UNIVERSITY OF TORONTO, DEPARTMENT OF ECE

ECE1762 - Homework 1

Xinyun Lv(1001091178), Yang Wang(1001319227)

September 19, 2015

1 PROBLEM I

Sort the following functions from asymptotically smallest to asymptotically largest.

$$2^{log_{10}n} \quad log_{lgn}n \quad lg(nlogn) \quad n \quad (\sqrt{2})^{lgn} \quad (lglgn)^{lglgn}$$

2 PROBLEM II

Show that any sequence of n^3+1 numbers contains either

- a strictly-increasing subsequence of length $n+1$.
- a strictly-decreasing subsequence of length $n+1$, or
- $n+1$ elements with the same value.

3 PROBLEM III

Solve the following recurrences. State tight asymptotic bounds for each function in the form $\Theta(f(n))$ for some recognizable function $f(n)$. Prove your answer. Assume reasonable but nontrivial base cases if none are supplied.

(a) $A(n) = 2A(n/4) + nloglogn$

(b) $B(n) = B(n/2) + logn$

(c) $C(n) = 3C(n/2) + nlogn$

(d) $F(n) = F(\lfloor logn \rfloor) + logn$

4 Problem IV

m balls are thrown into n bins (independently) so that each ball is equally likely to fall into any of the bins. Estimate as precisely as you can the smallest number m (as a function of n) so that the probability of all balls falling into different bins is smaller than $\frac{1}{n^c}$, for a fixed constant $c > 0$.

5 Problem V

Give a combinatorial argument to prove that

$$\sum_{i=0}^{n} \binom{n}{i} 2^i = 3^n$$

6 Problem VI

Consider a light ray entering two adjacent planes of glass on a table. At any meeting surface (between the two planes of glass, or between the top glass and the air) the light may either reflect (bounce) or continue straight through (refract). In the example below, the light ray bounces 7 times before it leaves the glass plane. The light always reflects between the bottom glass and the table. How many different paths can a light ray take if it bounces n times before it leaves the top glass plane? Give a recurrence relation to answer this question.