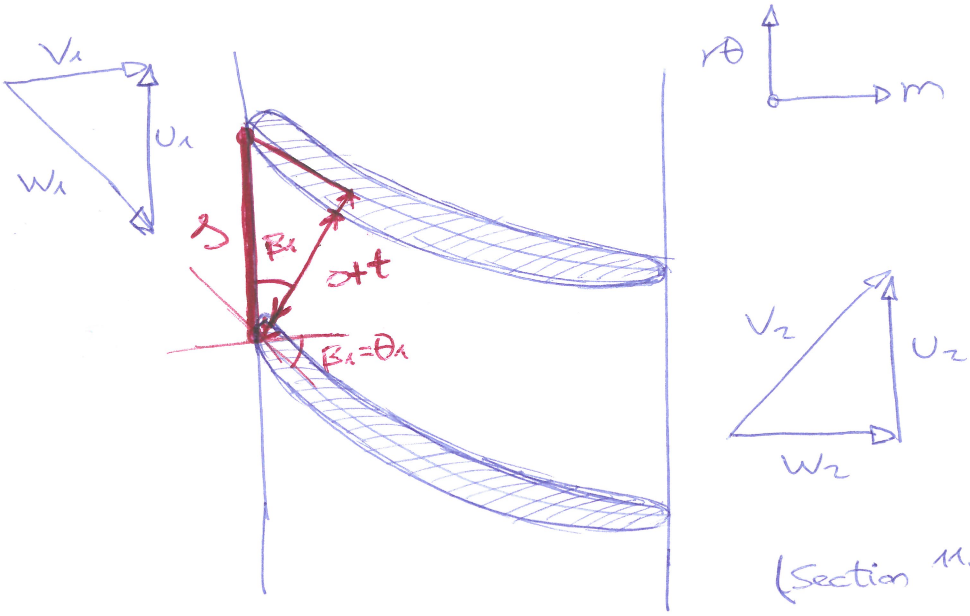
Impeller throat area derivation

The area at the throat near the impeller can be calculated using the cosine rule in combination with the blade thickness near the impeller leading edge:



Euler equation

$$W = V_{\theta_2} \cdot U_2 - V_{\theta_1} \cdot U_1$$

(Section 11.5 of Casey and Robinson 2021)

(A better estimation is possible if the β_{in} distribution is known) divide in intervals

The cosine rule gives:

$$s \cdot \cos(\theta_1) = o + t \qquad \text{where} \qquad s = \left(\frac{2\pi r}{Z}\right)$$

- s is the blade spacing
- o is the blade opening
- t is the blade thickness near the leading edge
- r is the radial coordinate
- Z is the number of blades:

Evaluating the ~~opening~~ throat area from the opening gives

Assuming arithmetic mean radius

$$A_{th} = Z \cdot o \cdot (r_t - r_h) = Z \cdot (r_t - r_h) \cdot \left[s \cdot \cos\theta_1 - t \right]$$

$$A_{th} = Z(r_t - r_h) \cdot \left[\frac{2\pi (r_h + r_t)}{2Z} \cos\theta_1 - t \right] = \pi (r_t^2 - r_h^2) \cos\theta_1 - t \cdot Z (r_h - r_t)$$

$$A_{th} = A_{in} \cos\theta_1 - t \cdot Z \cdot (r_h - r_t) = A_{in} \cos\theta_1 - t \cdot \frac{2 \cdot \pi (r_h + r_t)}{2 \cdot s} (r_h - r_t)$$

$$A_{th} = A_{in}^{\cos\theta_i} - \frac{t}{s}\pi(r_t^2 - r_h^2) = A_{in}^{\cos\theta_i} - \left(\frac{t}{s}\right)\cdot A_{in}$$

$$A_{th} = \left(\cancel{1}^{\cos\theta_i} - \frac{t}{s}\right)A_{in}$$

$$\boxed{A_{th} = \left(\cos\theta_i - \frac{t}{s}\right)A_{in}}$$

$A_{th} = (1-B_{th})\,A_{in}$ where $B_{th} = [1 - \cos\theta] + t/s$

Therefore the area at the throat is reduced with respect to the area at the impeller inlet because of:

- Metal angle θ_i
- ~~blockage due~~ Blade thickness ~~blockage~~ to spacing ratio (metal blockage)
- (Possibly additional aerodynamic blockage) δ^*

The acceleration from the impeller inlet to the throat can be estimated as:

assume $\Delta U = 0$

$$h_{in} + \frac{W_{in}^2}{2} - \cancel{\frac{U_{in}^2}{2}} = h_{th} + \frac{W_{th}^2}{2} - \cancel{\frac{U_{th}^2}{2}}$$

[conservation of rothalpy in the ~~relative~~ rotating frame]

$$\boxed{(1) \qquad h_{th} + \frac{W_{th}^2}{2} = h_{in} + \frac{W_{in}^2}{2}}$$

From conservation of mass we have that:

$$\dot{m} = \rho_{in} A_{in} W_{m,in} = \rho_{th}\cdot A_{th}\cdot W_{th}$$

Such that: $\rho_{in} A_{in} W_{in}\cdot\cos(\beta_{in}) = \rho_{th} A_{th}\cdot W_{th}$

$$\frac{W_{th}}{W_{in}} = \left(\frac{\rho_{in}}{\rho_{th}}\right)\cdot\left(\frac{A_{in}}{A_{th}}\right)\cdot\cos(\beta_{in}) \qquad i = \beta - \theta$$

$$\boxed{(2) \qquad W_{th} = W_{in}\cdot\left(\frac{\rho_{in}}{\rho_{th}}\right)\cdot\left[\frac{\cos(\beta_{in})}{\cos\theta_{in} - \left(\frac{t}{s}\right)}\right] = W_{in}\left(\frac{\rho_{in}}{\rho_{th}}\right)\cdot\frac{\cos(\theta_{in}+i)}{\cos\theta_{in} - \left(\frac{t}{s}\right)}}$$

W_{th} and $(\rho_{th} | h_{th})$ are computed from (1) and (2) assuming $[s_{in} = s_{th}]$

Iterative solution is required