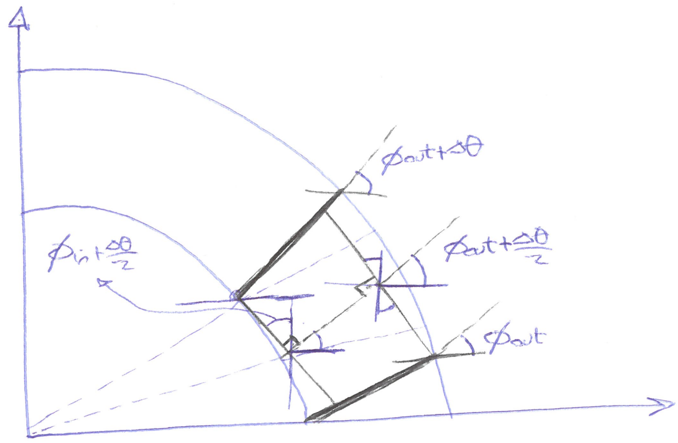
Exact calculation of the throat area in a radial cascade

In order to calculate the exact throat area of a radial cascade it is necessary to specify the shape of the camberline



The ~~the~~ throat width is best approximated by the intersection method. In this method, the channel meanline method, the is assumed to follow the direction perpendicular to $\phi_{out}+\frac{\Delta\theta}{2}$

⇒ At the leading edge we have:

β_{in}

$$\begin{cases} x_{throat} = x_{leading}^{\Delta\theta} + W \cdot \sin\left(\phi_{in} + \frac{\Delta\theta}{2}\right) = x_{camber}^{\theta_0}(U) \\ y_{throat} = y_{leading}^{\Delta\theta} - W \cdot \cos\left(\phi_{in} + \frac{\Delta\theta}{2}\right) = y_{camber}^{\theta_0}(U) \end{cases}$$

Where the superscript θ_0 denotes the blade starting at θ_0 and the subscript denotes the blade starting at $\theta_0+\Delta\theta$.

The coordinates of the camberline $(x_{camber}, y_{camber})[U]$ are a function of the parameter U.

The solution of the system of equations are the values $[U, W]$ that lead to the equality $\begin{cases} x_{throat} = x_{camber} \\ y_{throat} = y_{camber} \end{cases}$

⇒ Similarly at the trailing edge we have

$$x_{throat} = x_{trailing}^{\theta_0} - W \sin\left(\phi_{out} + \frac{\Delta\theta}{2}\right) = x_{camber}^{\Delta\theta}(U)$$

$$y_{throat} = y_{trailing}^{\theta_0} + W \cos\left(\phi_{out} + \frac{\Delta\theta}{2}\right) = y_{camber}^{\Delta\theta}(U)$$

Where $\phi_{out} = \beta_{out} + \theta_{out}$ and $\theta_{out} = \theta_0 + \hat{\theta}$ → polar change depending on each type of camberline

ϕ_{out} → computed from MATLAB camberline functions.

Alternatively, the throat area at the leading and trailing edges can be computed using the projection method.

In this case the leading/trailing edge point is projected into the adjacent camberline by computing the minimum distance between a point and a curve

$$\Rightarrow \quad \min_{u \in \mathbb{R}} \; \| \vec{r}_{camber} - \vec{r}_{\substack{leading \\ trailing}} \| = \sqrt{(x_c(u) - x_p)^2 + (y_c(u) - y_p)^2} \quad \text{s.t} \quad u \in [0,1]$$

This optimization problem can be easily solved numerically.

The limitation of this approach is that the resulting throat line is perpendicular to the camberline at the projected point, rather than being perpendicular to the channel midline with slope $\phi + \frac{\Delta\theta}{2}$