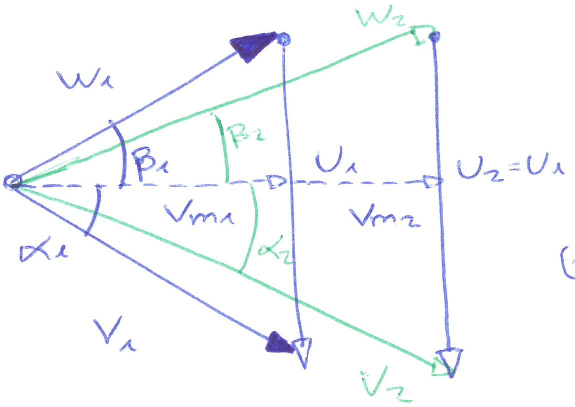
Incidence Loss according to Stanitz

Galvas (1973) Whitfield (1990)

When the flow approaches the leading edge of the impeller, the frontal area of the passage is reduced by the presence of the blades.

This involves a flow contraction that changes the inlet velocity triangle by accelerating the axial component of velocity. It is assumed that the tangential components of velocity do not change (no swirl is created or destroyed). Naturally, the absolute and relative flow angles (α and β) change due to the changes of axial velocity at constant swirl



$W_{1\theta} = W_{2\theta}$ means that

$$W_{m1} \tan(\beta_1) = W_{m2} \tan(\beta_2)$$

$$(1) \quad \boxed{\tan(\beta_2) = \frac{V_{m1}}{V_{m2}} \tan(\beta_1)}$$

Similarly we have that

$$V_{1\theta} = V_{2\theta}$$

$$V_{m1} \tan(\alpha_1) = V_{m2} \tan(\alpha_2)$$

$$(2) \quad \boxed{\tan(\alpha_2) = \left(\frac{V_{m1}}{V_{m2}}\right) \tan(\alpha_1)}$$

$\Rightarrow$ Mass balance requires:

$$\dot{m} = \rho_1 A_1 V_{m1} = \rho_2 A_2 V_{m2}$$

Assuming that the flow is incompressible:

$$(3) \quad \boxed{\frac{V_{m1}}{V_{m2}} = \left(\frac{\rho_2}{\rho_1}\right) \cdot \left(\frac{A_2}{A_1}\right) = \left(\frac{A_2}{A_1}\right)}$$

where

$$A_1 = 2\pi r H$$

$$A_2 = Z \cdot (s - t) \cdot H$$

$$A_2 = Z s H \cdot \left(1 - \frac{t}{s}\right)$$

$$A_2 = 2\pi R H \, (1 - t/s)$$

$$A_2 = A_1 (1 - t/s)$$

$$(4) \quad \boxed{\left(\frac{A_2}{A_1}\right) = \left(1 - \frac{t}{s}\right) = B}$$

The difference between the relative flow angle upstream the impeller and the relative flow angle at the impeller leading edge is:

$$(5) \quad \varepsilon = \beta_1 - \beta_2$$

Since the angles β_1 and β_2 are related according to:

$$(6) \quad \boxed{\tan(\beta_2) = B \tan(\beta_1)}$$

we have that:

$$\tan(\varepsilon) = \tan(\beta_1 - \beta_2)$$

$$\tan(\varepsilon) = \frac{\tan(\beta_1) - \tan(\beta_2)}{1 + \tan(\beta_1)\tan(\beta_2)}$$

(ε is only zero for $B = 1$ or $\beta_1 = 0$)

(ε is not the same as the incidence)

$$(7) \quad \boxed{\tan(\varepsilon) = \frac{\tan(\beta_1)(1-B)}{1 + B\tan^2(\beta_1)}}$$

This equation relates the deviation between β_1 and β_2 with the metal blockage

Equation (7) is not necessary to compute the incidence loss

Incidence loss

The incidence loss depends on the difference between the metal angle and the relative flow angle.

The model of Galvas (1973) models the loss as proportional to the squared sine of the incidence angle

$$\text{Loss} = \frac{\Delta h_L}{W^2/2} \sim \sin^2(i) \qquad i - \text{Incidence angle.}$$

The incidence angle can be defined in (2) different ways:

1) $i = \beta' - \theta = \arctan(B \cdot \tan(\beta_1)) - \theta$

$\tan(\beta') = B\tan(\beta_1)$

{ Incidence is defined as the mismatch between the relative flow angle at the leading edge and the metal angle.

2) $i = \beta_1 - \beta_{1,opt} = \beta_1 - \arctan\left(\frac{1}{B}\tan\theta\right)$

$(\beta_{1,opt} \Rightarrow \beta_2 = \theta)$

$\tan(\beta_{1,opt}) = \frac{1}{B}\tan(\beta'_{opt}) = \frac{1}{B}\tan\theta$

{ Incidence is defined as the mismatch between the upstream relative flow angle, and the hypothetical angle that would lead to no mismatch with the metal angle at the leading edge

The two definitions lead to the same result when $B=1$ $(t/s=0)$, but the results are not exactly the same when $B<1$ (small discrepancy)

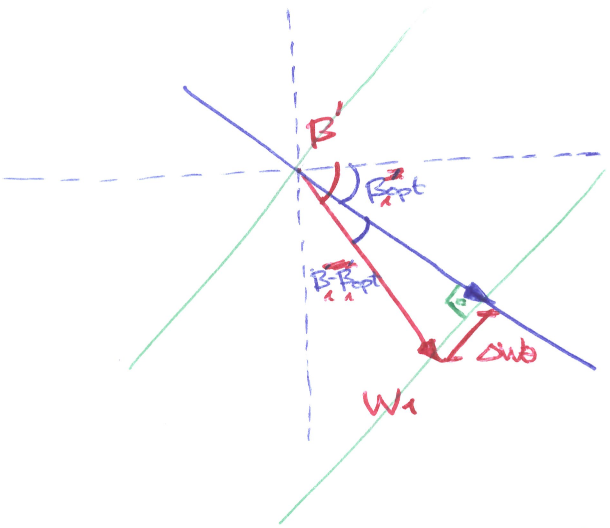
They are almost the same thing, but not exactly the same. The discrepancy increases as $\frac{t}{s}$ increases

---

Rationale for incidence loss model

When the rel angle of the flow is not equal to the blade angle there is an incidence.

The assumption is that the fraction of kinetic energy (normal) perpendicular to the optimal flow angle is lost: converted into internal (enthalpy) energy which manifests itself as an increase of entropy



Optimal direction (or metal angle)

$$\Delta W_\theta = W_1 \cdot \sin(\beta - \beta_{opt})$$

This model is limited in that it assumes the same loss at positive and negative incidence

$$\text{total enthalpy loss} = \frac{\Delta W_\theta^2}{2} = \frac{W_1^2}{2} \cdot \sin^2(\beta_1 - \beta_{1,opt})$$

Where $\tan(\beta_1)_{opt} = \left(\frac{1}{B}\right) \tan\theta$

Other authors, like Conrad, consider that the loss is only a fraction of the kinetic energy associated with the normal component of velocity, such that

$$\Delta h = f \cdot \frac{1}{2} W_1^2 \cdot \sin^2(\beta_1 - \beta_{1,opt}) \quad \text{where } f = [0{,}5,\ 0{,}7]$$