Resonance frequency based Techniques for measuring Young's modulus

ECTE458

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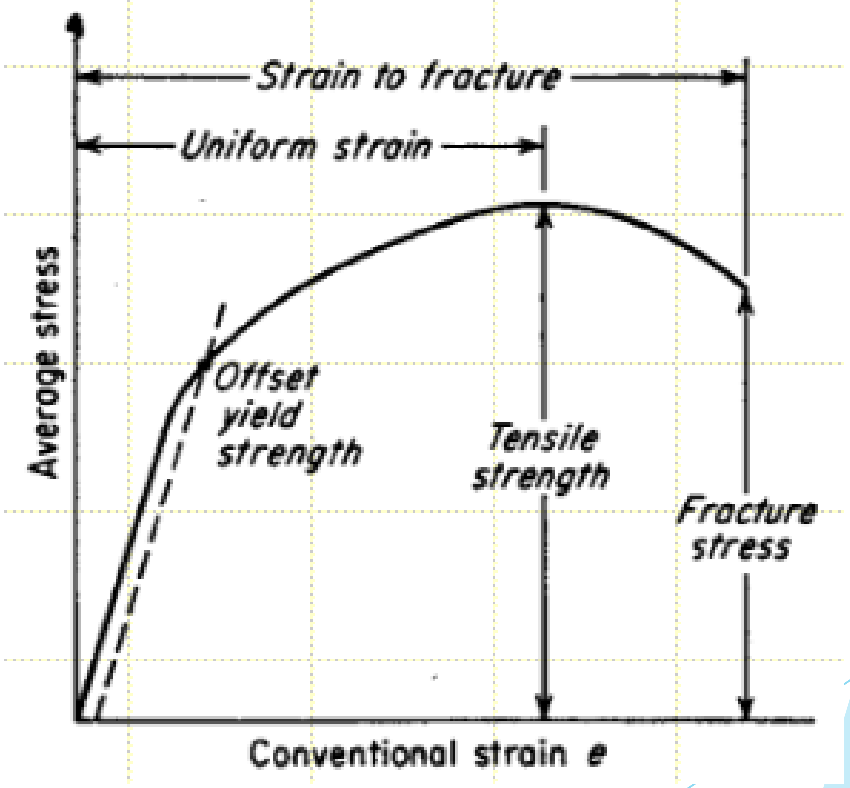
- The Concept and Measurement Principle of Young 's Modulus
- Existing techniques for measuring Young’s modulus(static & dynamic)
- Advantages self-mixing based methods

Concept of Young’s Modulus

- $E = \dfrac{Stress(force\ per\ unit\ area)}{Strain(deformation\ over\ initial\ length)}$

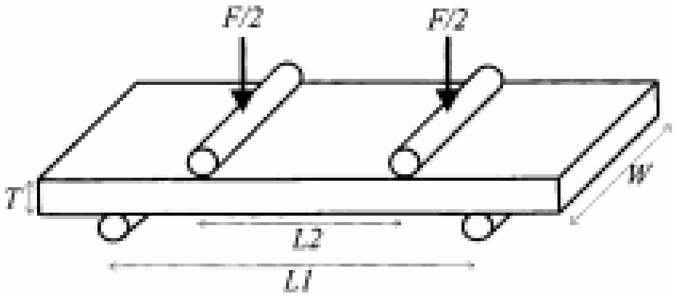
- The Young's modulus is the slope of the initial section of the curve (i.e. m in y = mx + b).



Why significant?

- Space elevator
- Skyscraper

Static methods

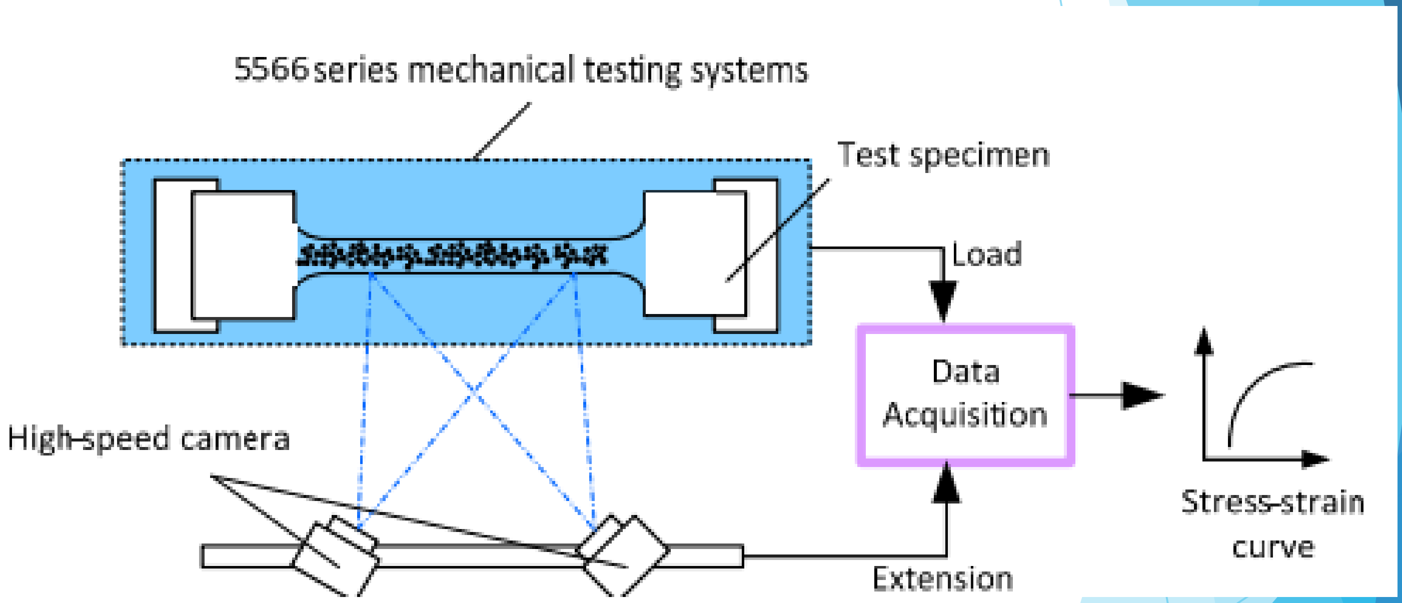


$$E = \frac{\sigma}{\varepsilon}$$

$$\sigma = \frac{3}{2}\frac{(L_1 - L_2)F}{WT^2}$$

and

$$\varepsilon = \frac{6(L_1 - L_2)T\Delta l}{L_1^3 - 3L_1L_2^2 + 2L_2^3}$$



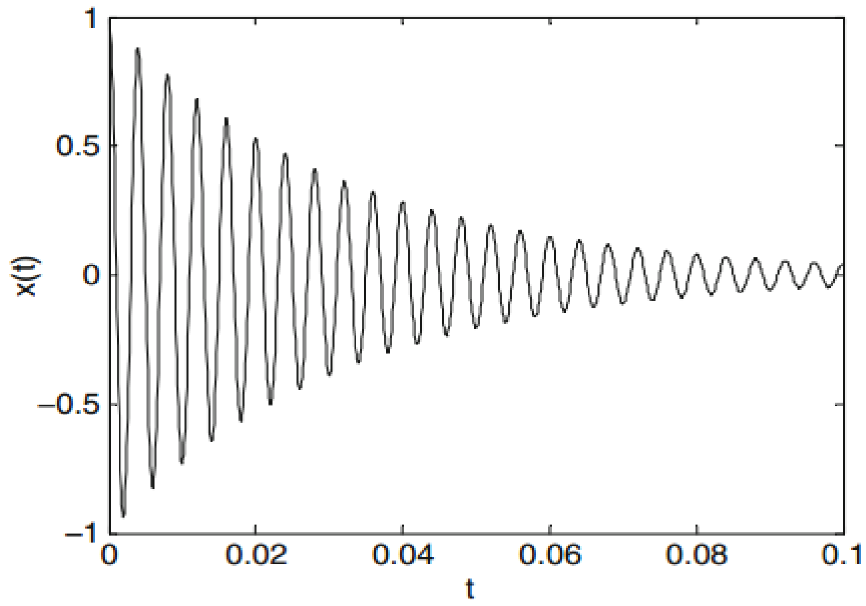
Principle of dynamic methods

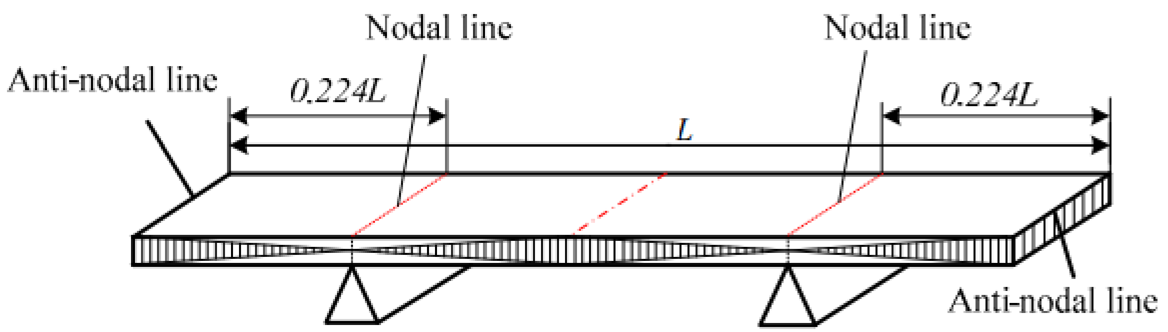
A transform from vibration frequency to Young’s Modulus.

Restricted Dimension → Restricted vibration model → Frequency → Young’s Modulus

1st-order mode vibration of a free-free rectangular specimen

- Encountered when a body is disturbed from its equilibrium position and a corresponding vibration occurs.
- Spring-mass-damper system





Principle of dynamic methods

- $E = 0.9465\left(\frac{mf_f^2}{b}\right)\left(\frac{L^3}{t^3}\right)T_1$
- For $\frac{\mathrm{L}}{\mathrm{t}} \geq 20$

  $T_1 = [1.000 + 6.585(t/L)$

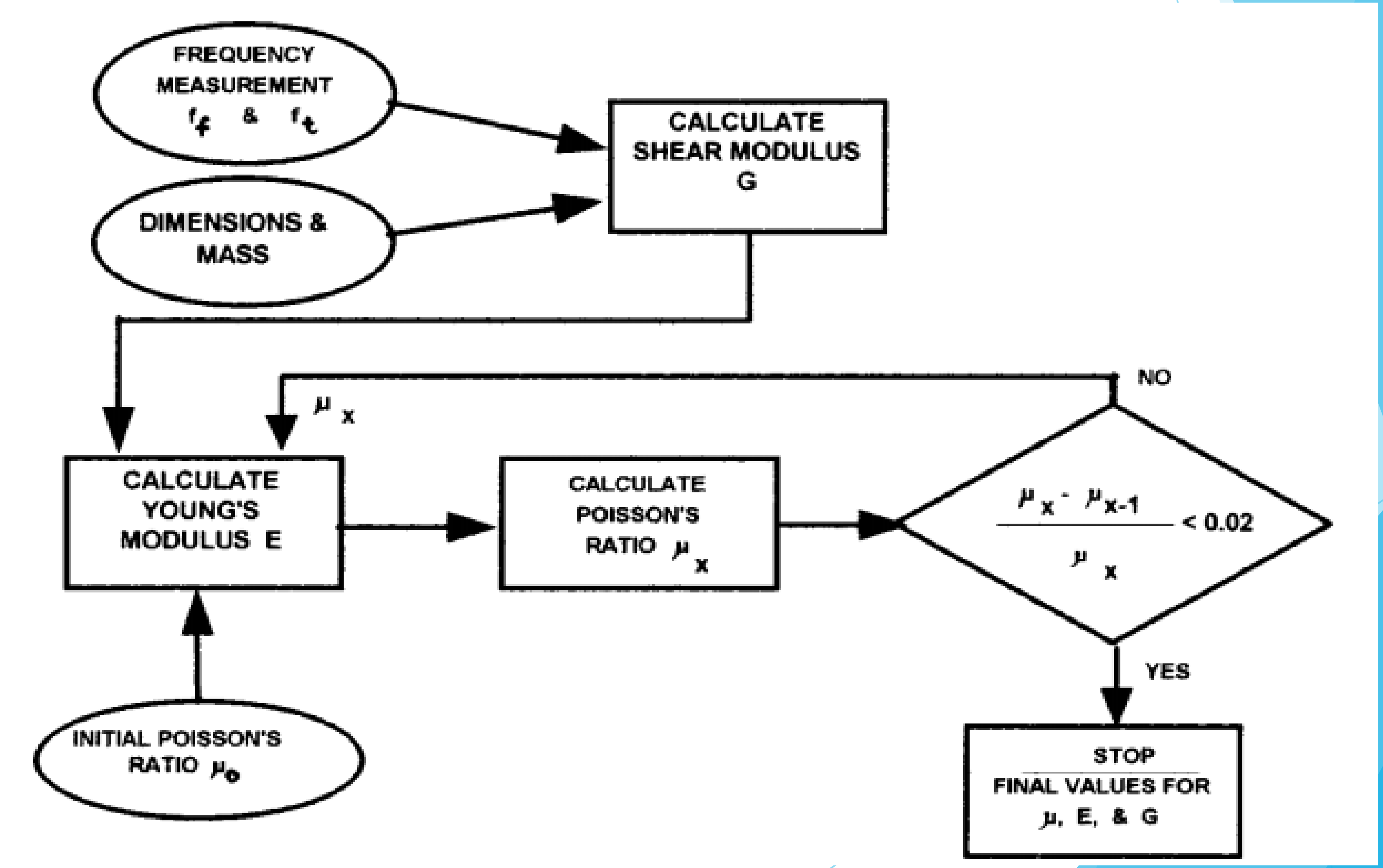
- For $\frac{\mathrm{L}}{\mathrm{t}} < 20$ + shear modul

  T_1

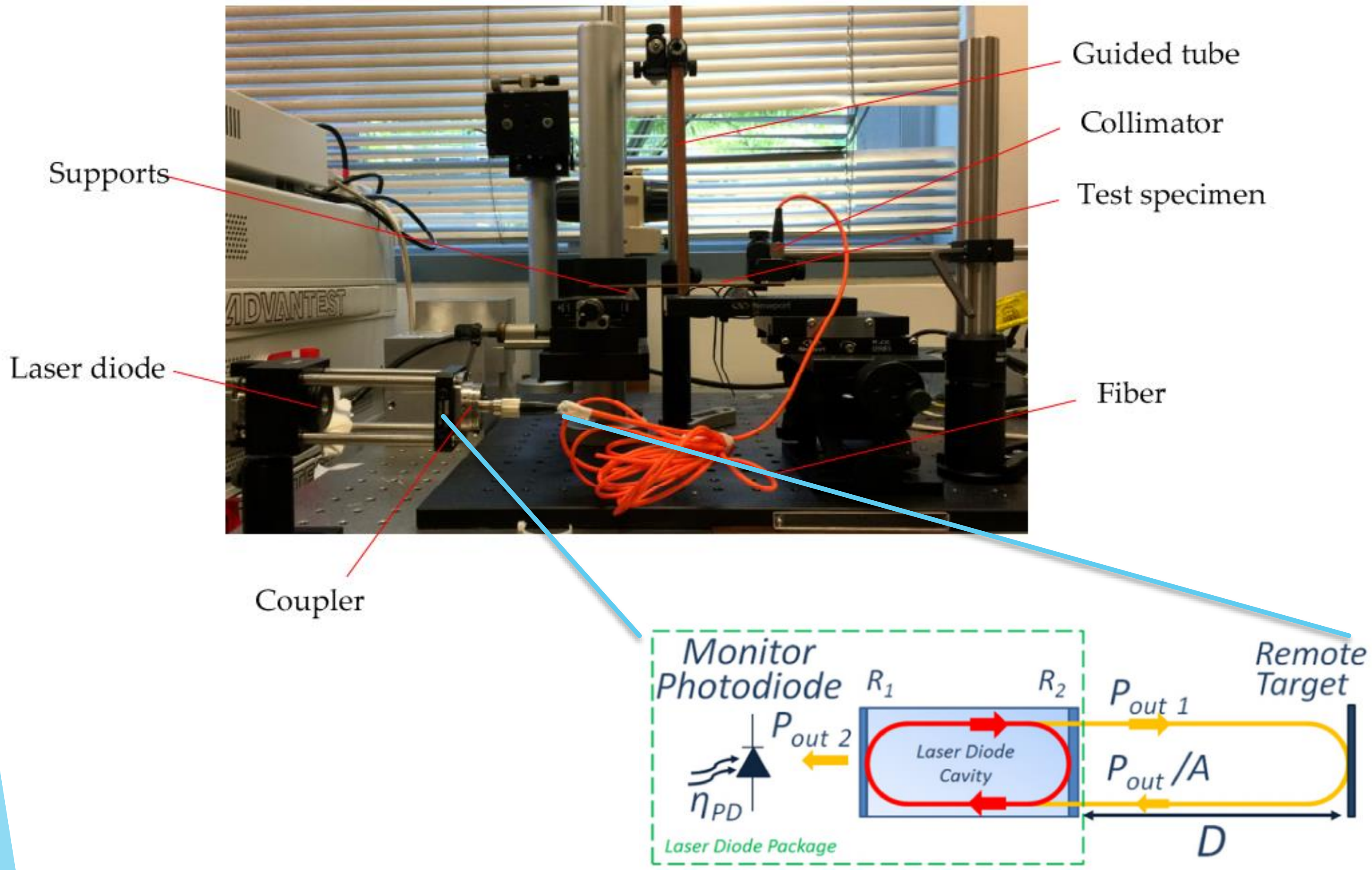
  $= 1 +$

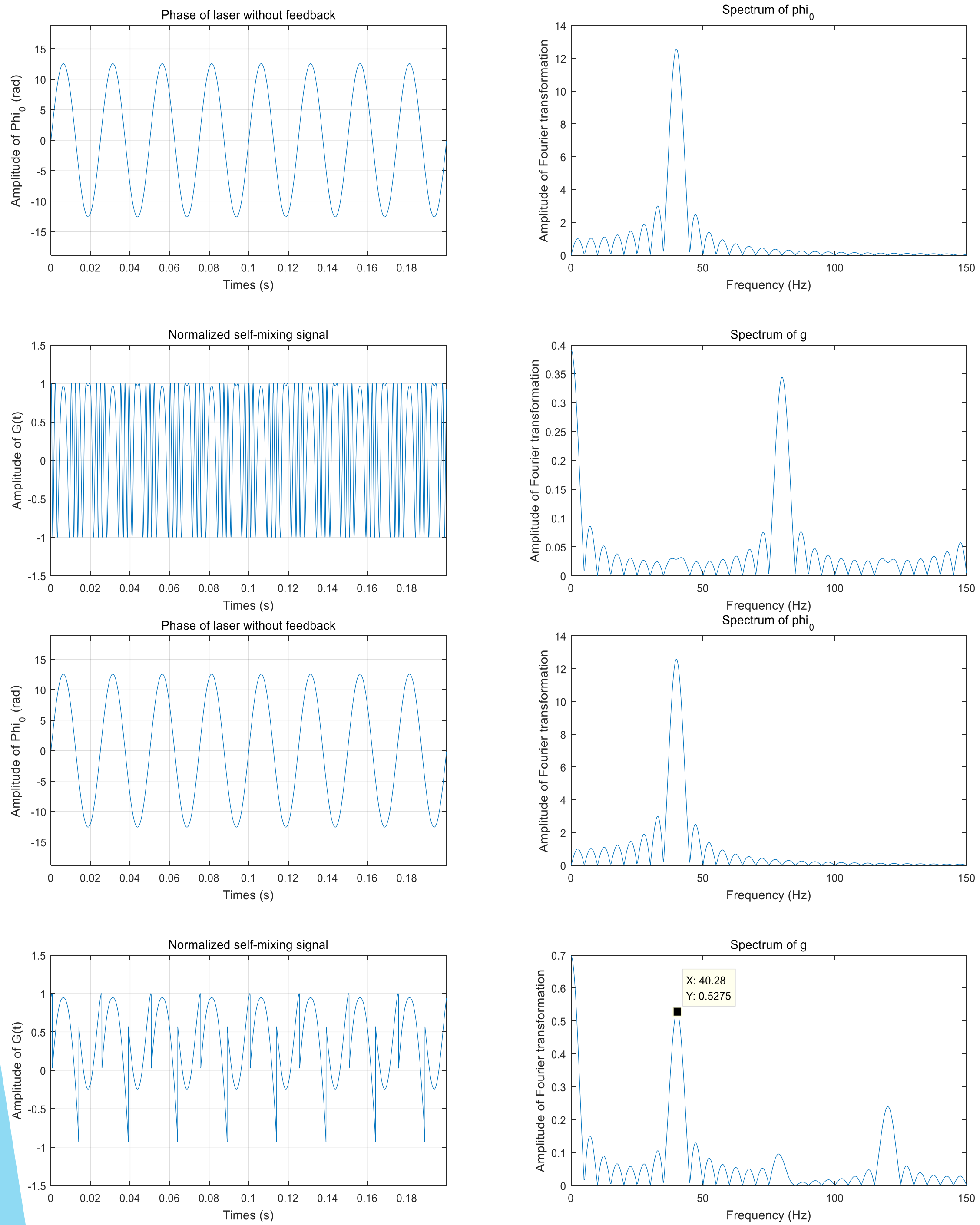
  $- \Big[\frac{}{1.0} $

- For $\frac{\mathrm{L}}{\mathrm{t}} < 20$ + no shear mo



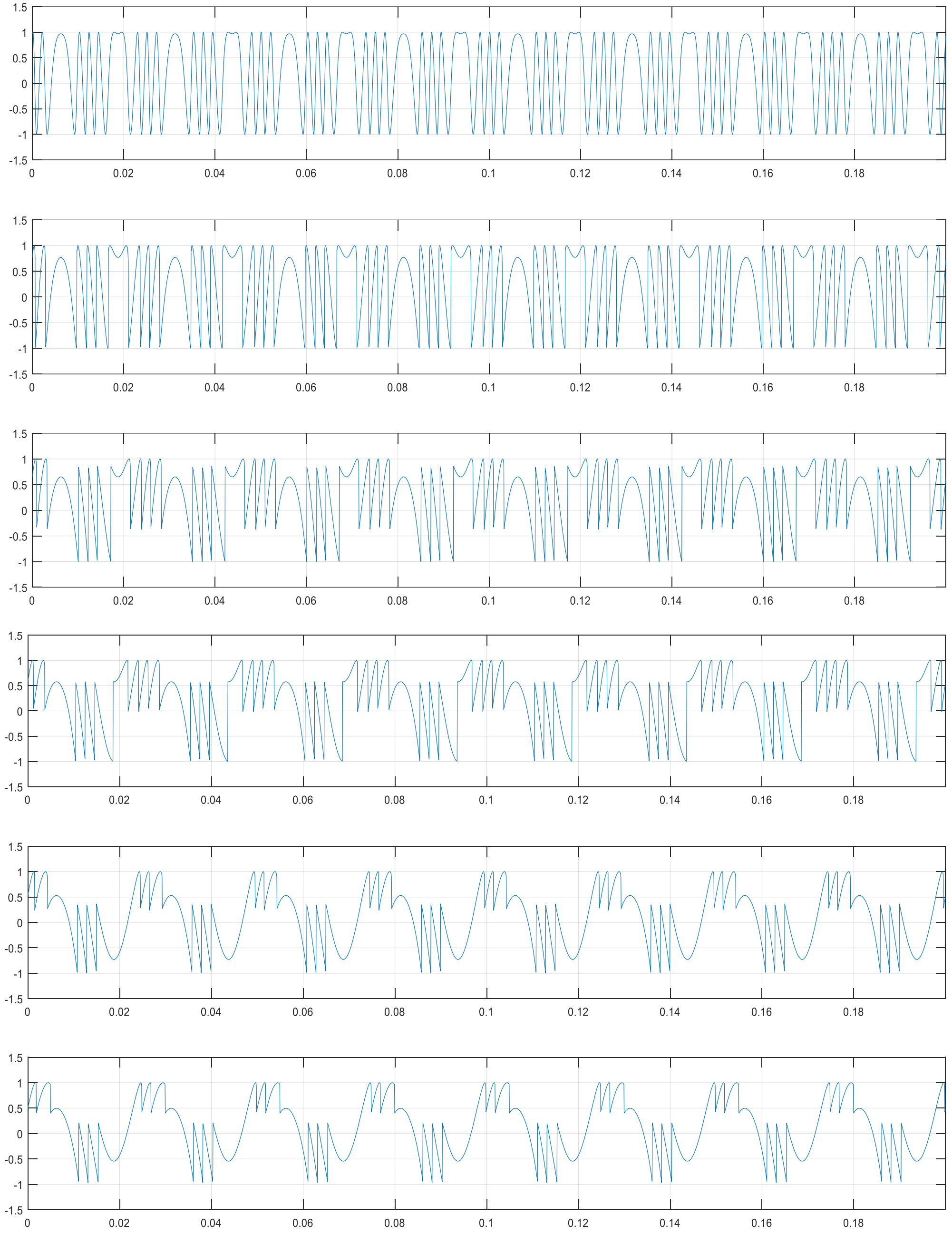
Self-mixing based methods





Simulation

- Vibration frequency (Hz)
- Sampling frequency (Hz)
- Sampling length
- Feedback Coupling factor(3)



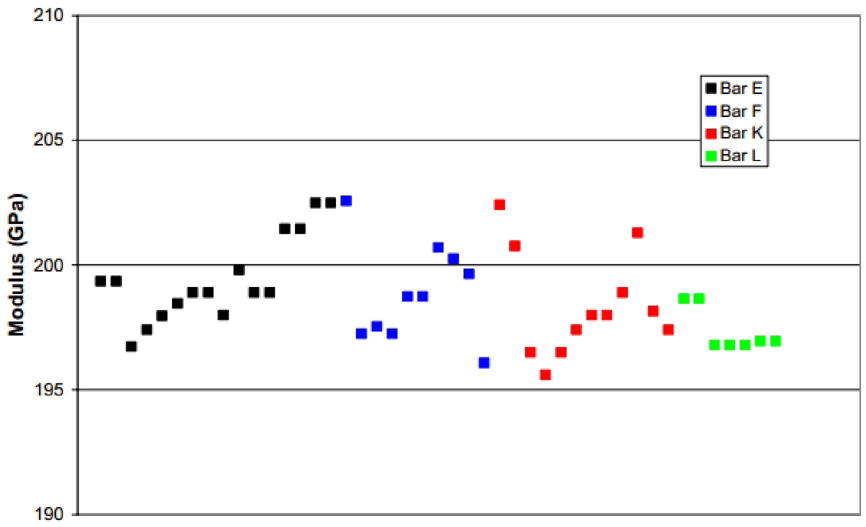
Optical feedback

- FFT spectrum of *G*(t) with different feedback factor
- A good feedback is 3-6

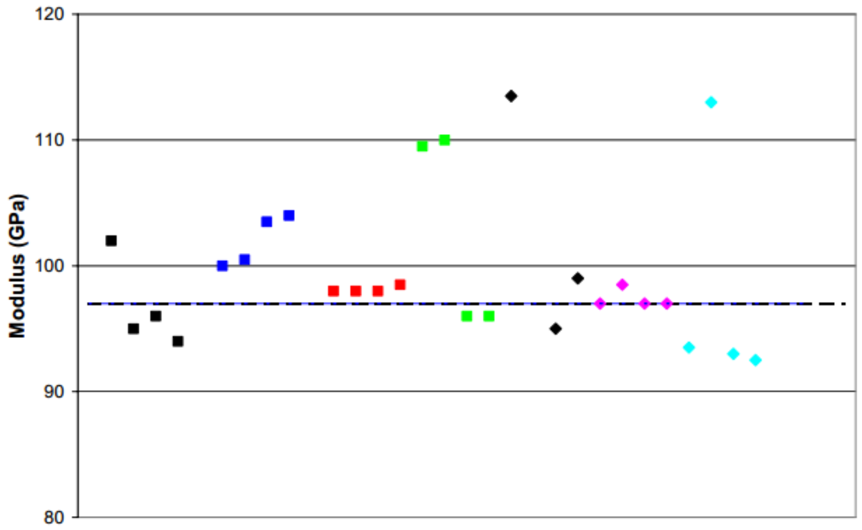
Intercomparison exercises for static methods

Unwin intercomparison exercise on mild steel

Uncertainty is ± 2, and standard deviations is ~2%



Recent VAMAS international intercomparison exercise on SiC



Data generated using copper test-piece using self-mix based method

	Weight (g)	Length (mm)	Width (mm)	Thickness (mm) - Young's Modulus (GPa)	Frequency (Hz)
Sample 1	1176	450	42	7.95-206; 8.00-202; 8.05-198	207
Sample 2	1174	450	42	7.95-206; 8.00-202; 8.05-198	207
Sample 3	1451	450	42	9.80-208; 9.85-204; 9.90-201	256
Sample 4	1454	450	42	9.80-208; 9.85-205; 9.90-202	256
Sample 5	1755	450	42	11.8-210; 11.85-207; 11.9-205	309
Sample 6	1743	450	42	11.8-208; 11.85-206; 11.9-203	309

Conclusion

- Concept of Young's Modulus
- Principle of static and dynamic methods
- Simulation of self-mixing based principle
- Intercomparison

Self-mixing based methods

- Good inherent accuracy
- Quick, simple and nondestructive

Static methods

- Large interlaboratory scatter
- Need averaging extensometry
- Specialist test