

ECE1762 - Homework 1

Xinyun Lv, Yang Wang

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1 PROBLEM I

Sort the following functions from asymptotically smallest to asymptotically largest.

$$2^{\log_{10} n} \quad \log_{\lg n} n \quad \lg(n \log n) \quad n \quad (\sqrt{2})^{\lg n} \quad (\lg \lg n)^{\lg \lg n}$$

2 PROBLEM II

Show that any sequence of $n^3 + 1$ numbers contains either

- a strictly-increasing subsequence of length $n + 1$.
- a strictly-decreasing subsequence of length $n + 1$, or
- $n + 1$ elements with the same value.

3 PROBLEM III

Solve the following recurrences. State tight asymptotic bounds for each function in the form $\Theta(f(n))$ for some recognizable function $f(n)$. Prove your answer. Assume reasonable but nontrivial base cases if none are supplied.

(a) $A(n) = 2A(n/4) + n \log \log n$

(b) $B(n) = B(n/2) + \log n$

(c) $C(n) = 3C(n/2) + n \log n$

(d) $F(n) = F(\lfloor \log n \rfloor) + \log n$

4 PROBLEM IV

m balls are thrown into n bins (independently) so that each ball is equally likely to fall into any of the bins. Estimate as precisely as you can the smallest number m (as a function of n) so that the probability of all balls falling into different bins is smaller than $\frac{1}{n^c}$, for a fixed constant $c > 0$.

5 PROBLEM V

Give a combinatorial argument to prove that