Meanline model for impeller without choking

⓪ Define boundary conditions $p_{01}, T_{01}, p_2, \Delta h_s$

① Define geometric parameters

Z_R, Z_S

$r_{1s}/r_2 \in [0.4 - 0.7]$

$r_{1h}/r_{1s} \in [0.25 - 0.7]$

$b_2/r_2 \in [0.02 - 0.8]$

$\beta_{1g} \in [-70°, -20°]$

$\beta_{2g} \in [-45°, 0°]$

② Define the specific speed and diameter (blade speed)

$$\psi = \frac{U^2}{\Delta h_s} \quad \text{or} \quad \lambda = \frac{U}{V_0} = \frac{\sqrt{2}}{2} \cdot \frac{1}{\sqrt{\psi}}$$

$$\Omega_s = \frac{\omega \cdot \cancel{\Delta h_s^{3/4}} \, \dot{V}_{2s}^{0.5}}{\Delta h_s^{0.75}} \rightarrow \text{compute } \Omega$$

③ Compute geometry

$r_2 = U_2 / \Omega$

$r_{1s} = r_2\left(\frac{r_{1s}}{r_2}\right)$

$r_{1h} = r_{1s}\left(\frac{r_{1h}}{r_{1s}}\right)$

$r_{1RMS} = \left(\frac{r_{1s}^2}{2} + \frac{r_{1h}^2}{2}\right)^{1/2}$

$\beta_{1g} = \theta_1$ known as DoF

$\beta_{2g} = \theta_2$ known as DoF

$A_1 = 2\pi(r_{1s}^2 - r_{1h}^2)$

$A_2 = 2\pi r_2$

④ Compute velocity triangles

Guess V_1

$U_1 = r_1 \cdot \Omega$

$V_{1m} = V_1 \cos(\alpha_1)$

$V_{1\theta} = V_1 \sin(\alpha_1)$

$W_{1m} = V_{1m}$

$W_{1\theta} = V_{1\theta} - U$

$\beta_1 = \arctan\left(\frac{W_{1\theta}}{W_{1m}}\right)$

Guess W_2 and β_2

$U_2 = r_2 \Omega$

$W_{m2} = W_2 \cos(\beta_2)$

$W_{\theta 2} = W_2 \sin(\beta_2)$

$V_{m2} = W_{m2}$

$V_{\theta 2} = W_{\theta 2} + U_2$

$\alpha_2 = \arctan\left(\frac{V_{\theta 2}}{V_{m2}}\right)$

⑤ Evaluate thermodynamic properties

$h_1 = h_{01} - V_1^2/2$

$\rho_1 = \rho_1(h_1, s_1)$

$h_2 = \left(h_1 + \frac{W_1^2}{2} - \frac{U_1^2}{2}\right) - \left(\frac{W_2^2}{2} - \frac{U_2^2}{2}\right)$

p_2 is known as DoF or fixed param

$\rho_2 = \rho(p_2, h_2)$

⑥ Evaluate constraints

Mass eqn $\dot m_1 = \dot m_2$ $\quad \begin{cases} \dot m_1 = \rho_1 V_{1m} A_1 \\ \dot m_2 = \rho_2 V_{2m} A_2 \end{cases}$

Loss eqn $\left(\frac{h_2 - h_{2s}}{W_2^2/2}\right) = \xi_{Loss\ model}$

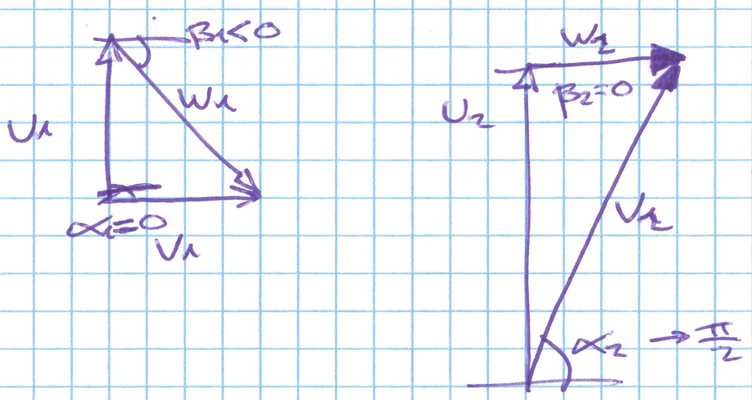
Slip factor model $\longrightarrow$ $V_{2\theta} = \sigma \cdot U_2 - V_{2m} \cdot \tan(\theta_2)$

$\beta_2 = \theta_2 - \delta$

Computed from slip model

$$\boxed{3 \text{ DoF} + 3 \text{ constraints}}$$

Velocity triangles for $\alpha_1 = 0$ $\quad \beta_2 = 0$



Best way to draw

$(m - r)$ plane with inlet velocity triangle and radii (vector into the page)

$(\theta - r)$ plane (180°) with outlet velocity triangle