Vaneless diffuser modeling

The diffuser of a centrifugal compressor is a radial annular channel with converging or parallel walls.

⇒ The vaneless diffuser can be modeled with an implicit system of ODEs representing the transport equations for mass, momentum (r and θ), and energy

$$A \frac{dU}{dm} = b$$

Where:
$$\begin{cases} A \in \mathbb{R}^{4\times 4} \\ U = [\rho, U_\theta, U_r, P] \in \mathbb{R}^4 \\ b \in \mathbb{R}^4 \\ m = r - r_{in} \quad \text{for a radial diffuser} \end{cases}$$

⇒ The velocity vector is given by:

$$\vec{V} = V_m \hat{e}_m + V_\theta \hat{e}_\theta = V_x \hat{e}_x + V_r \hat{e}_r + V_\theta \hat{e}_\theta$$

$$\begin{cases} V_m = V \cdot \cos(\alpha) \\ V_\theta = V \sin(\alpha) \\ V_r = V_m \, \sin\phi = V \cos\alpha \sin\phi \\ V_x = V_m \cos\phi = V \cos\alpha \cos\phi \end{cases}$$

$\phi = \frac{\pi}{2}$ for a radial vaneless diffuser

⇒ The geometry is defined by

Radius $\quad r(m) = r_{in} + m \sin\phi = r_{in} + m$

Width $\quad b(m) = b_{in} + 2m\tan(\delta) \quad$ Where δ is the divergence semiangle

Some geometric relations

Radius ratio → $RR = \frac{r_{out}}{r_{in}} = \frac{r}{r_{in}}$

Area ratio → $AR = \frac{b \cdot r}{b_{in} \cdot r_{in}}$

$$AR = \left(1 + \frac{2m}{b_{in}} \tan(\delta)\right)\left(\frac{r}{r_{in}}\right)$$

$\frac{b_{in}}{r_{in}}$ is often $[0{,}05 - 0{,}5]$ for centrifugal compressors

$$AR = \left(1 + \frac{2 r_{in}}{b_{in}}\left(\frac{r}{r_{in}} - 1\right)\tan(\delta)\right) \cdot \left(\frac{r}{r_{in}}\right)$$

$$AR = \left(1 + 2\left(\frac{r_{in}}{b_{in}}\right) \cdot (RR - 1)\tan(\delta)\right) \cdot RR$$

AR can be computed from $\left(\frac{r_{in}}{b_{in}}\right)$, $\tan(\delta)$ and RR

RR can be computed from AR solving a quadratic equation

$AR = RR = \left(\frac{r}{r_{in}}\right)$ for the special case when $\delta = 0$ (parallel walls)

Inputs to solve the model

Inlet stagnation state (p_0, T_0)

Inlet flow angle α_{in}

Inlet Mach number Ma_{in}

Diffusser geometry, $(b_{in}, r_{in}, \phi, \delta, r_{out})$

Friction coefficient $C_f \in [0{,}000, 0{,}020]$

Solving the inlet state

$$h_0 = h + \frac{V^2}{2}$$

$$h_0 = h(p, s_0) + \frac{1}{2}\left(a(p, s_0) \cdot Ma_{in}\right)^2$$

Guess p_{in} and solve iteratively (fzero)

$$\begin{cases} \rho_{in} = \rho(p_{in}, s_0) \\ a_{in} = a(p_{in}, s_0) \end{cases} \qquad \begin{cases} V_m = Ma_{in} \cdot a_{in} \cdot \cos(\alpha_{in}) \\ V_\theta = Ma_{in} \cdot a_{in} \cdot \sin(\alpha_{in}) \end{cases}$$

Special case → Inviscid incompressible flow

Conservation of mass $\rho_{in} V_{m\,in} r_{in} b_{in} = \rho V_m r b$

Conservation of angular momentum $V_{\theta in} \cdot r_{in} = V_\theta \cdot r$

$$\frac{V_\theta}{V_m} \cdot \left(\frac{\cancel{r}}{\rho \cancel{r} b}\right) = \left(\frac{V_\theta}{V_m}\right)_{in} \cdot \left(\frac{\cancel{r}}{\rho \cancel{r} b}\right)_{in}$$

$$\left(\frac{V_\theta}{V_m}\right) = \left(\frac{V_\theta}{V_m}\right)_{in} \left(\frac{\rho}{\rho_{in}}\right)\left(\frac{b}{b_{in}}\right)$$

$$\boxed{\tan(\alpha) = \tan(\alpha_{in}) \cdot \left(\frac{\rho}{\rho_{in}}\right) \cdot \left(\frac{b}{b_{in}}\right)}$$

The flow angle is preserved for incompressible, inviscid flow will no variation of channel width

If $\rho = \rho_{in}$ and $b = b_{in}$ then $\tan(\alpha) = \tan(\alpha_{in}) = \text{constant}$

This means that the flow angle does not change at different radii. As a result the pathlines (or streamlines) follow a logarithmic spiral:

- Streamline equation in polar coordinates:

$$r \frac{d\theta}{dr} = \frac{V_\theta}{V_r} = \tan(\alpha) = \tan(\alpha_i) = \text{constant}$$

$$d\theta = \tan(\alpha) \frac{dr}{r}$$

$$\boxed{\theta - \theta_{in} = \tan(\alpha) \ln\left(\frac{r}{r_{in}}\right) \iff r = r_{in} \exp\left(\frac{\theta}{\tan(\alpha_{in})}\right)}$$

$$\begin{cases} x = r\cos\theta = r_{in} \cos\theta \cdot e^{\theta/\tan(\alpha)} \\ y = r\sin\theta = r_{in} \sin\theta \cdot e^{\theta/\tan(\alpha)} \end{cases}$$

Inviscid compressible flow in a converging diffusser

If b decreases ($\delta < 0$) $\tan(\alpha)$ decreases (more radial) and the flow stabilizes

When ρ increases $\tan(\alpha)$ increases (more tangential) and the flow is less stable

Vaneless diffuser pressure recovery

Vaneless diffusser equations:

Mass $\qquad V_m\, \cancel{d\rho/dm} + \rho\, dV_m/dm = -\rho V_m/(br)\cdot d(br)/dm$

m-Momentum $\qquad \rho V_m \dfrac{dV_m}{dm} + \dfrac{dp}{dm} = \dfrac{\rho V_\theta^2}{r}\sin\phi - \cancel{\dfrac{2\tau_w}{b}}\cos(\alpha)$

θ-Momentum $\qquad \rho V_m \dfrac{dV_\theta}{dm} = -\dfrac{\rho V_\theta V_m}{r}\sin(\phi) - \cancel{\dfrac{2\tau_w}{b}}\sin(\alpha)$

For the ideal case of inviscid, incompressible flow:

$$
\begin{cases}
\dfrac{dV_m}{dm} = -\dfrac{V_m}{br}\cdot\dfrac{d(br)}{dm} \\[2ex]
\dfrac{dV_\theta}{dm} = -\dfrac{V_\theta}{r}\sin\phi = -\dfrac{V_\theta}{r}\cdot\dfrac{dr}{dm} \\[2ex]
\dfrac{1}{\rho}\left(\dfrac{dp}{dm}\right) = \dfrac{V_\theta^2}{r}\sin\phi - V_m\dfrac{dV_m}{dm} = -V_\theta\dfrac{dV_\theta}{dm} - V_m\dfrac{dV_m}{dm}
\end{cases}
$$

These equations can be solved as:

$$\left(\frac{V_{m2}}{V_{m1}}\right) = \left(\frac{A_2}{A_1}\right)$$

$$\left(\frac{V_{\theta 2}}{V_{\theta 1}}\right) = \left(\frac{r_2}{r_1}\right)$$

$$\int_1^2 \frac{dp}{\rho} = -\int_1^2 V_\theta\, dV_\theta - \int_1^2 V_m dV_m$$

$$\left(\frac{p_2-p_1}{\rho}\right) = \left(\frac{V_{\theta 1}^2 - V_{\theta 2}^2}{2}\right) + \left(\frac{V_{m1}^2 - V_{m2}^2}{2}\right) = \frac{V_1^2}{2} - \frac{V_2^2}{2}$$

$$C_p = \frac{(p_2-p_1)}{\frac{1}{2}\rho V_1^2} = 1 - \left(\frac{V_{\theta 2}^2}{V_1^2}\right) - \left(\frac{V_{m2}^2}{V_1^2}\right) \qquad \begin{cases} V_{m1}^2 = V_1^2\cos^2(\alpha_1) \\ V_{\theta 1}^2 = V_1^2\sin^2(\alpha_1) \end{cases}$$

$$\boxed{C_p = 1 - \cos^2(\alpha_1)\cdot\left(\frac{r_2}{r_1}\right)^2 - \sin^2(\alpha_1)\left(\frac{A_2}{A_1}\right)^2}$$

Incompressible
Inviscid