text{then} \begin{cases} V \to V_R \\ w \to w + b. \end{cases} \tag{4}$$

In other words, the membrane potential is reset to V_R while the adaptation current is incremented by b. More details can be found on the scholarpedia page.

1.2 Network of AdEx

From neudels it is possible to build models of networks. If in the brain numerous interactions may exist between cells, the historically identified one is the synaptic communication.

The coupling between neudels can be done through a model of synaptic activity. In the model considered here, the coupling between neudels is "conductance-based", meaning that spikes do not cause rise of currents in post-synaptic neudels, but of conductance, then creating currents through the interaction with membrane potential (nonlinear terms) of the following form:

$$I_{syn} = g_E(E_E - V) + g_I(E_I - V), \tag{5}$$

where E_E is the reversal potential of excitatory synapses and E_I is the reversal potential of inhibitory model of synapses. g_E and g_I are respectively the excitatory and inhibitory conductances, which increase by quantity Q_E and Q_I for each incoming spike. The increment of conductance is followed by an exponential decrease, with time constant τ_{syn}, according to the equation:

$$\frac{dg_{E/I}}{dt} = -\frac{g_{E/I}}{\tau_{syn}}. \tag{6}$$

In the network, neudels interact *only* through spiking events, which is the reason why we speak of *spiking networks*, even if neudels also have (continuous) internal variables like membrane potential.

Based on such (synaptic) interactions, it is possible to build networks by interconnecting neudels either with a given "controlled" structure or just with a probability of connection. The obtained model may be of very high dimension, in which case classical tools from low-dimensional dynamical systems, such as phase portrait studies, are not possible anymore. However we can still learn interesting aspects of dynamics with such models, it depends on what we chose to measure, and how. This will be the focus of the second project.

2 Evaluation

Note that if you are not comfortable with numerical methods and integration of differential equations, we would suggest to go for the first project, as it starts *from a simpler system.*

For the evaluation we ask you to send a report by email **before 14/01/2022**. The mark (for the group) will not depend at all on the choice of the project, but on the quality and clarity of the presentation of the results and, above all, on the relevance of the discussion. We are fully aware that this work can be confusing, because there is not a "right answer" or "an exact answer to a precise question". But this is in order to get out of the "too scholar" aspect and to get closer to everyday life of scientific research.

We expect the report to be **around 3-5 pages** (figures included, and if really needed you can add annexes), to re-explain the problem clearly in your own words, then to jointly present methods and results and, finally, to propose a scientific discussion.

3 First project : Adex on the test bed (on the birth of chaos)

3.1 Characterization of the single model

As said before, the main goal of a reduced neudel is to reproduce the shape of the time evolution of the membrane potential observed in single biological neurons. The fewer variables there are, the less computer resources it consumes, but a single variable is not sufficient to reproduce certain patterns such as bursts for example. This is why the AdEx model has two variables, which also conserves the advantage of allowing to visualize the phase space in two dimensions.

You can use the *AdEx_NC.py* code to visualize it, with the nullclines for each variable and the trajectory of a simulation. You can see the influence of the parameters on the dynamics by changing for example I_s, V_{reset}, E_L, etc ... It's a way to intuitively understand how the model works. By adjusting the parameters and modifying the topology of the phase space it is possible to obtain the desired patterns. But without many precautions, this type of approach of "fine tuning", can take us even further away from biological realities.

3.2 Relation to experiments (Should I burst?)

Keeping a close relation to experiments is a good recipe to avoid "fine tuning". For example, if the interest lies in understanding the transition from one dynamical regime to another, biological systems are known to support several.

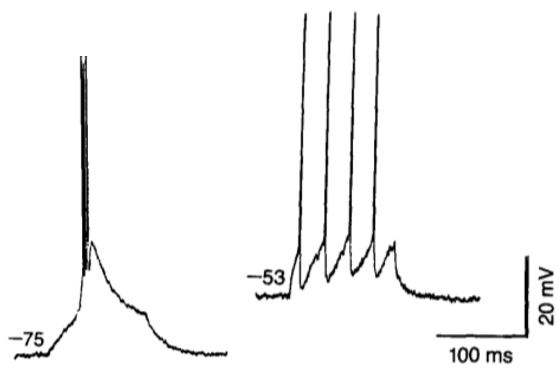

Figure 1: A thalamic cell response to current injection in the absence (left) and presence (right) of Acetylcholine (ACh).

For example, let's look at figure 1 (taken from [1]). Here, a neuron was stimulated with a current injection while being in one of two condition (linked to the absence or presence of the neuromodulator Acetylcholine). In one condition (left), the membrane potential resting state is very negative (around -75 mV), and responds to a brief stimulus with a burst. In the other condition (right), the membrane potential resting state is less negative (-53 mV), and responds to a brief stimulus with a regular series of spikes.

We provide two python scripts, *ADEX_TC_Ctl.py* and *ADEX_TC_ACh.py*, modeling these two conditions. Just three parameters of the AdEx neudel change between the two scripts, but the results are quite different.

- What are the main differences in the phase space?
- Are the neudel parameters for the resting state (*El*) similar to those in the figure? What happens if you change them?
- How is the bursting behavior achieved in *ADEX_TC_Ctl.py*?
- Why is the neudel in *ADEX_TC_ACh.py* spiking regularly?

3.3 Effects of interactions between neudels

As the single AdEx neudel no longer has any secret for you, we can then focus on the interaction between many neudels. Let's start with two. To do so, you can use the code called *AdEx_interact.py*. You can change the number of neudels with inhibitory or excitatory effects. You can try to vary parameters related to the strength of the coupling like Q_e, Q_i. It is thus possible to see that the overall system can evolve with different types of dynamical regimes. From these observations several questions can arise, here is a non-exhaustive list, others are of course very welcome:

- What can we say about the coupled system from the single neudel phase portrait?

- Are there levels of input (or interaction) for which such description is relevant to understand the global dynamics ?
- How to characterize the behavior of the global system?
- What if we increase the number of neudels in the network?

4 Second project: Network, scales and measures (on the nature of complexity)

In this project you directly jump to the network scale. The general idea of this project is that you propose a possible description and characterization of the activity of the network. To help you with that, here is a little introduction to possible measures (more are welcome! do not hesite to propose !), and also some questions to feed your thinking.

In the code proposed *Network_Brian2.py*, the network is made of two populations: 80% Regular Spiking (RS) and 20% Fast Spiking (FS) neudels, describing respectively excitatory and inhibitory neudels, for a total of 10000 neudels. It receives a Poissonian external input of incoming spikes. By changing parameters, such as connectivity between population, b value or external input, you may observe different regimes.

Each neudel being describe by 4 variables (2 from AdEx and 2 for the synaptic conductance), the system is now composed of 40000 equations. It is then very difficult to look at each variables or to represent the phase space in 40000 dimensions. We therefore need to find another way to represent, understand, and analyze the dynamics, through measures for example.

4.1 Global measurements of the network activities

From the network simulation it is possible to record the evolution of the variables, or the evolution of the mean and standard deviation of the variables. But we can also record elements of interest such as spikes, to obtain the raster plot, but also, to calculate an average firing rate through a time window. We then have also the question of time averaging in this case. The effect of "averaging" can make us lose important information, it depends on the dynamical regimes and the question asked (i.e. why are we using this model of network). We can therefore be interested in underlying scales.

4.2 From one to the whole

If we are interested in the activity of spike and the different possible regimes, measuring the coefficient of variation for each neudel and looking at the statistics maybe be interesting. However, it may not be enough to characterize the contribution of individual to the whole. Then a possibility is to look at the correlations between individual and global measures. But you can think in many other ways of characterizing the network activity.

4.3 General questions

- What can we call the network "behavior"? What can we measure in the network? How?
- Can we see correlations between the measures?
- What information do we lose, what information do we want to keep?
- How does the activity of a single neudel relate to the global behavior of the network?
- Does it depend on the global dynamical regime?

References

[1] David A McCormick. Cholinergic and noradrenergic modulation of thalamocortical processing. *Trends in neurosciences*, 12(6):215–221, 1989.

[2] Richard Naud, Nicolas Marcille, Claudia Clopath, and Wulfram Gerstner. Firing patterns in the adaptive exponential integrate-and-fire model. *Biological cybernetics*, 99(4-5):335, 2008.