COMPUTING INFLATION EXPECTATIONS FROM PRICES ON INFLATION CAP OPTIONS

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1. Introduction

Expectations of future inflation are core to the economy because they play a major role in setting today's inflation and output. Therefore, a large strand of literature has focused on measuring inflation expectations in order to feed models such as large-scale DSGEs or smaller workhorse models. In particular, measuring inflation expectations is key in many areas of empirical economics, for instance, when studying the effects of central banks' annoucements in monetary policy[1].
Since financial products generally bring revenues in the future, virtually all financial assets' present valuation encapsulates some degree of inflation compensation. Some assets are explicitly backed on future inflation rates and may be used to extract market participants' inflation expectations.
Most literature has focused on professional forecasters surveys or inflation-linked swaps in order to derive inflation expectations[2]. While the former provides low frequency explicit inflation expectations, the latter allows to derive high frequency implicit expectations- but both surveys and swaps provide point estimates, that is, generally, mean inflation expectations.
However, it is more useful to derive inflation expectations' full distributions rather than sole point estimates, as inflation expectations' higher moments may be calculated from such distributions. For instance, the variance should inform us on the level of disagreement or of uncertainty among market participants about the

[1]See Blinder and al (2008).
[2]See for instance Hubert (2016).

future inflation rate. Such distributions may be constructed from the observed prices of inflation options, but up to now, relatively little attention has been given to using these data in order to infer inflation expectations- although interest has been growing rapidly[3].

The research question this paper addresses is therefore: how to use observed prices of inflation options in order to compute inflation expectations? This paper uses observed prices of European 1-year options in order to derive risk-neutral implied probability density functions for inflation expectations over the one-year horizon.

While there has been recent research on providing researchers with procedures for deriving implied densities from option prices[4], available techniques usually break apart with negative interest rates and need to be adapted to markets' new conventions.

2. Model

There is a simple idea behind using inflation options' prices for constructing distributions of inflation expectations. Inflation options are contracts that give the right but not the obligation to receive a pre-defined inflation rate if the observed inflation rate in the future is above a certain strike rate. When giving value to such a financial product, market participants should incorporate some subjective probability that inflation will indeed be above the strike rate[5]. Therefore, using observed option prices at different strike prices but for the same expiration date, one should be able to recover market participants' implied probabilities given to

[3]For example, Smith (2012).

[4]See Bliss and Panigirtzoglou (2000), Andersen and Wagener (2002), Malz (2014), Blake and Rule (2015).

[5]For simplicity, I only describe the functioning of a cap inflation option, i.e, an inflation option that pays off if the observed inflation rate is above the strike price. The functioning of a floor option would be analogous.

the underlying inflation rate at each strike price and at a certain maturity. The relationship between the implied probability of the underlying inflation rate and the observed cap price is given by Cox and Ross (1976) as

$$C(t,T,K) = e^{-r(T-t)} \int_K^\infty w(S_t)(S_t - K)dS_t$$

which relates the price of a call option in time t, at a strike price K expiring at T, $C(t,T,K)$, to the density probability function of the underlying, $w(S_t)$. But Breeden and Litzenberger (1978) observe that

$$\frac{\partial^2 C(t,T,K)}{\partial K^2} = e^{-r(T-t)} w(K) \tag{1}$$

that is, twice differentiating the call price function $C(t,T,K)$ with respect to the strike price K yields the implied probability density function of the strike price K (up to a discount rate)- which is what we want to derive in order to compute (risk-neutral) inflation expectations.

3. Justification for why one needs a computational method to solve this model

However, the call-price function, that is the function that relates the price of an option to various strike prices, is intrinsically discrete because option prices are only observed at precise strike prices as shown in figure 1. For instance, option prices are observed only at common strikes such as 1%, 2%, or 0% in the current context of low inflation. Moreover, the number of observed strike prices is low - often less than 5 strikes - because cap options at strike prices that are too far away from the money- the strike price at which one exercise the option is far away from the relevant forward rate- are not liquid. Therefore, the main challenge in computing implied inflation expectations' density functions lies in constructing continuous call-price functions in order to apply (1).

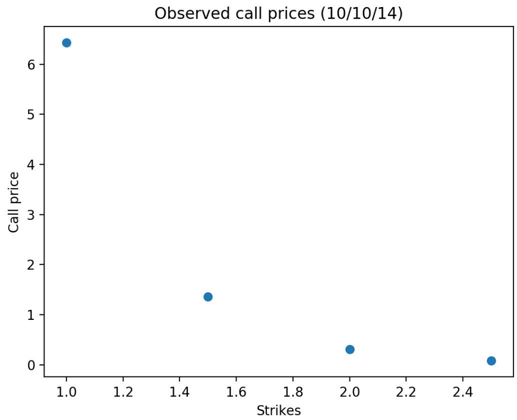


FIGURE 1. Observed 1-year call prices on 10/10/14.

Some points are note-worthy about the call prices displayed in figure 1, and justify our computational approach. First of all, each point represents the value of the 1-year option at a particular strike, and one does not observe any price at strikes below 1%, but one could certainly be interested in market participants' deflation expectations in late 2014. Secondly, it is clear that call prices are observed in discrete steps since there is no option that is traded at strikes such as 1.80%, but we need a continuous call-price function in order to apply (1). Finally, the call price function exhibits its usual shape: strikes that are above the underlying's price (the forward inflation rate here), out-of-the money strikes, are cheaper than in-the-money strikes that are below the forward rate, which have more value since they are more likely to be exercised.

Therefore, our aim will be to extrapolate the call-price function to far-out-of-the-money (high inflation strikes) and far in-the money (negative strikes) unobserved strike prices and to interpolate between strike prices. In order to do that, many techniques are available, see Blake and Rule (2015) or Smith (2012) for a review. Crucially, this paper will make use of B-spline interpolation in order to produce continuous call-price functions through observed option prices, to be twice-differenced in order to obtain risk-neutral density functions of inflation expectations.

4. Description of the computational approach used

One technique relies on using B-spline interpolation in order to interpolate the call-price function. But as noted in Malz (2014), since the call-price function exhibits both quasi-linear and exponential behaviour depending on the strikes, it is easier to interpolate the call-price function in the implied volatility/strike space, called the "volatility smile" (see figure 2). The volatility smile is a pricing and trading framework widely used in applied finance, and relates options' strikes to their implied volatilities, instead of their price. Implied volatilities are a measure of call price's sensitivity to the price of the underlying (here, the forward inflation rate). They may be computed using the standard Black-Scholes valuation model, but their precise calculation is beyond the scope of this paper as they are readily available on Bloomberg terminals[6].

Using Bloomberg data on weekly European 1-year inflation cap options from December 12th, 2014 to February 6th 2015, this paper employs a B-spline interpolation approach in order to interpolate and extrapolate the volatility smile. Implied inflation expectations' density functions are constructed in the following way:

(1) Observed discrete prices are converted into discrete implied volatilities.
(2) A cubic B-Spline with a knot vector containing equally spaced strikes is used in order to interpolate the volatility smile and extrapolate it to unobserved strikes (using the package `Interpolations.jl`).
(3) Continuous implied volatilities are converted back into continuous call prices

[6]Actually, traders do trade inflation options in implied volatility terms, not in terms of prices.

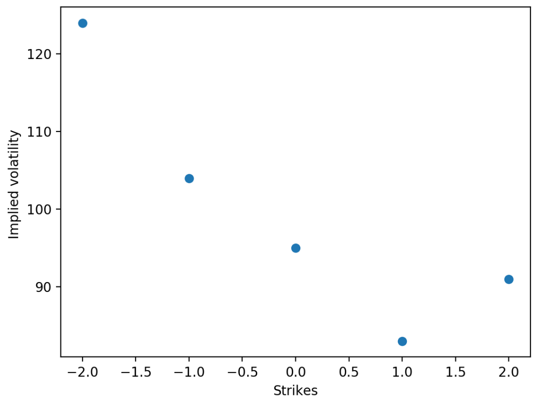


FIGURE 2. Observed 1-year implied volatilities on 12/12/14. Implied volatilities tend to form a "smile" with a minimum at the at-the-money strike, that is, the strike corresponding to the forward inflation rate.

using the Black-Scholes pricing model[7].

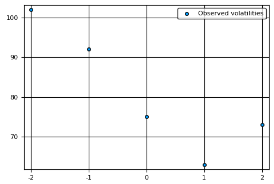
(4) The estimated continuous call-price function is twice-differentiated (using finite differences).

Figure 3 below summarises the procedure.

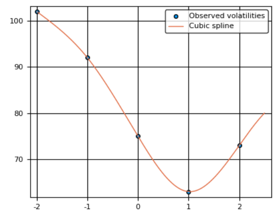
5. Results: how did the ECB's QE launch affect inflation expectations?

At its January 23rd 2015 press conference, after a long negative trend in European inflation expectations, the ECB announced they would embark into a large-scale quantitative easing programme in order to reach their inflation target. While the very accommodative policy was communicated during the press conference, it was also announced that the central bank would actually start buying government

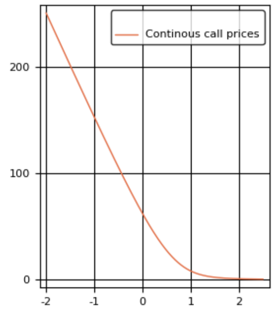
[7]The price C of a call option on underlying S maturing in t is given by $\mathrm{C(S,t)} = \mathrm{N(d_1)S} - \mathrm{N(d_2)Ke}^{-rt}$, where d_1 and d_2 depend on the forward rate, the strike, the implied volatility, the maturity date and the tenor of the option, and N is a normal cdf.



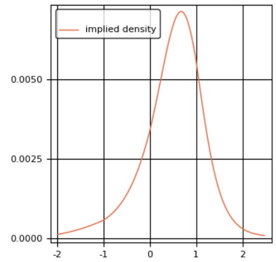
(A) Implied volatilities.



(B) A cubic spline interpolation.



(C) Continuous prices.



(D) Implied density.

FIGURE 3. Outline of the computational approach.

bonds only one month and a half later, on March 6th 2015. In this section, I address the change in market participants' inflation expectations between these two dates using cubic spline interpolation on 23/01/15 and 06/03/15 option prices.

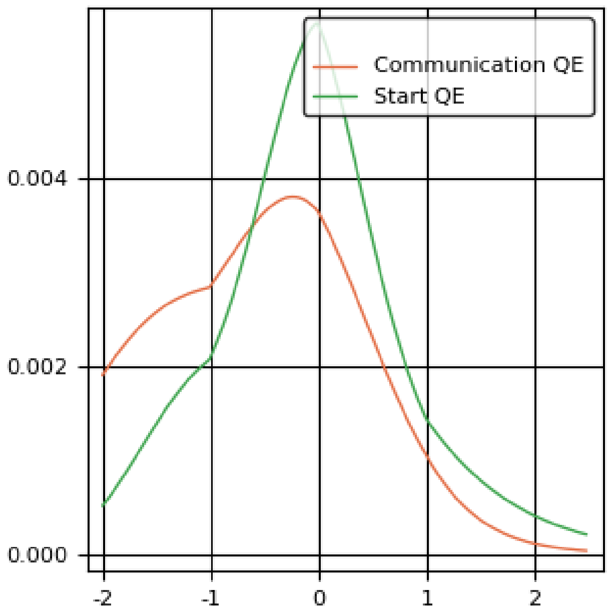


FIGURE 4. Two implied densities from 23/02/15 and 06/03/15 prices, using cubic spline interpolation on their volatility smiles.

While there is no clear shift towards higher inflation expectations in the distribution of inflation expectation on the first day of effective QE, it is very clear that the second distribution is much narrower around its mean than the first distribution. Indeed, the variance seems to have been lowered on the first day of QE's being effective in the bond market, supporting the idea that QE helped reduce market participants' uncertainty or disagreement about the inflation rate over the 1-year horizon.

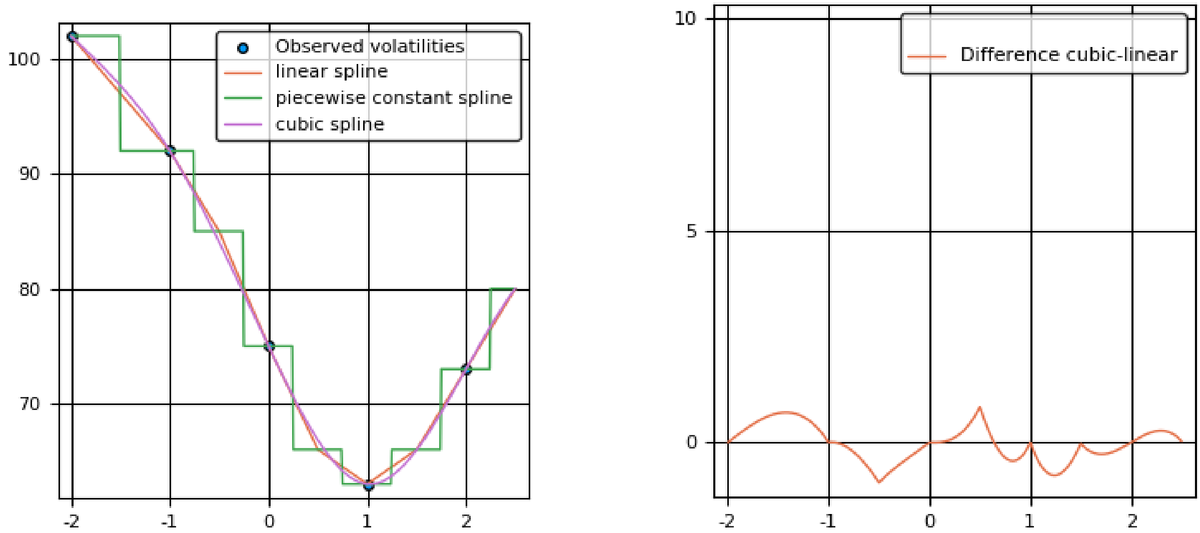
6. Discussion of the results

6.1. **Selection of the spline's order.** In this section, I address the question of the order of the spline to be used in order to interpolate the volatility smile. The key issue is to obtain sufficient level of smoothness in the estimated continuous price-call function to fit sufficiently well at all strikes. This is crucial particularly around at-the-money strikes for they will be the means of the density functions implied from the estimated continuous prices. I therefore conduct interpolation of the volatility smile on 10/10/14 data using constant, linear and cubic B-splines, and choose the one with the best fit. Figure 6 plots the result of the three interpolation methods of the volatility smile.

While the degree of smoothness around the knot obviously varies depending on the order of the spline, it is clear that the linear and cubic splines fit the data very similarly.

When it comes to the estimated "continuous" call-price functions, all three methods deliver extremely close results. Considering the very good fitting properties of the constructed splines as well as the need for maximum smoothness around at-the-money strikes, I choose to stick to the cubic spline when applying the method to new data.

6.2. **Comparison to other approaches.** Smith (2012) notes that it can be tricky to interpolate the volatility smile using spline interpolation as he finds better fit with other methods, such as the SABR, which is a parametric model of implied volatility derived from the Black-Scholes valuation model. But after



(A) 3 spline interpolations of the volatility smile.

(B) Spread cubic spline − linear spline.

FIGURE 5. Interpolation of the volatility smile.

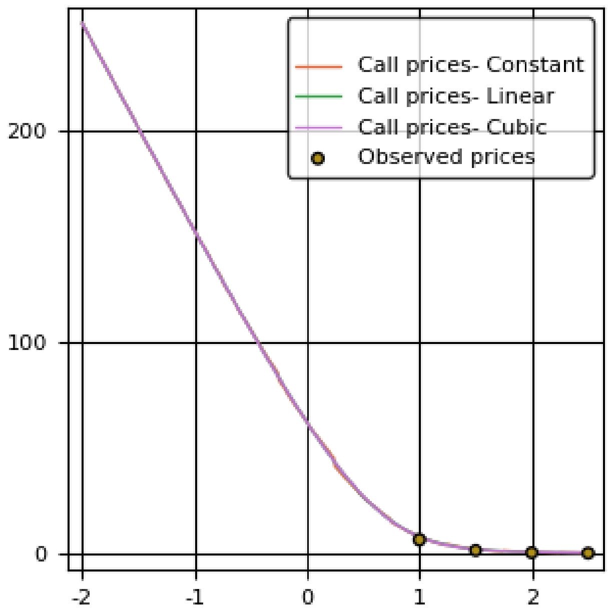


FIGURE 6. Three call-price functions.

adapting the SABR (in `Matlab` though) to my question, the two approches seem to fit very similarly - although the SABR is infinitely more cumbersome to run. In particular, due to its theoretical basis, it needs to be adapted in light of current negative strikes where a-theoretical splines run just fine. Figure 7 summarises the approach using the (shifted) SABR.

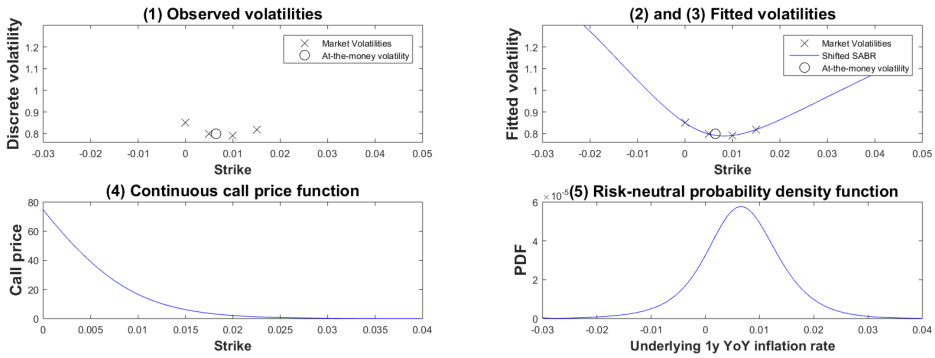


FIGURE 7. The SABR procedure follows the same logic as in spline interpolation.