Resonance Frequency Based Techniques for Measuring Young's Modulus

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Introduction

Elastic modulus is an intrinsic material property and a key parameter in engineering design and materials development. A wide range of test methods is available for measuring modulus, but there is currently some uncertainty within parts of the user community about the reliability of modulus data, to the extent that many use standard handbook values in their calculations and designs. This is not recommended and can be addressed through good experimental practice and careful measurement.

Both static and dynamic modulus methods are covered in this study. Results presented show that it is possible to obtain good modulus data from the tensile test, but this generally requires a separate and dedicated test set-up using high quality averaging strain measurement and data analysis procedures.

Static Methods

A number of examples are given to illustrate the typical variation in modulus obtained from tensile test intercomparison exercises. Of course the list is by no means exhaustive, and different groups around the world have carried out a large number of similar measurements, but the results do highlight some of the issues and difficulties associated with modulus measurement.

Figure 1. BCR tensile reference material

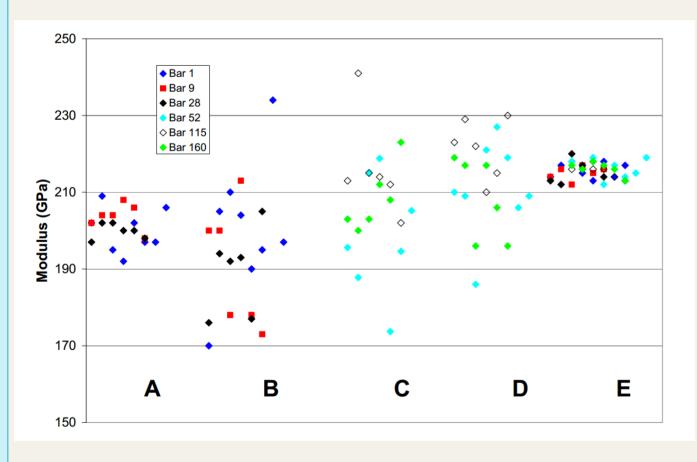


Figure 2. TENSTAND intercomparison exercise

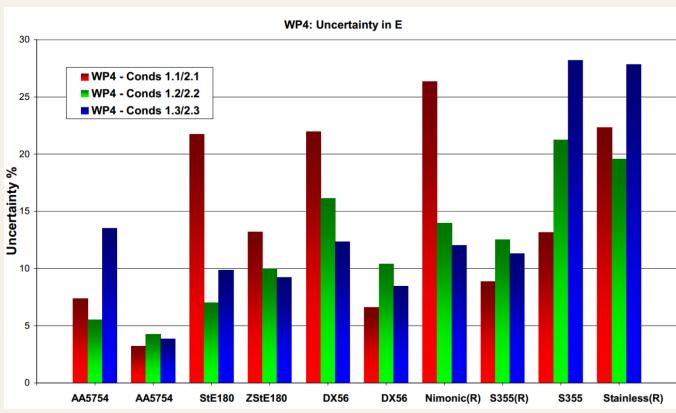


Table 1. Summary of data from intercomparison exercises

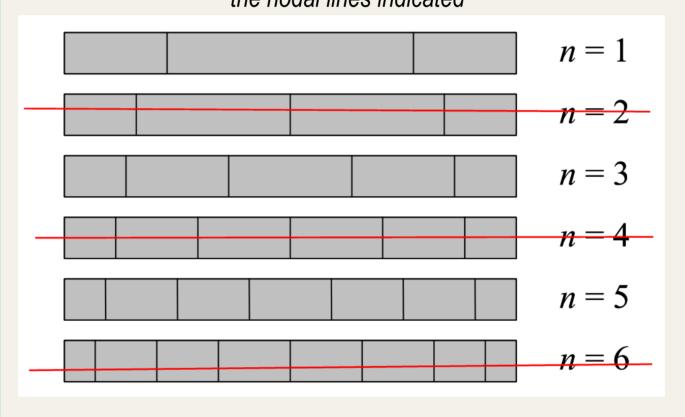
Reference	Year	Material	Uncertainty (±2SD)%
Unwin	1985	Mild steel	2
VAMAS	1997	SiC/AI MMC	6
BCR Report	2000	Nimonic 75	12
TENSTAND WP4	2005	Various	5-25
TENSTAND WP2	2005	Various-ASCII datafiles	1-6

Resonance and Impact Excitation Methods

These methods are essentially equivalent. Using usually a beam test-piece with uniform crosssection (round, square or rectangular), the characteristic vibration frequencies are determined either by continuously driving the vibration and sweeping the frequency to detect resonances, or by striking it, allowing it to 'ring', and then deconvoluting the recorded sound spectrum. The latter method is often termed the 'natural frequency' method. The same set of equations is employed for both methods in order to relate frequency to elastic modulus.

The methods and key issues are described in the following sections, with more detailed information relevant to particular set-ups or measurements – such as more detailed background on the equations and geometric correction factors, advice on selection of bar dimensions appropriate for different test modes, disc specimens and single crystal materials - given in the annexes.

Figure 4. Out-of-plane vibration modes of a 'free-free' beam with the nodal lines indicated



Calculation

For the fundamental flexure frequency of a rectangular bar:

$$\bullet E = 0.9465 \left(\frac{mf_f^2}{b}\right) \left(\frac{L^3}{t^3}\right) T_1$$

•For
$$\frac{L}{t} \ge 20$$

$$T_1 = [1.000 + 6.585(t/L)^2]$$

$$\begin{split} & \bullet \mathsf{For}^{\underline{L}}_{\frac{t}{L}} < 20 \ + \mathsf{shear} \ \mathsf{modulus} \\ & T_1 \\ &= 1 + 6.585(1 + 0.752 v + 0.8109 v^2) \left(\frac{t}{L}\right)^2 \\ & - 0.868 \left(\frac{t}{L}\right)^4 \\ & - \left[\frac{8.340(1 + 0.2023 \mu + 2.173 \mu^2) \left(\frac{t}{L}\right)^4}{1.000 + 6.338(1 + 0.1408 \mu + 1.536 \mu^2) \left(\frac{t}{L}\right)^2} \right] \end{split}$$

•For $\frac{L}{t}$ < 20 + no shear modulus

Self-Mixing Based Methods

Figure 5. Basic scheme of a self-mixing interferometer

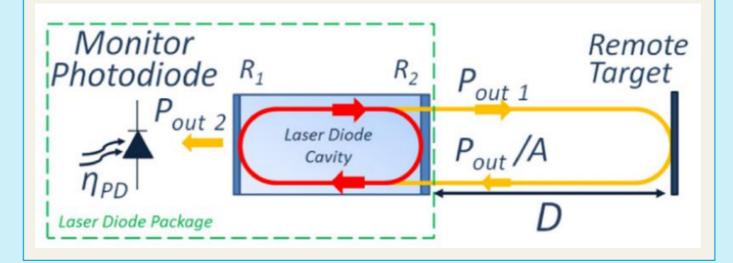
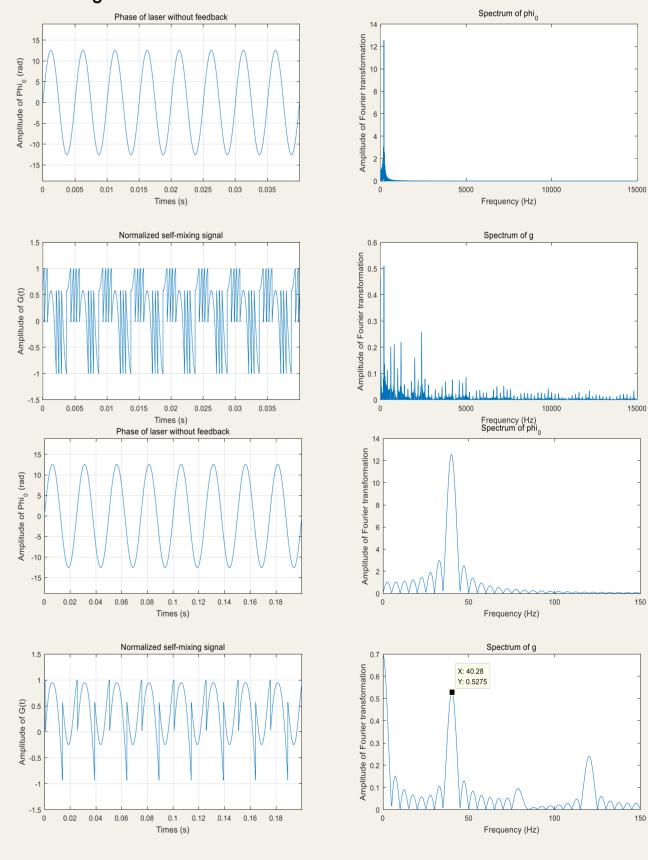


Figure 5. is a schematic diagram of the very early period technique of a interferometry in pervious laser diode were established in 1980 by Lang and Kobayashi. Self-mixing interferometry (SMI) allow a simple way to implement the detection of displacement at a scale of time and by this means, the resonance frequency of aimed specimen can be calculated. In this system, a laser diode cavity were used as laser source. The distance for the light transmitting is doubled as the distance between the laser diode package and the remote target, which is 2D that shown in Figure 5. The reflected light is directly back into the laser diode package.

Simulation

In order to verify the concept presented above, we firstly perform simulations with the aim to show the feasibility for measuring Young's modulus by the fiber-coupled SMLD. The specimen we used is a rectangular brass bar (with L = 138.35 mm, b = 12.06 mm, h = 2.23 mm, m = 30.65 g) and its Young's modulus is estimated as 120 Gpa from the literature. Thus, its fRO is calculated as 444 Hz. For simulations, the parameters associated to the SMLD are set as fs = 3 MHz (considering the bandwidth of the detection circuit used for experiments is 3 MHz), $\lambda 0$ = 785 nm, and we choose C = 3, α = 3, and the external cavity length is h0 = 0.5 m.

Figure 6. Simluation with Various Parameter Values



Conclusion

- 1, Choose relevant parameters carefully to fit the testpiece to make OOP signal clear and fringes sharp, therefore, ensures good start for signal processing.
- 2, The test-specimen is recommended to have a larger ratio between length and thickness. And the high rate can achieve a much simpler calculation.
- 3, For particular material use a dedicated test system, to fit the scope of test system with test-piece. By this way, to make dedicated system works in it's best qualitied scope.