NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2014-2015

CE1008/CZ1008 – ENGINEERING MATHEMATICS

Nov/Dec 2014 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer **ALL** questions.
3. This is an open-book examination.
4. All questions carry equal marks.

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1. (a) Evaluate $\lim_{x\to 0} \dfrac{|x^2 - x + 2|}{\cos(\pi)} + \dfrac{\dfrac{1}{x-1} + \dfrac{1}{x+1}}{x}$.

(5 marks)

(b) Evaluate $\dfrac{d}{dx}\left(x\lceil x\rceil\right)$, where $\lceil\ \rceil$ denotes the ceiling function.

(6 marks)

(c) Differentiate $y = \dfrac{x^2\cos(x)}{\ln(3e^{\cos(x)})}$.

(7 marks)

(d) Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{1}{n}\left(e^{\frac{i}{n}} + \left(\dfrac{i}{n}\right)^2\right)$.

(7 marks)

2. (a) Evaluate $\int t\sec^{-1}(t)dt$.

(6 marks)

(b) Evaluate $\int \sin(11x)\cos(7x)dx$.

(5 marks)

(c) Solve the differential equation $x\frac{dy}{dx}+2y=x^3$ with the conditions $x>0$ and $y(1)=1$.

(6 marks)

(d) A rectangle is to be inscribed in a semicircle of radius 5 as shown in Figure Q2. Obtain the dimensions of the rectangle such that its area is maximum, and determine the corresponding area.

(8 marks)

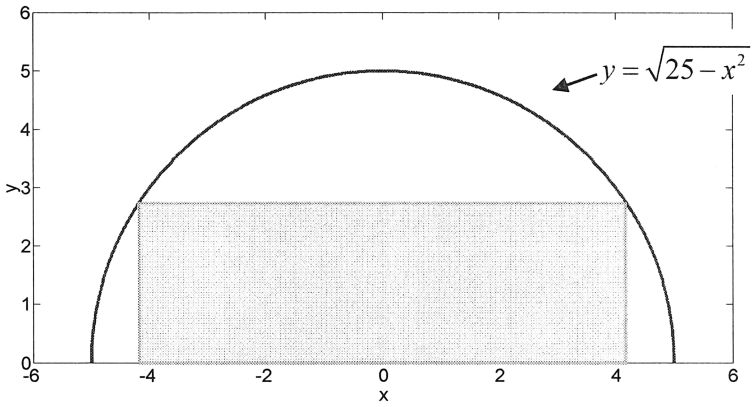

Figure Q2

3. (a) What is the difference between nominal and ordinal qualitative data?

(2 marks)

(b) Given the sample data set [3, 4, 6, 6, 7, 9, 11, 13, 20], determine the mode, the mean and the sample variance.

(3 marks)

(c) Draw a well-labelled Box-and-whisker plot for the data set given in Question 3(b) above.

(6 marks)

(d) Three students, A, B and C, are taking a Science quiz which will be graded as Pass (P) or Fail (F). The probabilities that A passes is 0.4, B passes is 0.5 and C passes is 0.6.

(i) Determine the sample space for the outcome of the grades for all three students.

(3 marks)

(ii) What is the probability that exactly 2 students pass the quiz?

(4 marks)

(e) A communication channel model is shown in Figure Q3 where the output Y is the sum of the input X and the random noise component W.

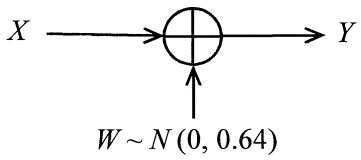

Figure Q3

(i) Given that X is a constant value, determine the probability distribution of Y.

(3 marks)

(ii) Given that X is a Bernoulli random variable with probabilities $P(X=-1)=0.4$ and $P(X=+1)=0.6$, determine the probability that the channel output Y is a positive value.

(4 marks)

4. (a) The distribution of the weights of 1000 students is normal with a mean of 55 kg and a variance of 25 kg^2. 100 random samples of size 16 are taken from this population. Determine the following:

(i) The mean and standard deviation of the sampling distribution.

(4 marks)

(ii) The number of sample means that fall between 53 kg and 56 kg.

(6 marks)

(b) It is claimed that at least 65% of the first-year students favor the new 'grade-free' system. A survey was carried out and found that only 54 in a sample of 100 students voted in favor of the 'grade free' system.

Would you agree with the claim?

You are required to write the null and alternate hypotheses, the test statistic and the *P*-value, and state your conclusions. Use the level of significance of 0.05.

(10 marks)

(c) Briefly explain the Bisection method that uses iteration to find the root of a function $f(x)$.

Name a numerical approach that is more efficient than the Bisection method in finding the root of a function.

(5 marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.