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An optimum set of loss models for performance prediction of centrifugal compressors

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Abstract: The present study has tested most of the loss models previously published in the open literature and found an optimum set of empirical loss models for a reliable performance prediction of centrifugal compressors. In order to improve the prediction of efficiency curves, this paper recommends a modified parasitic loss model. The performance analyses by using various empirical loss models are also compared with those by the two-zone modelling. Predicted performance curves by the proposed optimum set agree fairly well with experimental data for a variety of centrifugal compressors. The prediction method developed through this study can serve as a tool for preliminary design and assist the understanding of the operational characteristics of general purpose centrifugal compressors.

Keywords: centrifugal compressors, mean streamline analysis, loss correlations, two-zone modelling, performance prediction

NOTATION

b	impeller width
b^*	ratio of vaneless diffuser inlet width to impeller exit width
C_f	skin friction coefficient
C_p	specific heat at constant pressure
D	diameter
D_f	diffusion factor
D_{hyd}	impeller average hydraulic diameter
L_b	impeller flow length
L_θ	impeller meridional length
$\dot{m}$	mass flowrate
P	pressure
PR	stage total-to-total pressure ratio
r	radius
T	temperature
U	tangential impeller speed
V	absolute velocity
W	relative velocity
W_{ui}	tangential component of impeller inlet relative velocity
Z	number of blades
α	absolute flow angle from meridional direction
γ	specific heat ratio
Δh	enthalpy change (J/kg)
ε	clearance
ε_{wake}	wake fraction of blade-to-blade space
η_s	stage isentropic efficiency
ν	kinematic viscosity
ρ	fluid density

Subscripts

0	total condition
1	impeller inlet
2	impeller exit and vaneless diffuser inlet
3	vaneless diffuser exit
act	actual condition
bld	blade loading
cl	clearance
df	disc friction
Euler	Euler
h	impeller inlet hub
inc	incidence
int	internal condition
lk	leakage
m	root mean squared position
mix	mixing
m1m	meridional direction at impeller inlet
rc	recirculation
sf	skin friction
t	impeller inlet tip
u	tangential direction
vld	vaneless diffuser

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1 INTRODUCTION

Although the computational fluid dynamics procedures have progressed remarkably in the analysis of turbomachinery, reliable turbulence models for complex turbulent flows such as the passage flow between blades with high surface curvature under rotation are not yet available and practical design and off-design performance analysis of centrifugal compressors still rely on the conventional empirical loss correlations.

During the past several decades, the empirical loss correlation method has been persistently developed, which is well documented in standard textbooks such as those by Dixon (**1**), Whitfield and Baines (**2**), Yahya (**3**) and Lakshminarayana (**4**). Recent studies using the same method have been carried out by Aungier (**5**), Thanapandi and Prasad (**6**), Takagi *et al.* (**7**) and Denton (**8**) among many others. Takagi *et al.* (**7**) formulated loss models for leakage flow and disc friction based on their measurements of centrifugal pumps at three different specific speeds. Thanapandi and Prasad (**6**) surveyed a number of available loss correlations and found a satisfactory set of models for almost the full range of operating conditions of low specific speed submersible pumps. Recently Aungier (**5**) presented a modular performance prediction procedure which has been validated against about one hundred different stages with a wide range of flow coefficients and pressure ratios up to about 3.5. In the first stage of the present study, all these prediction procedures based on the same numerical scheme have been tested against well-documented Eckardt impellers and the authors' own experimental one (see Fig. 1 for meridional cross-sections and Table 1 for detailed specifications of the KIMM impeller). Unfortunately, however, the test results revealed that their claims on the merits of their procedures were not objectively proved.

The present study is aimed at finding an optimum set of loss correlations among all available empirical models for each respective loss mechanism. In order to carry out this task, frequently quoted empirical loss models in the open literature were first collected and listed in Table 2. All of the 144 possible combinations from Table 2 for six internal loss mechanisms were investigated for their prediction performance of the total pressure ratios of the test impellers mentioned above. An optimum set of internal loss models was then selected and used to find the best set of parasitic models

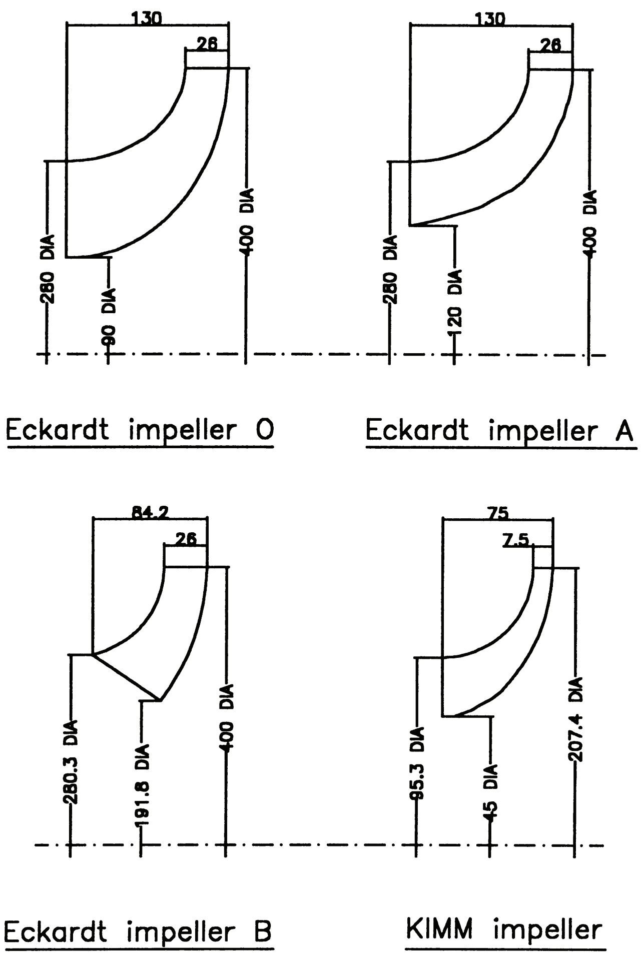

Fig. 1 Meridional cross-sections of four impellers

Table 1 Specifications of KIMM impeller*

Impeller	Inlet tip diameter (mm)	95.3
	Inlet hub diameter (mm)	45
	Discharge diameter (mm)	207.4
	Discharge width (mm)	7.5
	Number of blades	12
	Length in axial direction (mm)	75
	Blade thickness (mm)	3
	Blade angle at inlet tip (deg)	56.26
	Blade angle at inlet hub (deg)	34.22
	Blade angle at discharge (deg)	30
Vaneless diffuser	Outlet diameter (mm)	370
	Inlet and outlet width (mm)	8

*Design mass flowrate is 0.3 kg/s at 15 000 r/min

Table 2 List of internal loss models

Loss mechanism	Loss model
Incidence loss	Aungier (**5**) Galvas (**9**) Conrad *et al.* (**10**)
Blade loading loss	Aungier (**5**) Coppage *et al.* (**11**)
Skin friction loss	Coppage *et al.* (**11**) Jansen (**12**)
Clearance loss	Aungier (**5**) Jansen (**12**) Krylov and Spunde (**13**)
Mixing loss	Aungier (**5**) Johnston and Dean (**14**)
Vaneless diffuser loss	Coppage *et al.* (**11**) Stanitz (**15**)

Table 3 List of parasitic loss models

Loss mechanism	Loss model
Disc friction loss	Daily and Nece (**16**)
Recirculation loss	Aungier (**5**) Coppage *et al.* (**11**) Jansen (**12**) Present*
Leakage loss	Aungier (**5**) Takagi *et al.* (**7**)

*Explained in Table 5.

for disc friction loss, recirculation loss and leakage loss that leads to the most accurate efficiency prediction. Table 3· shows the list of parasitic loss models tested in the present study.

The predictions using the empirical loss models were also compared with those obtained by the two-zone modelling analysis.

2 SELECTION OF INTERNAL LOSS CORRELATIONS

Even if the flow between the impellers is assumed to be under ideal frictionless conditions, the relative eddies in the impeller blade passage can make the impeller exit relative flow angle differ from the impeller exit blade angle. A measure of deviation can be calculated using the concept of a slip factor. There are a number of empirical formulae that can be used to estimate the slip factor (**1**–**4**). However, based on the comparative study of Wiesner (**17**), the present study adopted the slip factor formulation of Wiesner (**17**), without any modification, to calculate the tangential absolute velocity component at the impeller outlet. The performance prediction of centrifugal compressors is conducted by an iterative solution to converge on the fluid density at the impeller exit. The impeller exit fluid density obtained through the computation procedure with the selected combination of loss models is compared to the estimated fluid density. Once the iterative solution is to be converged within a specified tolerance, the compressor stage total-to-total pressure ratio can be calculated as follows:

$$\mathrm{PR} = \left[\left(\frac{\Delta h_{\mathrm{Euler}} - \sum \Delta h_{\mathrm{int}}}{C_{\mathrm{p}} T_{01}}\right) + 1\right]^{\gamma/(\gamma-1)} \tag{1}$$

where $\sum \Delta h_{\mathrm{int}}$ represents the summation of all internal losses in the compressor.

The physical origins of various loss factors in turbomachines are not described in this paper; their specific effects and contributions to the entropy increase in turbomachines are given in the literature (**2**, **8**). The representative distributions of specific loss mechanisms in turbomachines are given in Thanapandi and Prasad (**6**).

In the present study, all possible 144 combinations from Table 2 for six different internal loss mechanisms were tested. By comparing their prediction accuracies, three sets of the most desirable combinations were selected which are shown in Table 4. For clarity of presentation, only these three sets are discussed with the test data. Experimental data to be compared are three Eckardt impellers (**18**, **19**) and the KIMM impeller (see Table 1 for its specifications).

Figures 2 to 4 show comparisons of predictions and test data for three Eckardt impellers O, A and B. The loss combination set I-1 gives poor predictions compared to the other two sets, I-2 and I-3, but it moderately computes the performance trends of Eckardt impeller B. Sets I-2 and I-3 are of comparable combinations for Eckardt's test data. Figure 2 shows that set I-3 is the best combination. In Fig. 3, set I-2 is a better combination than set I-3 except for the

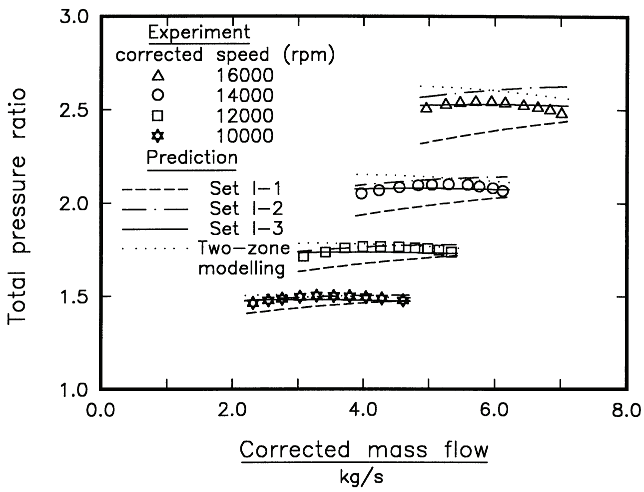

Fig. 2 Total pressure ratio of Eckardt impeller O

Table 4 Three sets of internal loss correlations

Loss mechanism	Set I-1	Set I-2	Set I-3
Incidence loss	Aungier (**5**)	Aungier (**5**)	Conrad *et al.* (**10**)
Blade loading loss	Coppage *et al.* (**11**)	Aungier (**5**)	Coppage *et al.* (**11**)
Skin friction loss	Coppage *et al.* (**11**)	Jansen (**12**)	Jansen (**12**)
Clearance loss	Krylov and Spunde (**13**)	Aungier (**5**)	Jansen (**12**)
Mixing loss	Aungier (**5**)	Aungier (**5**)	Johnston and Dean (**14**)
Vaneless diffuser loss	Coppage *et al.* (**11**)	Stanitz (**15**)	Stanitz (**15**)

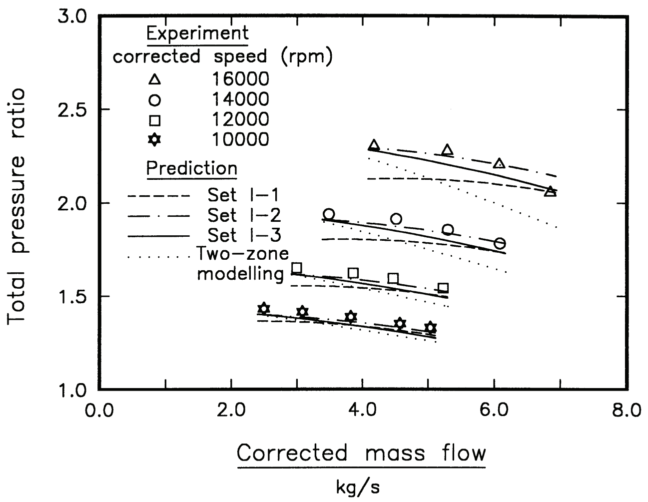

Fig. 3 Total pressure ratio of Eckardt impeller A

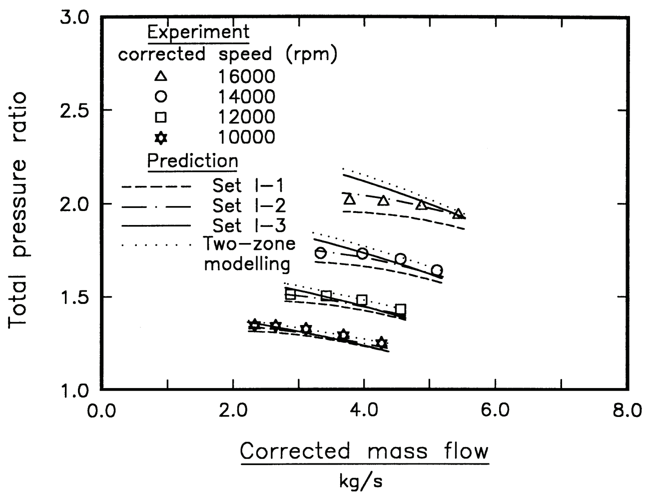

Fig. 4 Total pressure ratio of Eckardt impeller B

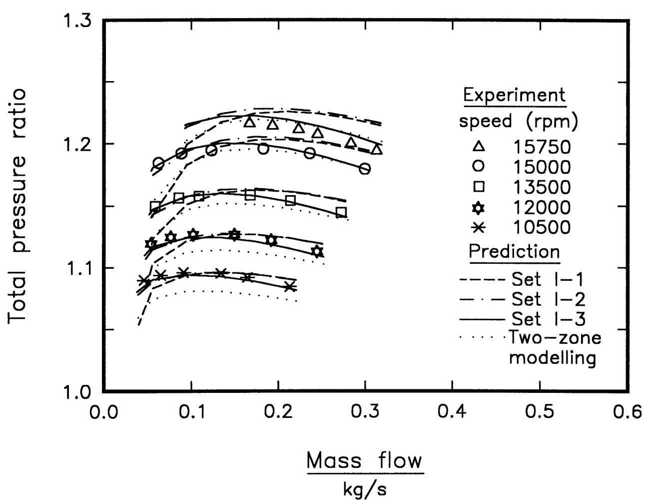

Fig. 5 Total pressure ratio of KIMM impeller

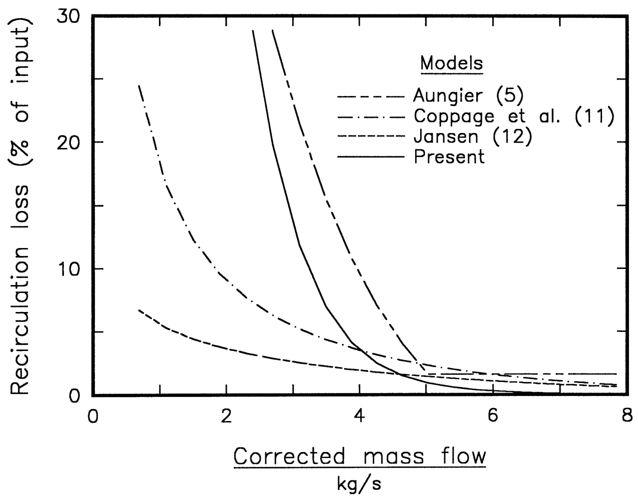

Fig. 6 Comparison of recirculation loss models

high mass flow region at 16 000 r/min. On the other hand, set I-2 yields the best result in the prediction of Eckardt impeller B, as shown in Fig. 4.

A performance prediction of the KIMM impeller total pressure ratio is given in Fig. 5. Set I-3 of loss combination produces an excellent prediction as compared with the other sets throughout the operating ranges. Consequently, this study suggests set I-3 of loss combination as the best set of internal loss models to predict the total pressure ratio of centrifugal compressors.

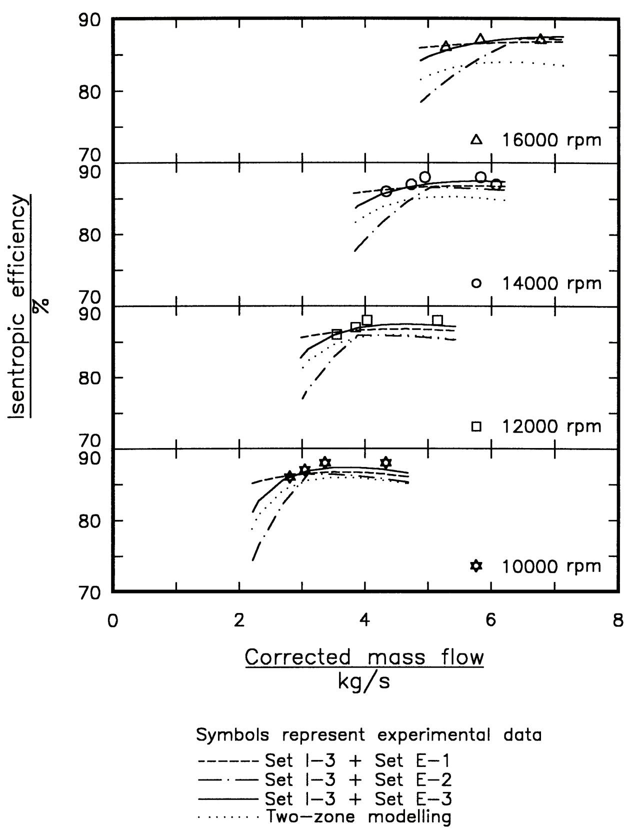

Fig. 7 Isentropic efficiency of Eckardt impeller O

Table 5 Three sets of parasitic loss correlations

Loss mechanism	Set E-1	Set E-2	Set E-3
Disc friction loss	Daily and Nece (**16**)	Daily and Nece (**16**) [quoted by Japikse (**20**)]	Daily and Nece (**16**)
Recirculation loss	Coppage *et al.* (**11**)	Aungier (**5**)	Present*
Leakage loss	Aungier (**5**)	Aungier (**5**)	Aungier (**5**)

*$8 \times 10^{-5} \sinh(3.5\alpha_2^3) D_f^2 U_2^2$
where

$$D_f = 1 - \frac{W_2}{W_{1t}} + \frac{0.75\Delta h_{\text{Euler}}/U_2^2}{(W_{1t}/W_2)[(Z/\pi)(1 - D_{1t}/D_2) + 2D_{1t}/D_2]}$$

3 SELECTION OF PARASITIC LOSS CORRELATIONS

The compressor stage isentropic efficiency is computed by the following equation:

$$\eta_s = \frac{\Delta h_{\text{Euler}} - \sum \Delta h_{\text{int}}}{\Delta h_{\text{act}}} \tag{2}$$

where the denominator represents the actual input enthalpy change including the parasitic works. In addition to the internal losses, the parasitic losses give rise to an increase in impeller discharge stagnation enthalpy without any corresponding increase in pressure (**2**). The parasitic losses are assumed to be made up of disc friction loss, recirculation loss and leakage loss.

Among eight possible combinations from Table 3, three sets of the most desirable parasitic losses selected through the present study are given in Table 5, which includes a refined recirculation loss correlation proposed in this study. Because the recirculation loss dominates the disc friction loss and the leakage loss under the entire operating conditions, the predicted efficiency curves are strongly influenced by the recirculation loss model. Figure 6 compares the recirculation loss models in terms of the percentage of input power versus corrected mass flow. Jansen's model (**12**) is a modified form of Coppage *et al.*'s model (**11**). As can be seen from the efficiency predictions of centrifugal compressors in Figs 7 to 9, both of these models underestimate the recirculation loss in the low flow region. Aungier (**5**) introduced the blade stall limit of Lieblein (**21**) into the recirculation loss model to improve the previous model at the low

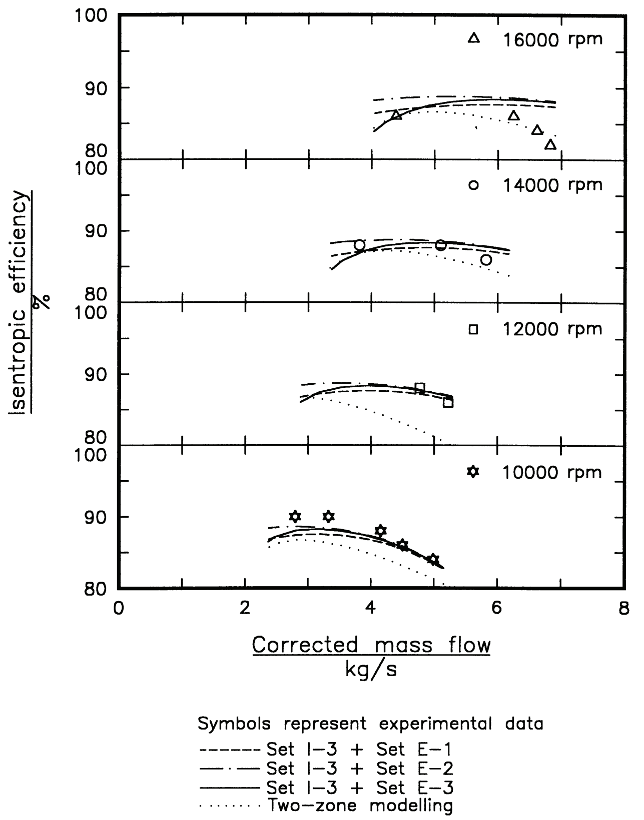

Fig. 8 Isentropic efficiency of Eckardt impeller A

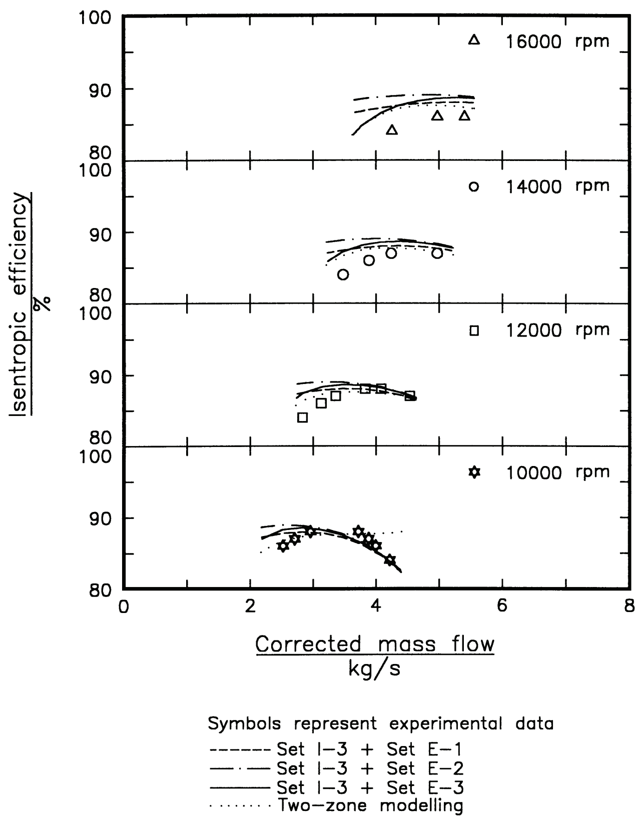

Fig. 9 Isentropic efficiency of Eckardt impeller B

Table 6 An optimum set of loss models for centrifugal compressors

Loss mechanism	Loss model	Reference
Incidence loss	$\Delta h_{\mathrm{inc}} = f_{\mathrm{inc}} \dfrac{W_{\mathrm{ui}}^2}{2}$ where $f_{\mathrm{inc}} = 0.5$–0.7	Conrad *et al.* (**10**)
Blade loading loss	$\Delta h_{\mathrm{bld}} = 0.05 D_{\mathrm{f}}^2 U_2^2$	Coppage *et al.* (**11**)
Skin friction loss	$\Delta h_{\mathrm{sf}} = 2C_{\mathrm{f}} \dfrac{L_{\mathrm{b}}}{D_{\mathrm{hyd}}} \overline{W}^2$ where $\overline{W} = \dfrac{V_{1\mathrm{t}} + V_2 + W_{1\mathrm{t}} + 2W_{1\mathrm{h}} + 3W_2}{8}$	Jansen (**12**)
Clearance loss	$\Delta h_{\mathrm{cl}} = 0.6 \dfrac{\varepsilon}{b_2} V_{\mathrm{u}2} \left\{ \dfrac{4\pi}{b_2 Z} \left[\dfrac{r_{1\mathrm{t}}^2 - r_{1\mathrm{h}}^2}{(r_2 - r_{1\mathrm{t}})(1 + \rho_2/\rho_1)} \right] V_{\mathrm{u}2} V_{\mathrm{m}1\mathrm{m}} \right\}^{1/2}$	Jansen (**12**)
Mixing loss	$\Delta h_{\mathrm{mix}} = \dfrac{1}{1 + \tan^2 \alpha_2} \left(\dfrac{1 - \varepsilon_{\mathrm{wake}} - b^*}{1 - \varepsilon_{\mathrm{wake}}} \right)^2 \dfrac{V_2^2}{2}$	Johnston and Dean (**14**)
Vaneless diffuser loss	$\Delta h_{\mathrm{vld}} = C_{\mathrm{p}} T_{02} \left[\left(\dfrac{P_3}{P_{03}} \right)^{(\gamma-1)/\gamma} - \left(\dfrac{P_3}{P_{02}} \right)^{(\gamma-1)/\gamma} \right]$	Stanitz (**15**)
Disc friction loss	$\Delta h_{\mathrm{df}} = f_{\mathrm{df}} \dfrac{\bar{\rho} r_2^2 U_2^3}{4\dot{m}}$ where $\bar{\rho} = \dfrac{\rho_1 + \rho_2}{2}$, $f_{\mathrm{df}} = \begin{cases} \dfrac{2.67}{Re_{\mathrm{df}}^{0.5}}, & Re_{\mathrm{df}} < 3 \times 10^5 \\ \dfrac{0.0622}{Re_{\mathrm{df}}^{0.2}}, & Re_{\mathrm{df}} \geqslant 3 \times 10^5 \end{cases}$, $Re_{\mathrm{df}} = \dfrac{U_2 r_2}{\nu_2}$	Daily and Nece (**16**)
Recirculation loss	$\Delta h_{\mathrm{rc}} = 8 \times 10^{-5} \sinh(3.5\alpha_2^3) D_{\mathrm{f}}^2 U_2^2$	Present (Table 5)
Leakage loss	$\Delta h_{\mathrm{lk}} = \dfrac{\dot{m}_{\mathrm{cl}} U_{\mathrm{cl}} U_2}{2\dot{m}}$ where $U_{\mathrm{cl}} = 0.816\sqrt{(2\Delta P_{\mathrm{cl}}/\rho_2)}$, $\Delta P_{\mathrm{cl}} = \dfrac{\dot{m}\{r_2 V_{\mathrm{u}2} - (r_1 V_{\mathrm{u}1})_{\mathrm{m}}\}}{Z\bar{r}\bar{b}L_\theta}$, $\bar{r} = \dfrac{r_1 + r_2}{2}$, $\bar{b} = \dfrac{b_1 + b_2}{2}$, $\dot{m}_{\mathrm{cl}} = \rho_2 Z \varepsilon L_\theta U_{\mathrm{cl}}$	Aungier (**5**)

flow region. However, Fig. 6 reveals that his model displays a physically unrealistic abrupt change at the stall point in the recirculation loss prediction. On the other hand, Japikse (**20**) suggested an empirical recirculation loss correlation curve (‘bucket’ model) as a function of the non-dimensional mass flowrate based on many experimental data deduced from various impeller tests. Because, however, he did not provide an empirical formula to represent the ‘bucket’ curve, his model curve is excluded in the present test. Common to all these recirculation loss models is the fact that they are expressed as a function of the diffusion ratio (**11**) and the impeller exit flow angle. A number of the present test calculations showed that the recirculation loss must be in between those of Coppage *et al.* (**11**) and Aungier (**5**) in the low flow region. A new recirculation model is developed by adopting higher power to the flow angle contribution. In order to connect the loss distributions smoothly between low and high flow regions, a hyperbolic functional form is employed in the present model (Table 5). For the disc friction loss, Daily and Nece’s model (**16**) has been most commonly quoted, which is also adopted in this paper.

There have been many empirical models for prediction of the leakage flowrate. Typical ones are those listed in Table 3. A difference between them is that the conventional model [e.g. that of Takagi *et al.* (**7**)] includes the geometric variables such as clearance gap and sealing configuration, whereas Aungier's model (**5**) is nearly independent of the sealing type. Again, all of eight possible combinations from Table 3 have been tested against the stage isentropic efficiency data of Eckardt impellers.

Figures 7 to 9 show the three closest stage isentropic efficiency predictions among them. These three sets are listed in Table 5 . The internal loss model set I-3 was used in these computations. Figure 7 clearly shows that set E-3 of the present parasitic loss models yields the best agreement with experimental data. However, in Figs 8 and 9, prediction accuracies of all three sets are more or less similar, but with set E-3 being a little better. From these comparisons, the model set E-3 is recommended as the best combination for predicting the parasitic losses.

Correspondingly, this paper suggests that set E-3 of the new parasitic loss models be used to compute the actual enthalpy changes [equation (2)] together with the internal loss model set I-3.

4 COMPARISON WITH THE TWO-ZONE MODELLING ANALYSIS

As a reference for this study, the two-zone modelling analysis has been made to predict the total pressure ratio and the isentropic efficiency.

The flow leaving the impeller of a centrifugal compressor is not uniform. In order to describe the disturbed flow, the two-zone modelling analysis has often been made. The two-zone modelling analysis is based on either the classical jet-wake analysis (**22**), the entropy gain-oriented approach (**2**), the computation of the through-flow jet size (**23**) or the jet-wake calculation model according to the diffusion ratio model in the impeller (**20**).

The present study adopted a systematic two-zone modelling based on a combination of the latter three concepts. The computational results using this method are included in Figs 2 to 9. Although the two-zone modelling analysis takes a more detailed flow field into account, it yields less accurate predictions than the recommended set of empirical loss models for all test impellers in the present study. Consequently, it is found that more refinement of the two-zone modelling, particularly for the jet-wake flow area ratio, the secondary flow strength (the wake mass flow fraction) and the diffusion model for impellers, has to be accomplished.

5 CONCLUSIONS

The mean streamline analysis procedure has been utilized to find the best combination of internal as well as parasitic loss models for prediction of the complete performance map of centrifugal compressors. The set of loss models recommended by this study is found to predict the performance curves of centrifugal compressors with acceptable accuracy. A new model for the recirculation loss which is the most influential part of the parasitic works is introduced in this study to improve the prediction of the stage isentropic efficiency. The empirical loss correlations suggested in this paper are specifically summarized in Table 6. As a reference, the predictive capability of the mean streamline analysis procedure is compared with that of the two-zone modelling. It was found that although the two-zone modelling takes into account the non-uniform flow field at the blade outlet, more refinement of the present two-zone modelling is necessary in order to predict the compressor performance at the same level of accuracy as the mean streamline analysis procedure. The predictive procedure presented herein can be used as a conceptual design tool in the preliminary design phase of centrifugal compressors.

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