Disk Friction Losses in Turbomachinery

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1 Introduction

In this note, we introduce a simple physical model to predict the torque and power loss due to disk friction in terms of angular speed, disk rim radius, fluid density, and torque coefficient. In addition, the semi-empirical correlation proposed by Daily and Nece 1960 to compute the torque coefficient in terms of the the disk Reynolds number and the spacing to radius ratio is presented.

2 Definition of disk friction

Disk friction (also known as windage) is a loss mechanism present in all turbomachinery applications that is caused by viscous dissipation due to the velocity gradients between rotating disks and stationary walls, see Figure 1. This viscous torque reduces the amount of power that is available at the shaft couple with respect to the power that is transmitted from the fluid to the blades. The effect of disk friction ranges from being negligible in some cases to being one of the main sources of loss affecting the performance of the turbine in other cases.

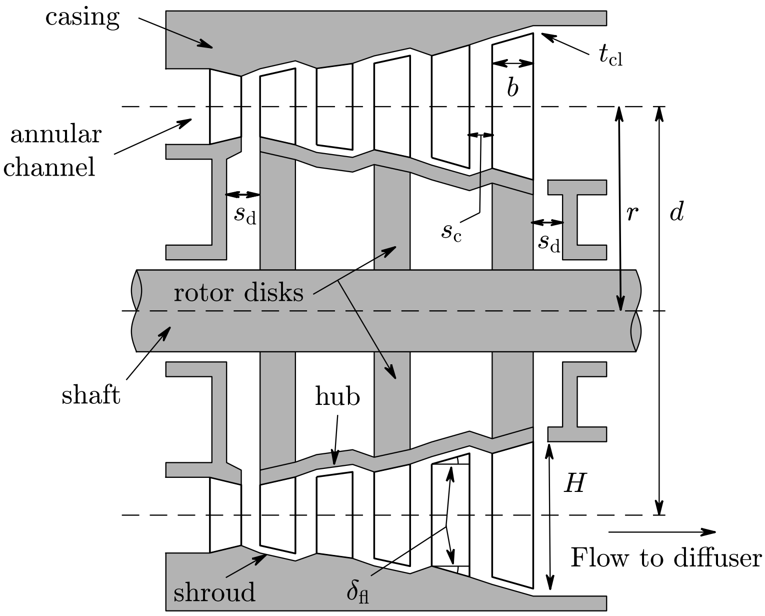


Figure 1: Axial-radial view of a three stage axial turbine.

3 A simple disk friction model

Disk friction losses are difficult to quantify accurately because the flow pattern occurring between rotating disks and stationary walls is complex. However, in most turbomachinery applications, it is sufficiently accurate to estimate the disk friction losses using semi-empirical methods such as the one proposed by Daily and Nece (1960). This method is summarized in Glassman (1973, Chapter 8).

The torque due to viscous stress on *both sides* of a rotating disk is given by Eqs. (1) and (2), where differential area element is given by $\mathrm{d}A = 2\pi r\,\mathrm{d}r$, the viscous stress is given in terms of a skin friction coefficient C_f such that $\tau = \frac{1}{2}\,C_\mathrm{f}\,\rho\,\omega^2 r^2$, and the relation between the torque coefficient C_m and skin friction coefficient is given by $C_\mathrm{m} = \frac{4\pi}{5}\,C_\mathrm{f}$. To perform the integration we have assumed mean uniform values for the skin friction coefficient and density. In addition, the lower limit of the integral was set to zero and the upper limit to the rim radius $R = r$. In a turbomachine, the lower limit of the integral would not be zero, but it would be a finite value given by the shaft radius. However, since the shaft radius is usually very small with respect to the rim radius, the error incurred assuming that the lower limit of the integral is zero is negligible.

$$M = 2\int_0^R r\,\tau\,\mathrm{d}A = 4\pi\int_0^R r^2\,\tau\,\mathrm{d}r = 2\pi\int_0^R C_\mathrm{f}\,\rho\,\omega^2 r^4\,\mathrm{d}r = \frac{2\pi}{5}\,C_\mathrm{f}\,\rho\,\omega^2 R^5 \tag{1}$$

$$\boxed{M = \frac{1}{2}\,C_\mathrm{m}\,\rho\,\omega^2 R^5} \tag{2}$$

The power dissipated due to disk friction is given by the product of angular speed and disk friction torque and it is given by Eq. (3). Note that, for a given blade speed, the disk friction loss decreases as the diameter decreases and the angular speed increases.

$$\dot{W}_\mathrm{windage} = \tau\,\omega = \frac{1}{2}\,C_\mathrm{m}\,\rho\,\omega^3 R^5 \tag{3}$$

Daily and Nece (1960) suggested a correlation for the torque coefficient in terms of disk Reynolds number, defined as $\mathrm{Re} = \rho\,\omega\,R^2/\mu$, and the ratio of axial spacing to rim radius ratio s_d/R. Daily and Nece (1960) suggested the existence of four flow regimes[1] and proposed a correlation to fit the experimental measurements for each of these regimes:

- Regime I – Laminar flow, small clearance:

$$C_\mathrm{I} = \frac{2\pi}{\mathrm{Re}}\left(\frac{s_\mathrm{d}}{R}\right)^{-1} \tag{4}$$

- Regime II – Laminar flow, large clearance:

$$C_\mathrm{II} = \frac{3.70}{\mathrm{Re}^{1/2}}\left(\frac{s_\mathrm{d}}{R}\right)^{1/10} \tag{5}$$

- Regime III – Turbulent flow, small clearance:

$$C_\mathrm{III} = \frac{0.080}{\mathrm{Re}^{1/4}}\left(\frac{s_\mathrm{d}}{R}\right)^{-1/6} \tag{6}$$

- Regime IV – Turbulent flow, large clearance:

$$C_\mathrm{IV} = \frac{0.0102}{\mathrm{Re}^{1/5}}\left(\frac{s_\mathrm{d}}{R}\right)^{1/10} \tag{7}$$

[1] In regimes I and III the axial gap is small and the boundary layers on the rotor disk and casing are merged while in regimes II and IV the sum of the thickness of the boundary layers on the rotor and the casing is less than the axial gap and there exists a core of rotating fluid with uniform velocity.

The flow regime of the flow for a given Reynolds number and axial gap can be found computing the loss coefficient for each regime and checking which one has the largest value. In other words, the torque coefficient is given my the by the maximum torque coefficient of the four regimes, Eq. (8).

$$C_{\mathrm{m}} = \max(C_{\mathrm{I}}, C_{\mathrm{II}}, C_{\mathrm{III}}, C_{\mathrm{IV}}) \tag{8}$$

The correlation for the torque coefficient as a function of the disk Reynolds number and the axial spacing to radius ratio is shown in Figure 2. It can be observed that the torque coefficient decreases monotonously with the Reynolds number and that the influence of the axial spacing to radius ratio depends on the flow regime.

Daily and Nece (1960) developed their correlation from experimental tests using water and four types of oil as working fluid the experimental data was within the ranges of Reynolds number $10^3 < \mathrm{Re} < 10^7$ and axial spacing to radius ratio $0.0127 < s_{\mathrm{d}} < 0.217$. However, Daily and Nece (1960) suggest that their correlation can also be used to predict the torque coefficient for $\mathrm{Re} > 10^7$.

For the case of turbomachinery applications, where the Reynolds number is expected to be high ($\mathrm{Re} > 10^7$), it seems reasonable to adopt a design value $\left(\frac{s_{\mathrm{d}}}{R}\right) = 0.05$ for the axial gap to thickness ratio because it gives a relatively low torque coefficient for a wide range of Reynolds numbers.

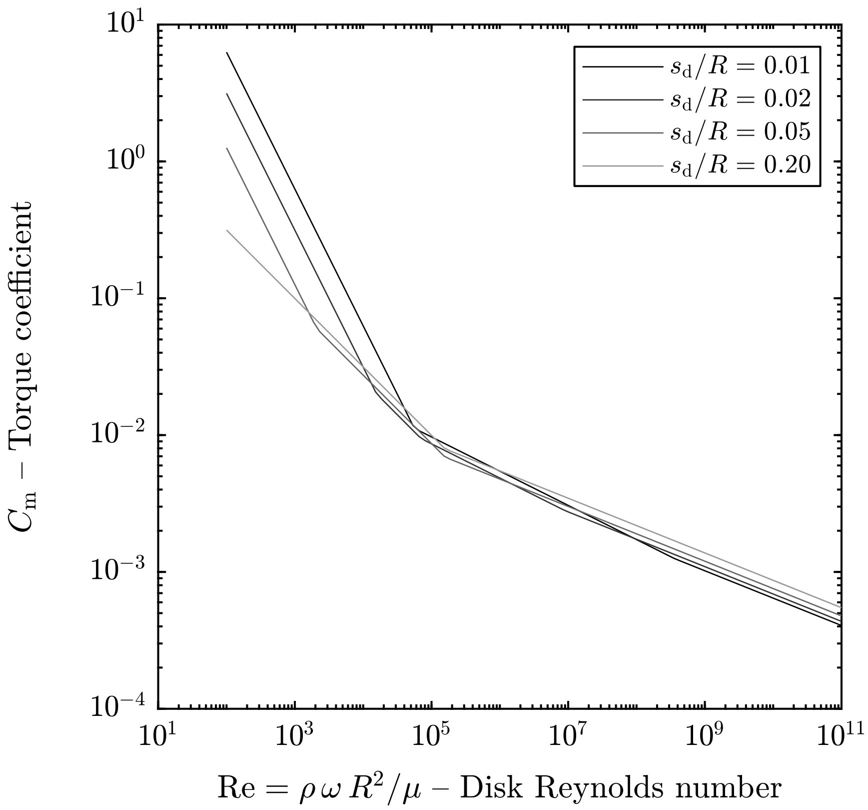


Figure 2: Correlation for the torque coefficient as a function of the disk Reynolds number and the axial spacing to rim radius ratio proposed by Daily and Nece (1960).

4 Disk friction in turbomachinery

4.1 Lost torque and entropy generation interpretations of disk friction loss

As discussed in Denton (1993), windage losses can be interpreted in terms of viscous torque or in terms of entropy generation. Denton argues that it is more sound to regard the windage loss as an entropy generation mechanism. The entropy generated due to viscous friction between the rotating disks and the stationary walls must find its way into the flow and be present at the turbine outlet. As a result of the entropy generation due to friction, the temperature of the fluid increases and the work exchanged with any downstream turbomachinery stage (compressor or turbine) increases. This is because the isobars of the enthalpy-entropy plane diverge as temperature increases. The view of windage loss in terms of lost torque is limited because it does not account for this reheat effect.

4.2 A quick thought about the entropy generation interpretation

4.2.1 Axial turbines

This section discusses a possible implementation of disk friction losses in a mean-line turbomachinery model using the entropy generation interpretation of windage loss.

Consider the two control volumes shown in Figure 3. If we assume that the energy exchange between CV1 and CV2 is equal to the power dissipated by disk friction and that the entropy exchange is equal to the entropy generated due to disk friction we can obtain expressions for the enthalpy Eq. (9) and the entropy Eq. (10) at the inlet of the rotor, where the power dissipated is given by Eq. (11), and the entropy generated is given by Eq. (12). The temperature and density of the fluid in the axial cavity can be approximated as those of the static state upstream of the rotor.

$$h_2 = h_1 + \frac{\dot{W}_{\text{windage}}}{\dot{m}} \tag{9}$$

$$s_2 = s_1 + \frac{\dot{\sigma}_{\text{windage}}}{\dot{m}} \tag{10}$$

$$\dot{W}_{\text{windage}} = \frac{1}{2} C_{\text{m}}\, \rho_{\text{f}}\, \omega^3 R^5 \tag{11}$$

$$\dot{\sigma}_{\text{windage}} = \frac{\dot{W}_{\text{windage}}}{T_{\text{f}}} = \frac{1}{2\,T_{\text{f}}} C_{\text{m}}\, \rho_{\text{f}}\, \omega^3 R^5 \tag{12}$$

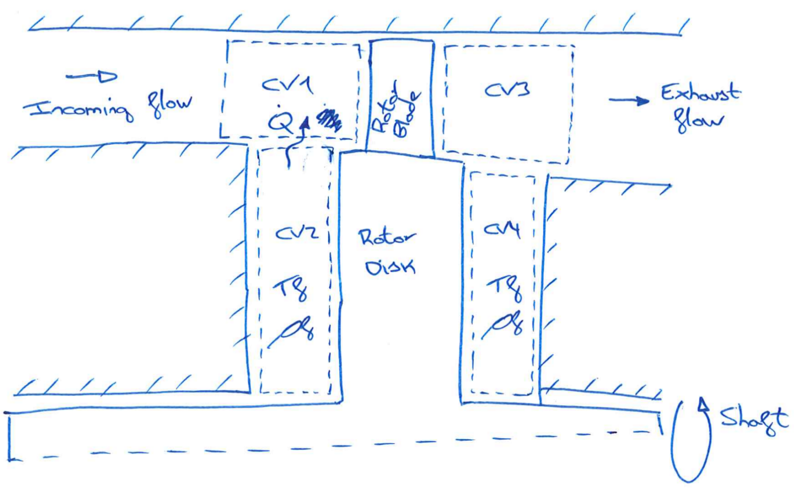
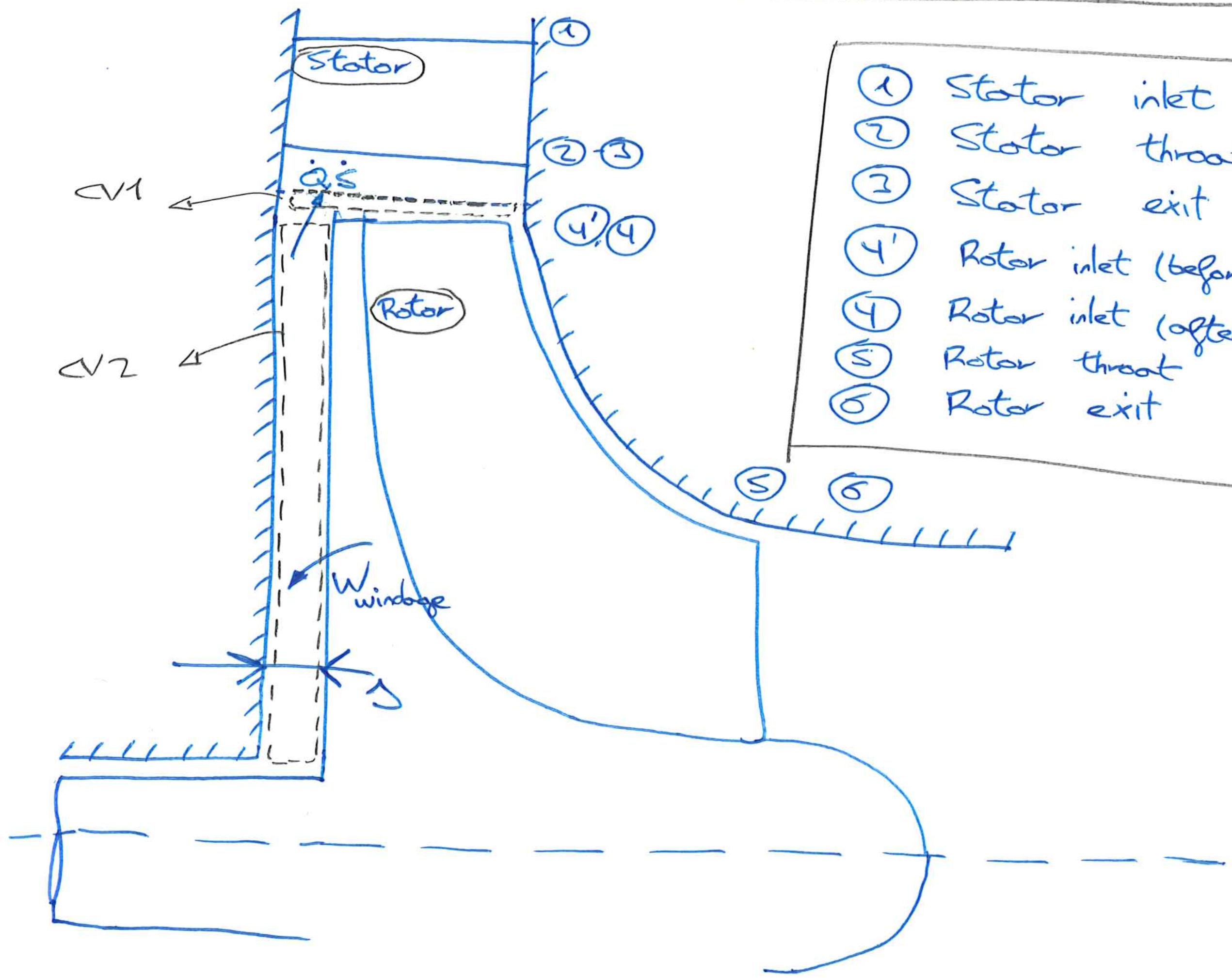


Figure 3: Sketch illustrating windage losses in axial turbines

Windage as an entropy generation mechanism in radial turbines



1 Stator inlet
2 Stator throat
3 Stator exit
4' Rotor inlet (before windage)
4 Rotor inlet (after windage)
5 Rotor throat
6 Rotor exit

Control volume analysis

CV1 → Interspace before the rotor where the mixing occurs

CV2 → Gap beteen rotor and wall where the viscous friction (windage loss) occurs

Modelling approach

Model the fluid mixing at control volume 1 as a heat flow rate $\boxed{\dot{Q}}$ and entropy flow rate $\boxed{\dot{S}}$ coming from control volume 2

Set up energy/entropy balances to find $\dot{Q}$ / $\dot{S}$

Control Volume analysis

$\Rightarrow CV_1 \begin{cases} \text{Energy} & \dot{m}\cdot(h_4'-h_4)+\dot{Q}=0 \\ \text{Entropy} & \dot{m}\,(s_4'-s_4')+\dot{S}=0 \end{cases}$

$\Rightarrow CV_2 \begin{cases} \text{Energy} & \dot{Q}=\dot{W}_{windage} \\ \text{Entropy} & \dot{S}=\dfrac{\dot{Q}}{T_g} \end{cases}$

(assuming no heat addition to the walls)

(entropy generated due to friction that goes into the main flow)

From these equations we can find (h_4, s_4):

$$\boxed{\begin{aligned} h_4 &= h_4' + \frac{\dot{W}_{windage}}{\dot{m}} \\ s_4 &= s_4' + \frac{1}{T_g}\cdot\left(\frac{\dot{W}_{windage}}{\dot{m}}\right) \end{aligned}}$$

Where (h_4', s_4') is known from state 3 and the interspace equations and $\dot{W}_{windage}$ is given by:

$$\boxed{\dot{W}_{windage} = T_{windage}\cdot\omega = \frac{1}{2}C_m\cdot\rho_g\cdot\omega^3 R_4^5}$$

With $C_m = C_m\left(\frac{s}{R_3}, \text{Reynolds}\right)$ given by the Daily and Nece (1960) correlations.

The quantities (ρ_g, T_g) can be approximated as $\begin{cases} \rho_g \simeq \rho_4' \\ T_g \simeq T_4' \end{cases}$

References

Daily, J. W. and R. E. Nece (1960). “Chamber Dimension Effects on Induced Flow and Frictional Resistance of Enclosed Rotating Disks”. In: *Journal of Basic Engineering*, pp. 217–230.

Denton, J. D. (1993). “The 1993 IGTI Scholar Lecture: Loss Mechanisms in Turbomachines”. In: *Journal of Turbomachinery* 115.4, pp. 621–656.

Glassman, A. J. (1973). *Turbine Design and Application*. NASA Special Publications.