Politecnico di Milano

Prova finale: introduzione all'analisi di missioni spaziali
AA 2015-2016

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Elaborato n. B15

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Data di consegna: 01/05/16

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1. Introduction

In this report there have been assigned two different orbits, characterized by different parameters, which are located on two distinct planes. There are two points defined on these orbits: an initial point on the first orbit and a final point on the other one. Our task is to move a satellite from the starting to the final point.

It does not exist a pre-set strategy to do this. In order to change the orbit we have to modify its parameters and we can do it in different ways and with different orders of manoeuvre.
Moreover, to move from an orbit to another, if they do not have a common point, we decided to use only bi-tangent transfers.

The report is divided in three major sections. In the first section we describe the initial orbit, in the second the final orbit and in the last one four possible transfers which are finally compared.
The comparison is made analysing the total speed variation ΔV and the total time necessary to the transfer.
All the functions/algorithms named in this paper are written using Matlab code.

2. Initial orbit characterisation

2.1 Determination of initial orbit parameters from assigned position and velocity:

The start point is set in a specific position of our initial orbit with an assigned velocity, as follows:

- rx = 1.98155900 10^3 km
- ry = -8.87313210 10^3 km
- rz = -5.46193620 10^3 km
- vx = -4.8518 km/s
- vy = 3.1500 10^-2 km/s
- vz = -3.8897 km/s

The values of speed and position modules are

- r = 1.0606 10^4 km
- v = 6.2186 km/s

With the algorithm orbital_parameters we have found the initial orbit's orbital parameters:

- Semi-major axis a = 10923 km
- The module of the eccentricity vector e = 0.1745
- Angle of inclination i = 2.2938 rad (131,425°)
- Longitude of ascending node Ω = 2.3493 rad (134,605°)
- Anomaly of the perigee ω = 2.3193 rad (132.886°)
- True anomaly ϑ = 1.5794 rad (90,493°)

2.2 Characteristics of the orbit:

Looking at the data we deduce that the initial orbit is an ellipse, calculating the orbit energy in the starting position we obtain:

E = v^2/2 –mu/r = -18.2464 km^2/ s^2

That's correct because effectively it is an elliptical orbit, so it has a negative energy on the whole orbit.

We have calculated the period of the orbit with the formula:

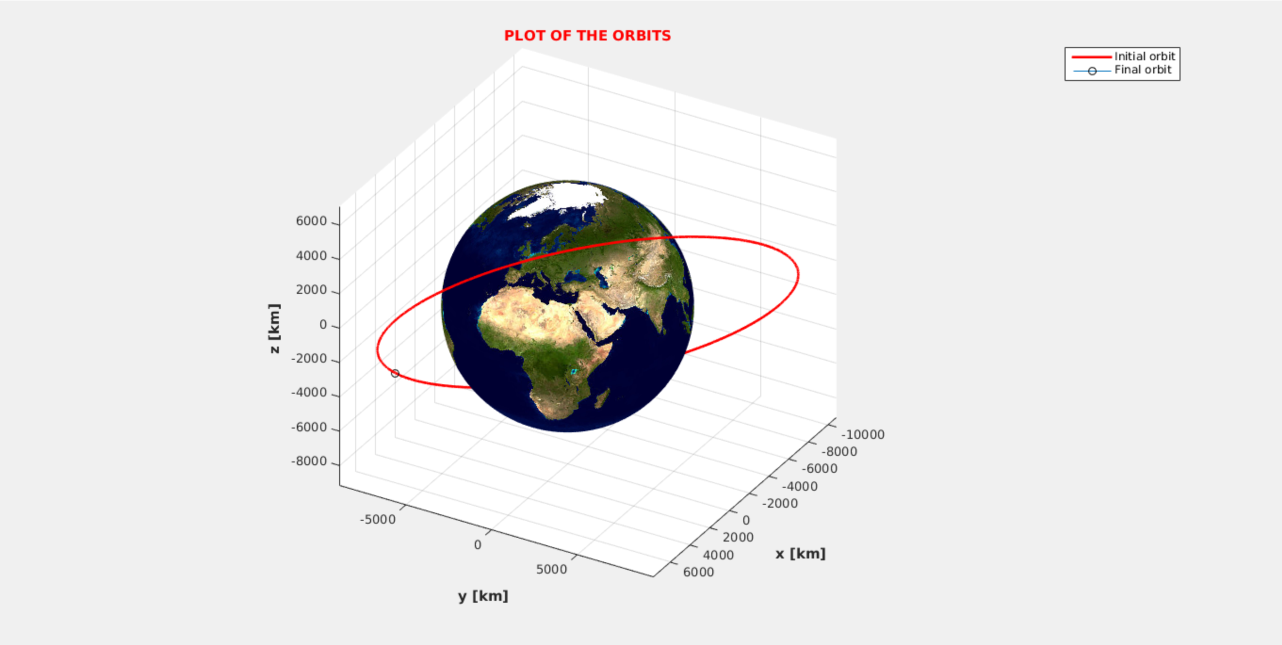
T = 2*pi*sqrt(a^3/mu)

Obtaining a result of 11361 seconds (3.156 hours).

The value of semi-latus rectum is: P = a*(1-e^2) =10590 km

The perigee is 9016.9 km far from the focal point, and the satellite has a speed of 5.20554 km/s, whereas the apogee point is 12829.1 km far and has a speed of 5.0644 km/s.

2.3
Graphical representation of the orbit.



3. Final orbit characterisation

3.1 Determination of final position and velocity from assigned final orbital parameters:

We know the six characteristics parameters of the orbit:

- Semi-major axis a = 3.61058246 10^4 km
- The eccentricity module e = 0.14010
- Angle of inclination i = 2.7709 rad (158.761°)
- Longitude of the ascending node Ω = 2.1961 rad (125.827°)
- Anomaly of the perigee ω = 0.80360 rad (46.043°)
- True anomaly ϑ = 1.48030 rad (84.815°)

Starting from these data, we are able to calculate the position and velocity for the final given point. To do it, we have calculated r and v that respectively identify the position and velocity in the perifocal system. Finally, thanks to a rotation matrix, we have changed the reference system in the geocentric system.

The obtained position is **r** = [33364; -4116; 9577] km, while the velocity vector is
v = [0.2714; -3.3519; -0.6771] km/s

The values of speed and position modules are

- r = 3.4955 10^4 km
- v = 3.4034 km/s

3.2
Looking at the data we deduce that the initial orbit is an ellipse, calculating the orbit energy in the starting position we obtain:

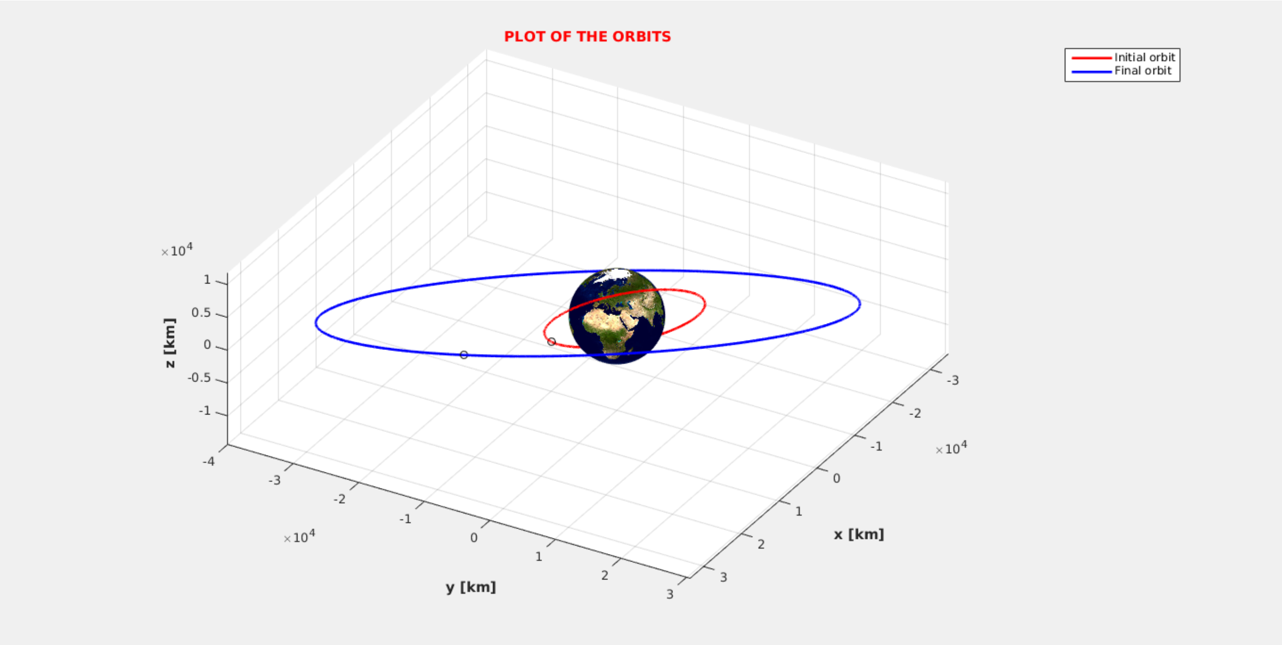
E = v^2/2 –mu/r = -5.6117 km^2/ s^2

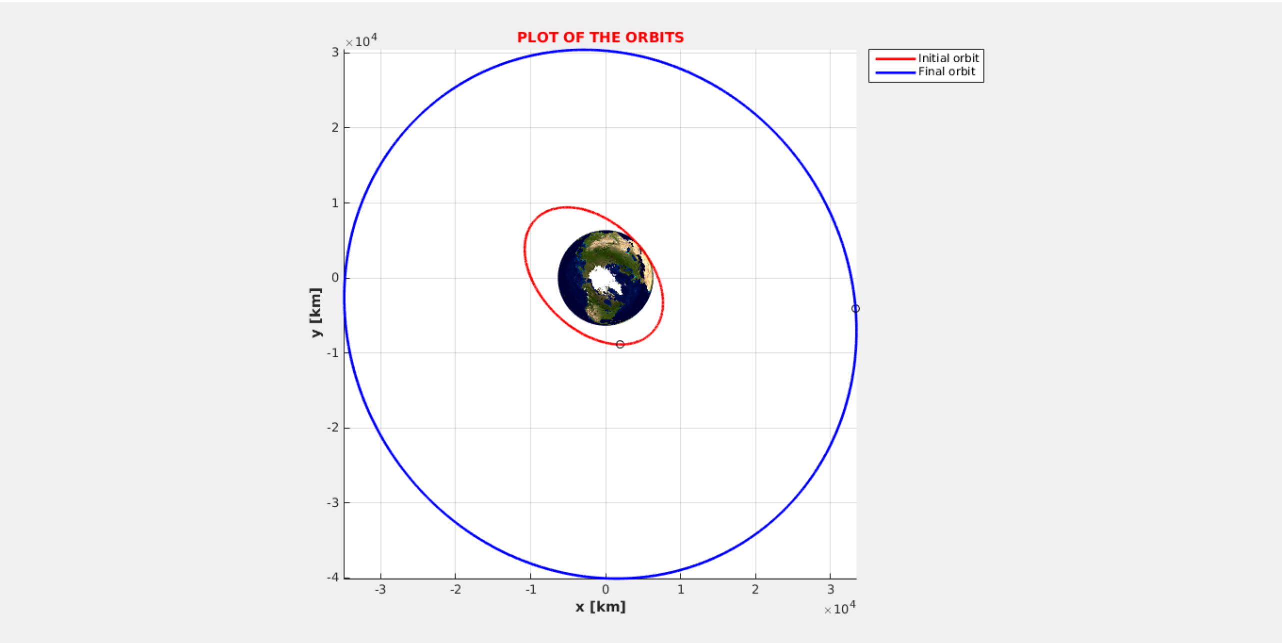
It can be calculated the period of the orbit, obtaining a result of 68277 seconds (about 19 hours).

The value of semi-latus rectum is: P = a*(1-e^2) =35397 km

The perigee is 31047 km far from the focal point, and the satellite has a speed of 3.8259 km/s, whereas the apogee point is 41164 km far and has a speed of 2.8856 km/s.

3.3
Graphical representation of the orbit.





4. Transfer trajectory definition and analysis

4.1

After the characterisation of initial and final orbit, the next goal is to move the satellite between these two orbits. Therefore we have to change all the satellite's orbital parameters, like the eccentricity ‘e’, the major semi axis ‘a’, the inclination angle ‘i’, the longitude of ascending node ‘Ω’, the argument of periapsis ‘ω’ and the true anomaly ‘ϑ’. In this paragraph we want to show the generic procedure to do that.

1. Plane changing manoeuvre. In this case, we have changed the orbital plane's inclination i and the longitude of ascending node ‘Ω’.
   The velocity impulse is given by $\Delta V_1 = 2V_\vartheta \sin(\alpha/2)$. After that, we have that three orbital parameters are changed, these are i_i, Ω_i, ω_i. It can be noticed that it’s also changed the argument of periapsis ‘ω’, this is due to the simultaneous change of i_i and Ω. The final parameters are:
   $\{a_i, e_i, i_{final}, \Omega_{final}, \omega_i, \vartheta\}$
2. Argument of periapsis changing manoeuvre. We wait until the object reaches the right position, then we change ‘ω’ with the impulsive manoeuvre $\Delta V_2 = 2|V_r|$. After that, the resultant orbital parameters are:
   $\{a_i, e_i, i_i, \Omega_i, \omega_{final}, \vartheta\}$
3. Bitangent transfer. This is a manoeuvre characterised by two impulsive ignitions, one at the apogee/perigee of the initial orbit and the other one at the perigee/apogee of the final orbit. This changes the orbit's geometrical shape (a and e).
   $\{a_{final}, e_{final}, i_i, \Omega_i, \omega_i, \vartheta\}$
4. Waiting, in order to move the satellite on its orbit without changing it. The only parameter that changes is the value of the true anomaly ϑ.
   $\{a_i, e_i, i_i, \Omega_i, \omega_i, \vartheta_{final}\}$

   The order and number of these manoeuvre depends only on the strategy chosen for that transfer.

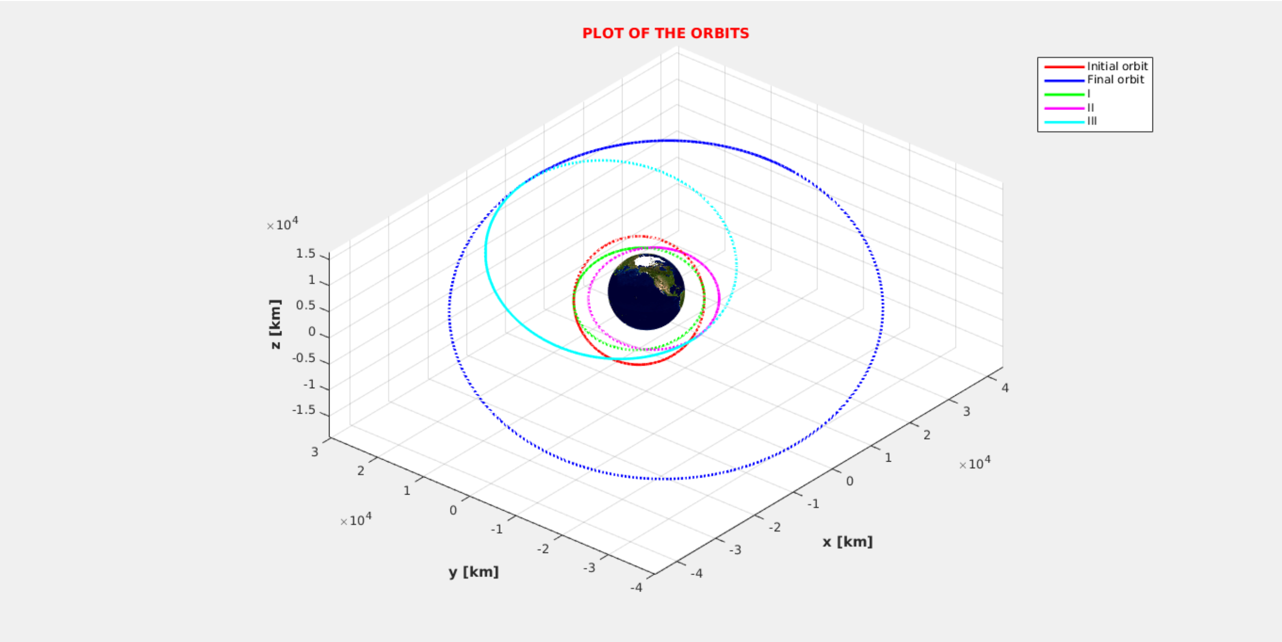
Only one kind of strategy doesn’t use this manoeuvers, because all the parameters are changed together with a unique transfer orbit. This strategy is called ‘Direct Transfer’ and it will be analysed in the further paragraph.

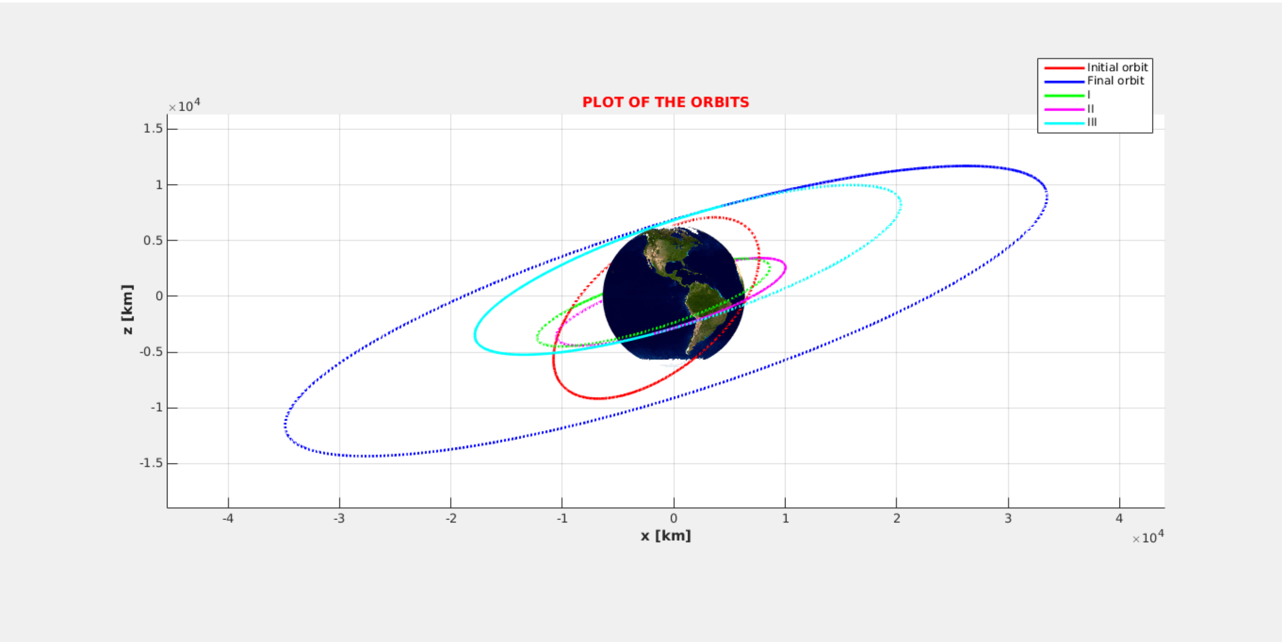
4.2 Discussion of the possible transfer strategies:

4.2.1a First strategy – change of plan/ω, change of shape

In this strategy, the satellite will be transferred from the starting orbit to the final one using the manoeuvres previously explained.
The first manoeuver used is the one that change the inclination plane, but in order to achieve this rotation the satellite must be located in one of the two intersections of the orbits, so the satellite must travel along its first orbit for about 1.5 hours
The plane change is the most expensive manoeuver of this strategy, this trouble is the same of the following transfer, but it will be contained from the third strategy due to a less speed in the point of rotation.
The second manoeuver is about the change of argument of periapsis, known as ω, that happen at the first intersection of the orbits, and this means that the time necessary to reach it is less than the second intersection, even if there's no difference from the impulse that must be applied.
The final manoeuver is a bitangent one, starting from the perigee. After the satellite moves to the apogee of this transfer orbit, it reaches the final point just following the final orbit until the value of ϑ is the one we expect to be.
This strategy requires less than 12 hours bringing the satellite into its final position.

Manoeuver	**Time (s)**	**ΔV (km/s)**
Start position-Change of plane	5166	-
Change of plane	-	2.5482
Transit	3069	-
Change ω	-	1.3747
Transit	4786	-
Bitangent first impulse		1.5667
Bitangent transit	16168	-
Bitangent second impulse		1.0859
Reach the final point	13092	
TOTAL	**42284**	**6.5756**





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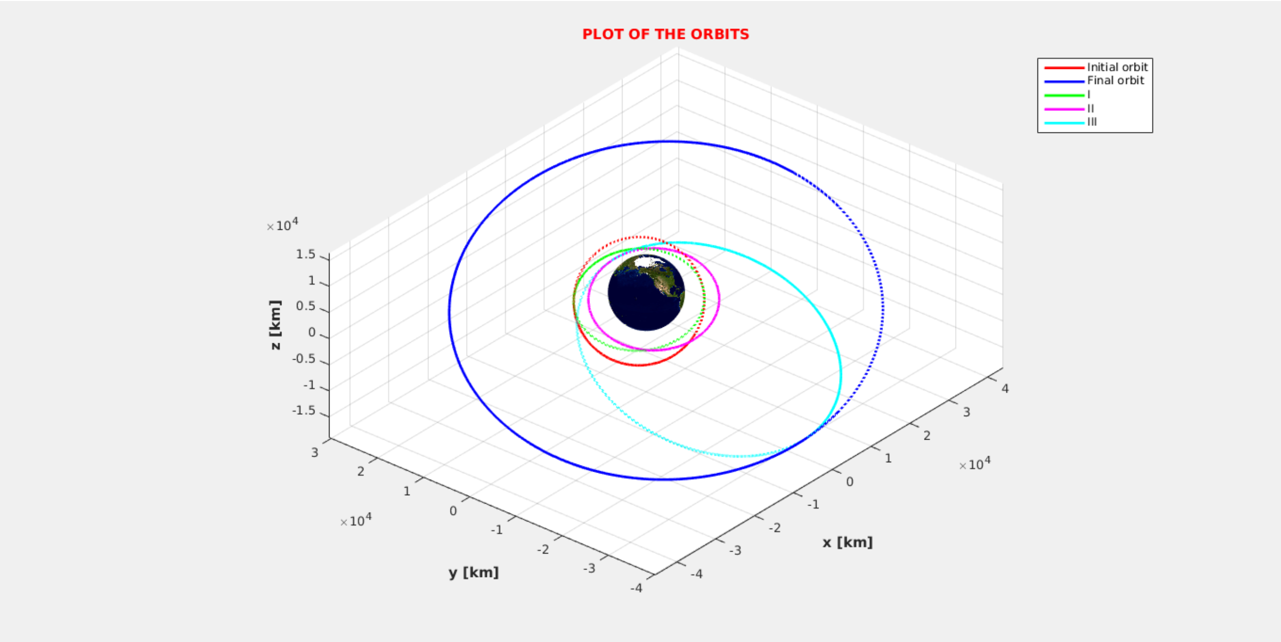
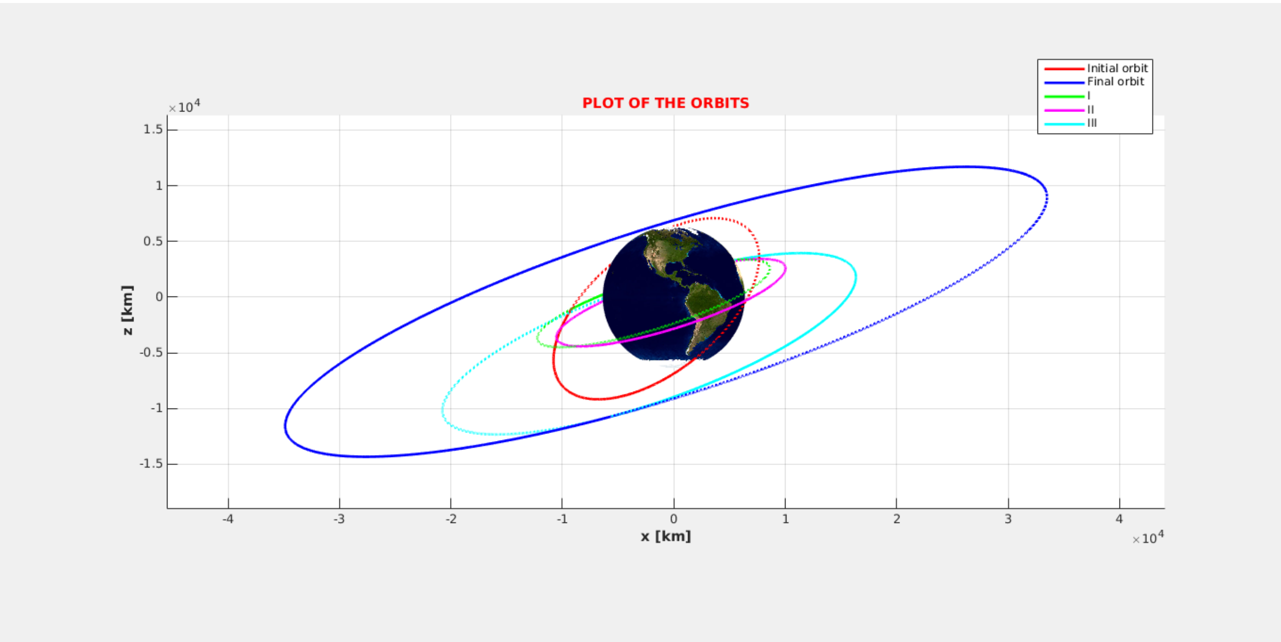
4.2.1b Second Strategy – change of plan/ω, change of shape

This strategy is quite similar to the previous one, the order of the manoeuvres is the same, but the difference is in the place where the maneuverers are executed. This means a difference in total costs of time and energy.
In particular, the manoeuver where is executed the change of argument of periaxis, ω, is the same of the previous one even if, as said before, this doesn't mean that the cost would have been different. The satellite will be in a better position in order to do the second manoeuver, a bitangent one, that changes the dimensions and shape of its orbits.
As it can be seen in the following table, the final time is almost doubled, but the complex cost in terms of energy is reduced by 0.3 Km/s.
This manoeuver requires about a day to be achieved.

Manoeuver	**Time (s)**	**ΔV (km/s)**
Start position-Change of plane	5166	-
Change of plane	-	2.5482
transit	7933	-
Change ω	-	1.3747
transit	3970	-
Bitangent first impulse		1.3108
Bitangent transit	19776	-
Bitangent second impulse		1.0202
Reach the final point	47230	
TOTAL	**84079**	**6.2539**





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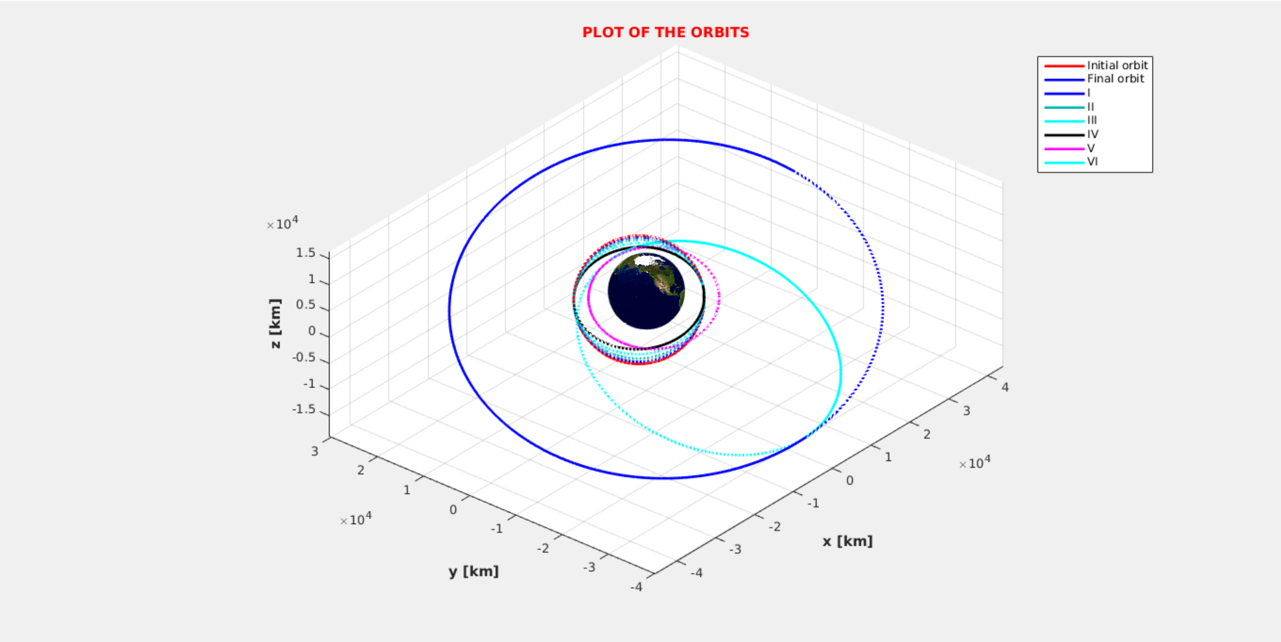
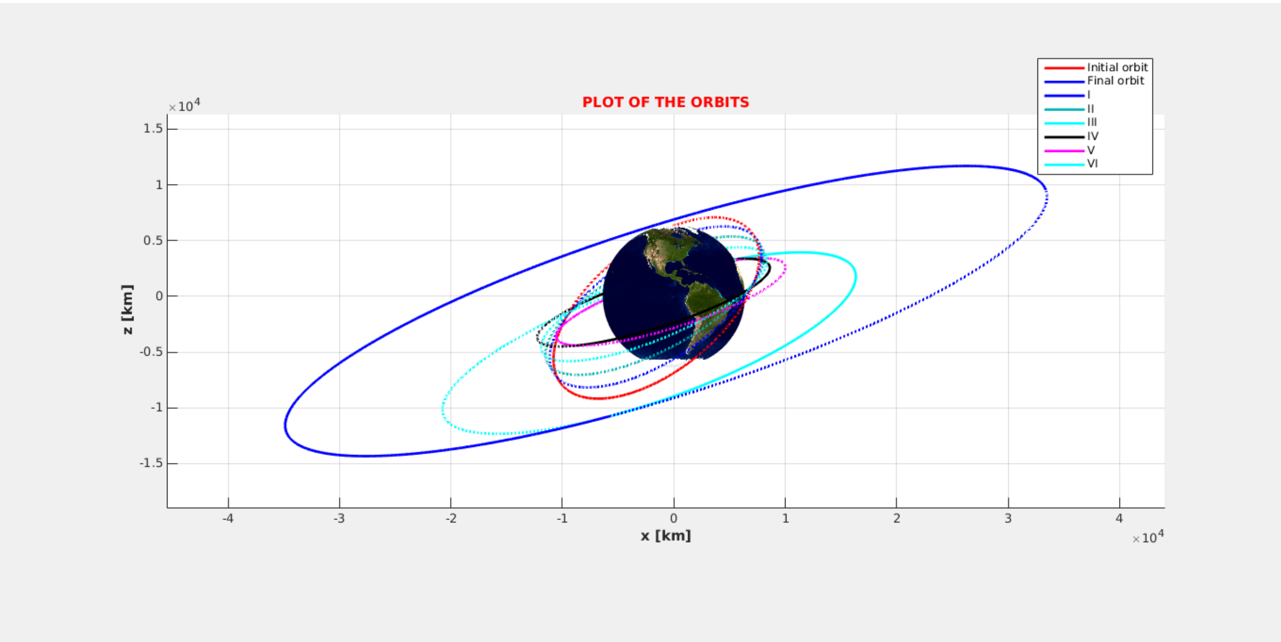
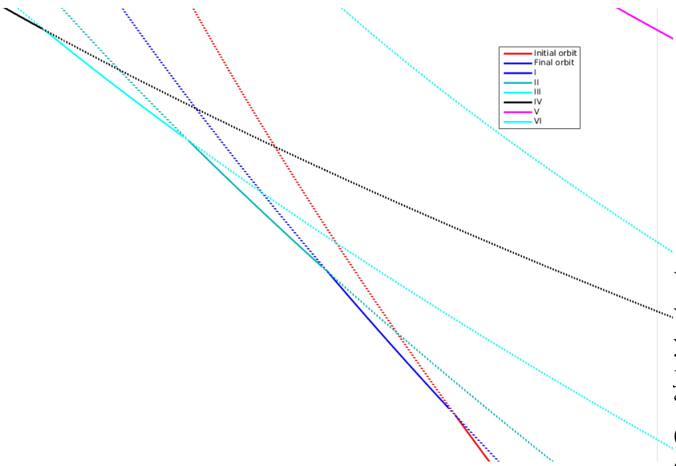
4.2.1c Third Strategy – change of plan in four steps, change of ω, change of shape
This transfer is quite similar to the second one because it has the same manoeuvers with the same orders, except the change of plan.
In the second strategy the manoeuver of plane changing is the most expensive of all, with a total impulse of 2.55 km/s, and this means that the satellite must increase its speed of about 9200 km/h in a short amount of time. Starting from the second principia of dynamic, F=m*a, in order to increase the speed in a very short time it's necessary to have an engine able to deliver a big force.
Obviously an engine able to deliver such a big power must be bigger than a normal one, or it would take too long to rotate the satellite.
Starting from this statement, we decided to 'chop' the first manoeuver in order to have 4 different plan changings instead of one.
The complexive value of time and impulse needed are lightly different, in particular the complexive impulse increase of 0.03 km/s, that is a negligible difference, but the value of every single plan changing is a bit more than ¼ of the total one.

Manoeuver	**Time (s)**	**ΔV (km/s)**
Start position-Change of plane	4961	
First change of plane		0.6428
transit	125	
Second change of plane		0.6431
transit	137	
Third change of plane		0.6445
transit	148	
Fourth change of plane		0.6471
transit	7933	
Change ω		1.3780
transit	3970	
Bitangent first impulse		1.3108
Bitangent transit	19776	
Bitangent second impulse		1.0202
Reach the final point	47230	
TOTAL	**84069**	**6.2865**







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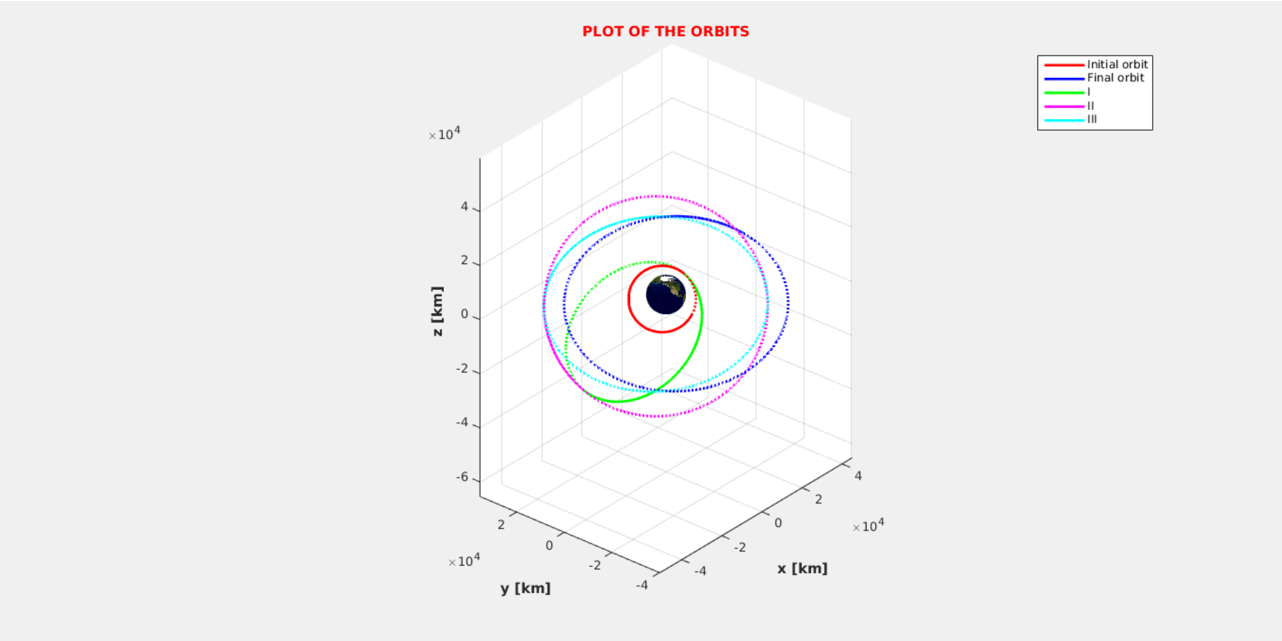
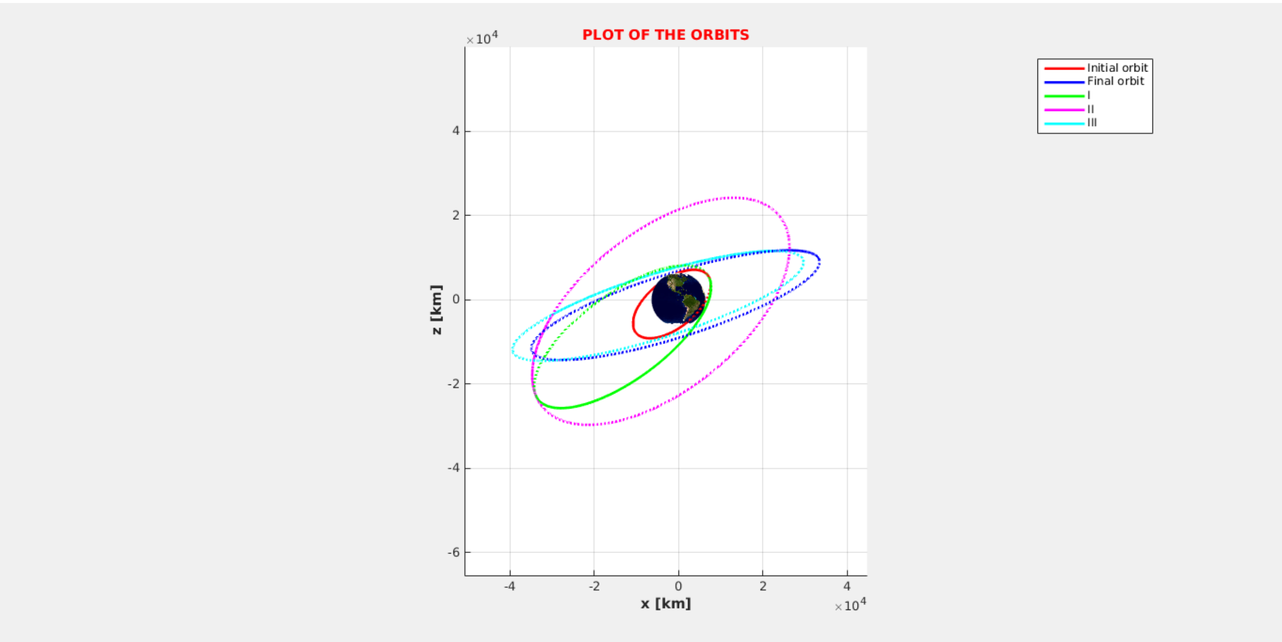
4.2.2 Fourth Strategy – change of shape, change of plan/ω

In this strategy the manoeuvers done are the same of the first two transfers, but they are executed in a different way. In particular, the change of plan is headed after the change of shape of the orbit, and this allow us to use less energy to rotate the satellite and change its periaxis argument.
This advantage of this strategy is given by the lower speed of the satellite in the external orbit, before its rotation. The cost of the rotation and change of periaxis argument is bounded to the radial/tangent speed of the satellite, and in the further orbit these speeds are lower than the inner orbit, giving us a big advantage to reduce the propellant needed.
As shown in the following label, this strategy needs only 4.37 km/s of total impulse; this means about 30-40% less propellant. In particular, the impulse needed to change plan is reduced from 2.54 km/s to 1.43 km/s, with a significant advantage.
Obviously, doing these manoeuvers so far from the focal point means slower velocity and higher period, and this can be noticed with a complex time of 64600 seconds, that means about 18 hours.

Manoeuver	**Time (s)**	**ΔV (km/s)**
Start position-bitangent orbit	9132	
Bitangent first impulse		1.3108
Bitangent transit	19776	
Bitangent second impulse		1.0202
Transit	9743	
Change of plane		1.4360
Transit	18623	
Change ω		0.6038
Reach the final point	7320	
TOTAL	**64594**	**4.3708**





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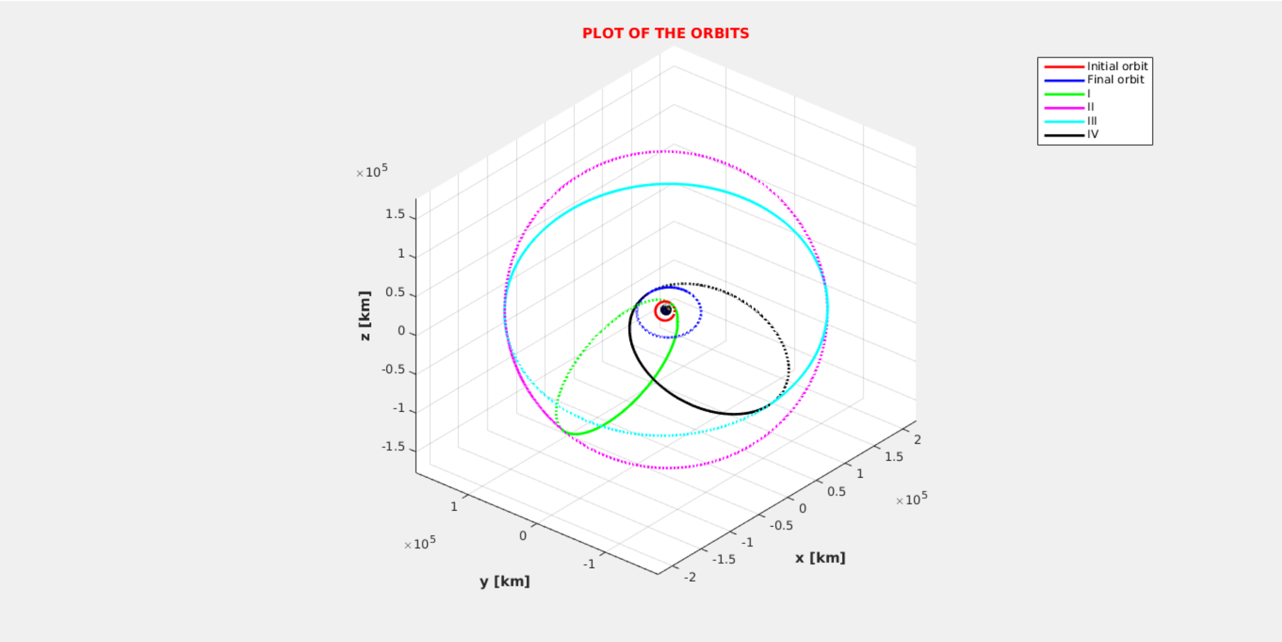
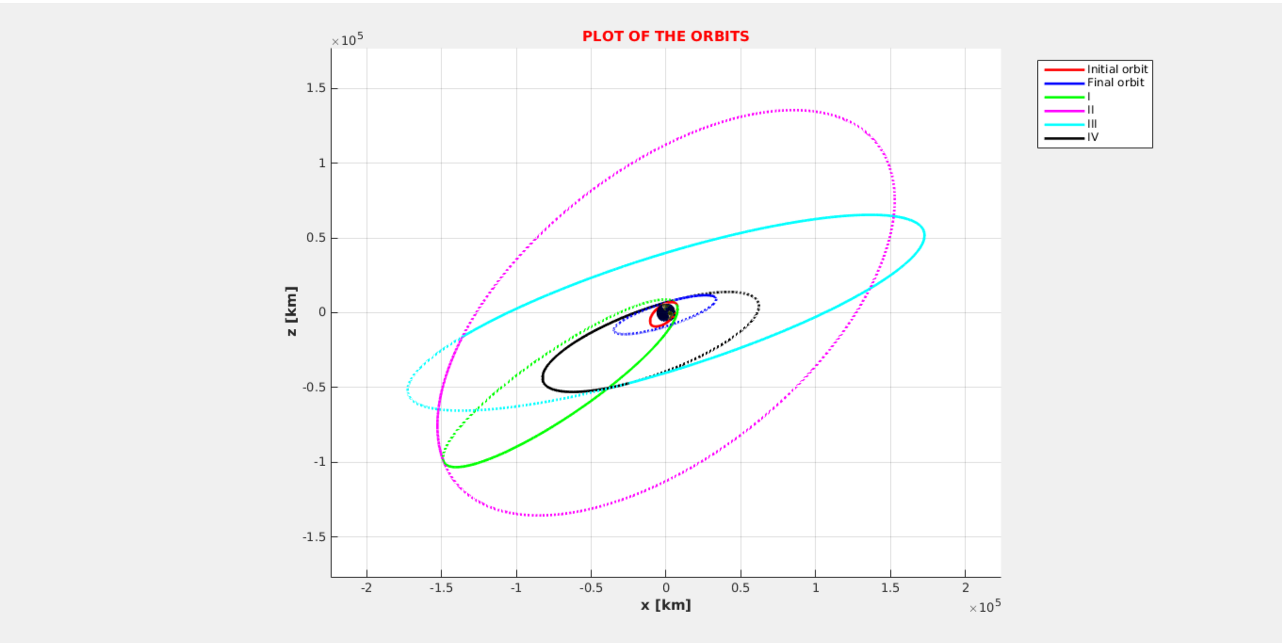
4.2.3 Fifth Strategy – Bielliptical transfer

This strategy uses a bielliptical transfer, and has the aim to bring the satellite very far from the focal point with a bitangent transfer, take advantage from this to change plan and then return to an inner orbit that is the final one.
This manoeuver allows us to change plan with a very low velocity, in fact it can be seen in the following table that the cost of plane changing is only of 0.71 km/s, that is really lower than the previous ones.
Obviously that orbit can be reached only with an expensive transfer, only the first bitangent orbit costs more than the half of the whole impulse.
The farthest orbit is circular, and it has a radius that can be freely chosen. The bigger is the orbit, the lowest will be the energy cost, but it will also take way more time to reach the final point.
With a compromise, we decided to settle the radius of the orbit at five time than the semi major axis of the final orbit, so is about 180’000 km.
It could have been chosen also a bigger orbit, like the lunar transfer orbit with a radius of 384’400km, but the satellite would have taken too much time (more or less three times the actual one).
Anyway, the nearness of the initial and final orbits makes unnecessary this transfer: the total impulse is higher than the one of the previous strategy, and the time needed is the highest of every strategy analysed in this report, almost 11 days.

Manoeuver	**Time (s)**	**ΔV (km/s)**
Start position-bitangent orbit	9132	
Bitangent first impulse		1.971
Bitangent transit	145180	
Bitangent second impulse		1.0276
Transit	85117	
Change of plane		0.7121
Transit	508790	
Bitangent first impulse		0.6809
Bitangent transit	171210	
Bitangent second impulse		0.8549
Reach the final point	13092	
TOTAL	**932520**	**5.2466**





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4.2.4 Sixth Strategy – Direct Transfer

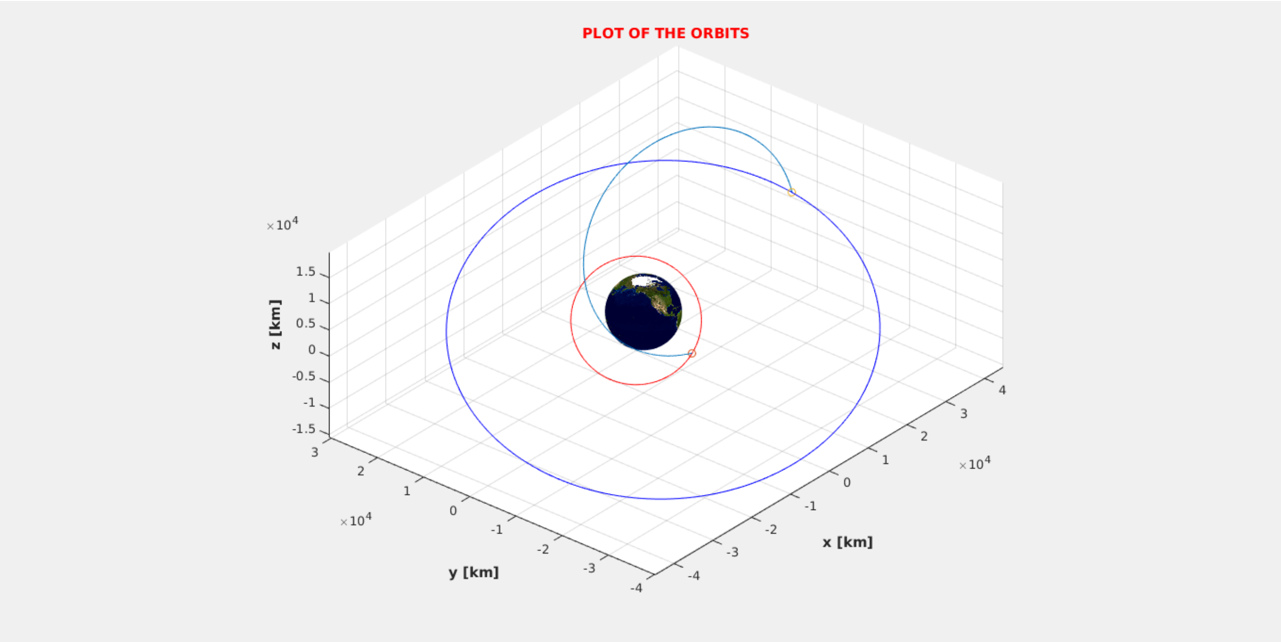
This strategy consists of a double impulse: the first one for reaching the transfer orbit and the second one for entering the final one.
This kind of transfer require a big impulse but is the fastest of all because the orbital parameters are changed all together with an unique transfer orbit.
In the following part of the paragraph is described the procedure to follow in order to obtain the parameters of the transfer orbit.
By making a cross product between the initial and the final position (r1,r2) we obtain the direction of the h vector, that defines the plane between the two positions.
Then we start calculating some of the orbital parameters which are unknown:
- inclination i: i=atan2(vect1,vers_h(3))
- orbital node n direction: n=cross(z,vers_h)/norm(cross(z,vers_h))
- ascending node OM : OM=OM=atan2(N(2),N(1))

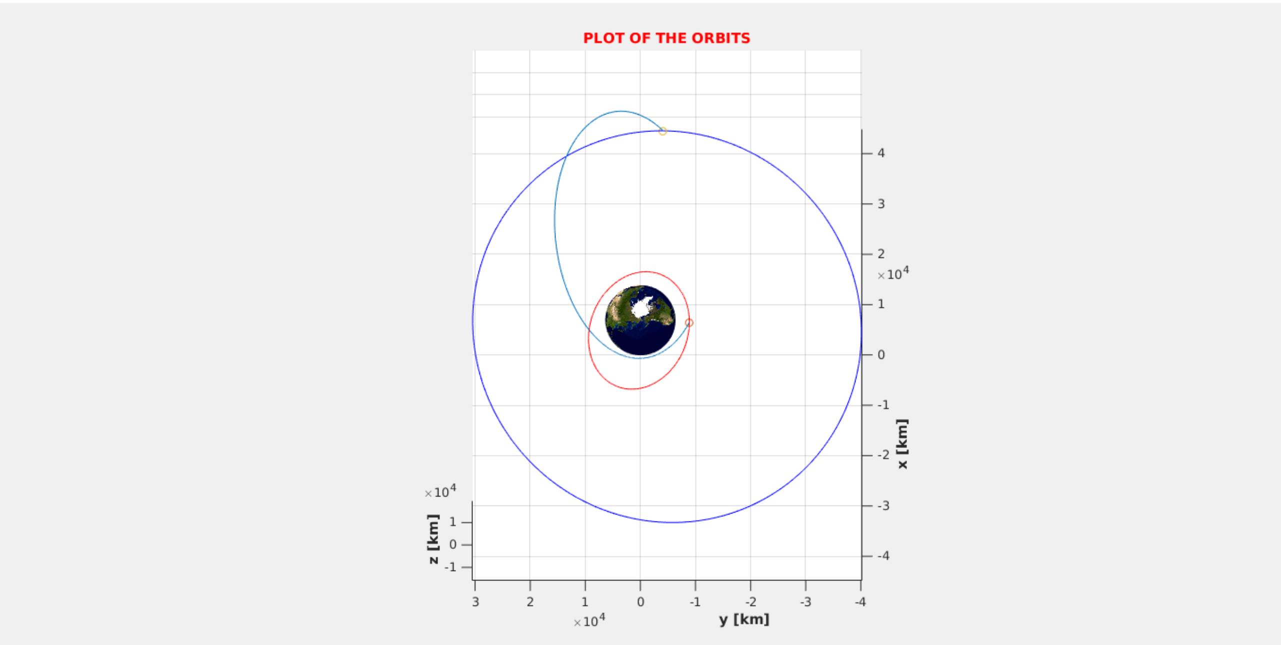
We have four conditions and five parameters to be determined.
So we need to fix one of these parameters: w, which represents the argument of periapsis, with a for cycle we can find the others and then choose the best transfer orbit, for Dv or time.
We write the 3 vectors that define the 3 directions of our reference system : x,y,z
With the definition of 3 rotation matrix we obtain only one rotation matrix (RT) that we can use to define the direction of the eccentricity vector:
- vers_e=RT*x'

In order to obtaining the eccentricity (e) and the semi major axis (a) we need to calculate the initial and final anomaly with a dot between the initial/final position and the eccentricity versor vers_e.
- eccentricity e: e=(norm(r2)-norm(r1))/(norm(r1)*cos(theta_i)-norm(r2)*cos(theta_f))
- semi major axis a: a=norm(r1)*(1+e*cos(theta_i))/(1-e^2)

Then with parorb2rv function we can find the initial and the final speed as outputs and the first and the second impulse of the manoeuvre as the norm of the sum of two speed vectors.
. Dv_1=norm(vi-v1)
. Dv_2=norm(v2-vf)
. Dv=Dv_1+Dv_2
For obtaining the time of the manoeuvre we use [Dt]=CalcoloTempi(a,e,theta_i,theta_f,mu)
Obviously we need to check that our orbit is not too close to the Earth.
We define a minimum distance from Earth as 300 km.
Finally we can start the two scripts, one for the best Dv and the second one for the best transfer time, but imposing the minimum distance from the Earth, the best resulting transfer orbit is the same, with a total Dv=7.7188 km/s and a transfer time of 23903 s, which are approximately six hours and half.

Manoeuver	Time (s)	ΔV (km/s)
Bitangent first impulse		5.3611
Bitangent transit	23903	
Bitangent second impulse		2.3577
TOTAL	**23903**	**7.7188**





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5. Conclusions

In this report have been analysed 6 strategies, with different kind of manoeuvres and different outcomes on the overall cost and time needed.
Here's a summary table with time and energy costs:

Strategy	Time (s)	Time (h)	ΔV (km/s)	Δt*ΔV
Strategy 1a: standard min. time	42284	11.745	6.5756	77.23
Strategy 1b: standard min. impulse	84079	23.355	6.2539	146.06
Strategy 1c: low impulses	84069	23.352	6.2865	146.80
Strategy 2: external plane change	64594	17.943	4.3708	74.43
Strategy 3: Bielliptical transfer	932520	259.03	5.2466	1359.03
Strategy 4: Direct Transfer	23903	6.640	7.7188	51.25

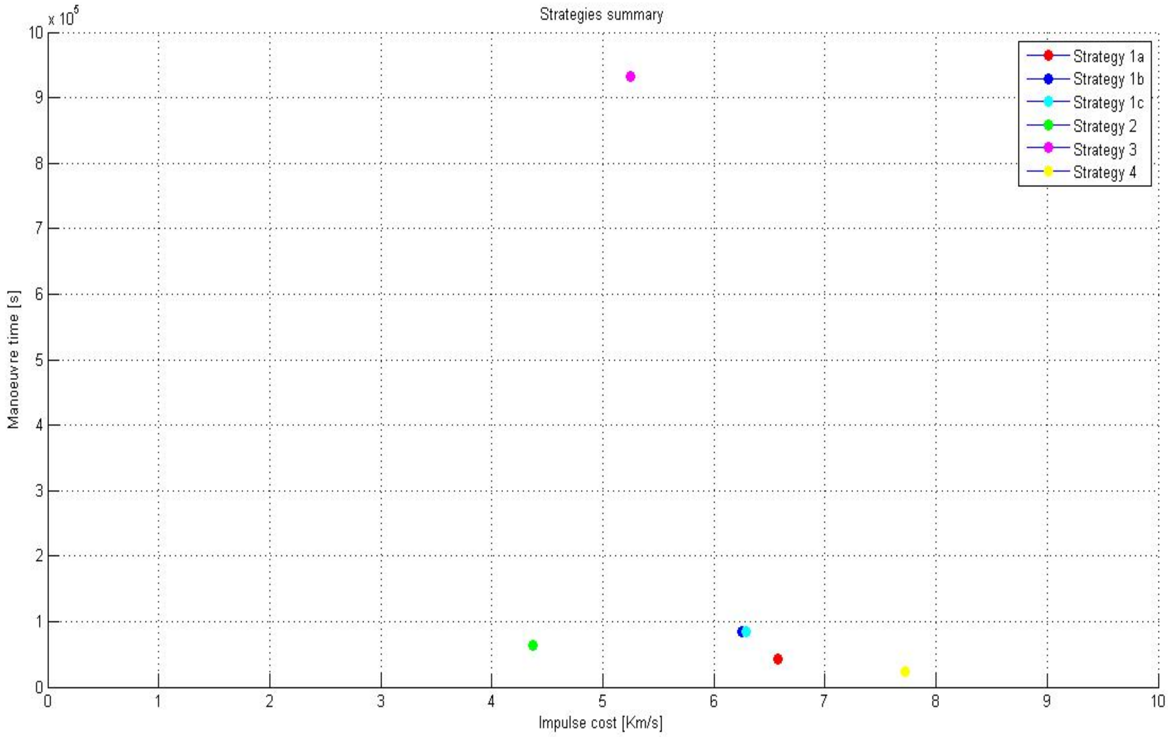
As it can be seen in the table, the first three transfer are quite similar due to the choose of the same kind of transfers. The first one is faster and more expensive than the other two due to the choose of a different point of manoeuver, but the value of **ΔV** is slightly less in the second one.
The third strategy (1c) can be deployed in case of small engine, because the highest costing manoeuver is executed in four different steps.
The fourth strategy is the one with less need of propellant, in fact has the minimum total impulse, and moreover it takes only 18 hours to place the satellite in the final point
The bielliptical transfer isn't as cheap as we expected, and this is given by the low radius ratio of the initial and final orbits, and also due to the lower rotation angle. This transfer is useful mostly in case of plane changing because in the external orbit the satellite reach very low velocities, and this makes the rotation cheaper than usual. Moreover, this is the slower strategy analyzed, with a total time of about 21 days.
The last one is the direct transfer strategy, as previously said, it takes a few hours to bring the satellite in the right position, but it's the most expensive at all.
These strategies could be analyzed under another parameter, that bond time and energy costs: this parameter is obtained as product of time needed and **ΔV**, and it can allow us to find a good compromise (ndr. the unit of measure of this parameter is a length, but it doesn't makes sense. It must be considered only as a number). This parameter is lower for the last transfer because of its dt, and also in the 'external plane changing' (2) due to its low impulse needs. Another strategy with a low parameter is the first one (1a), even if it's n not the best for **ΔV** or time, but its costs of time and energy are pretty low

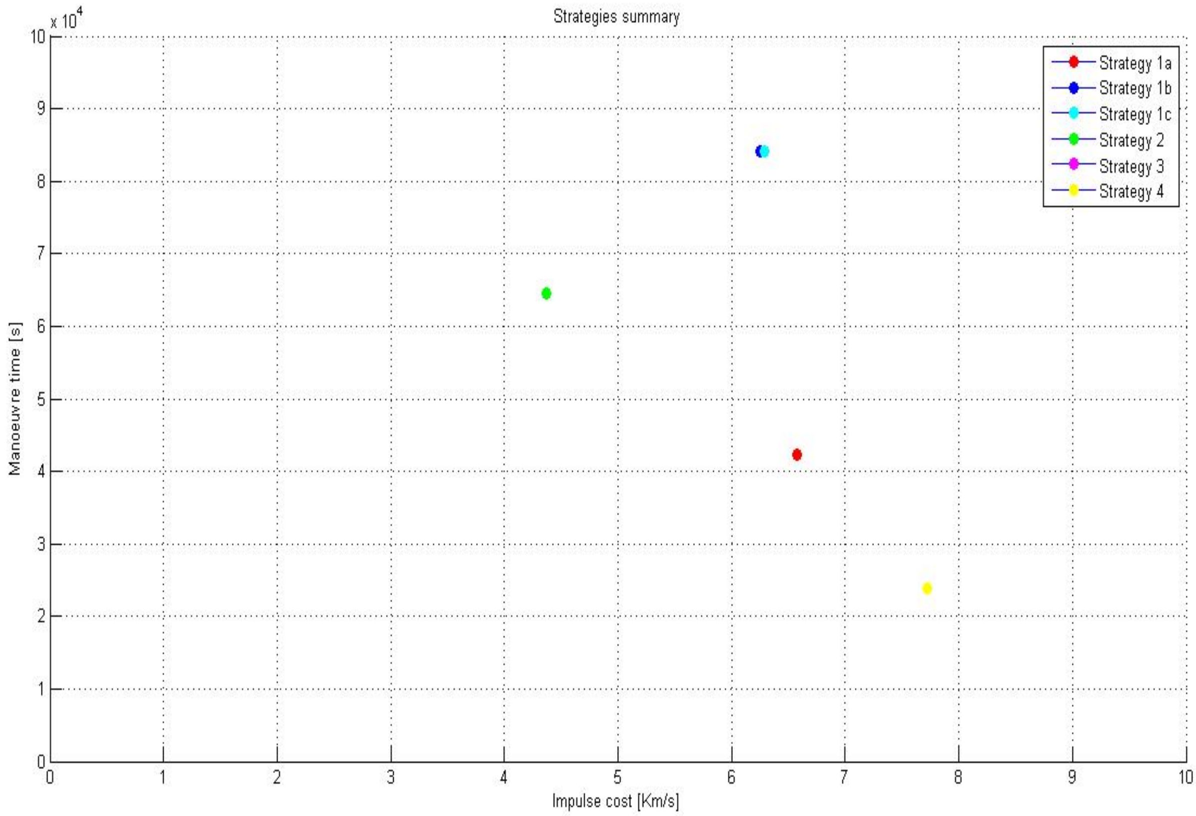
To sum up, the three strategies that could be chosen and that we recommend are:

- The strategy 1c, with low impulses in case of small engine
- The strategy 2, because it has the lower total impulse
- The strategy 4, because it's the fastest transfer and his parameter is the lowest one

Obviously, in case the time doesn't matter, we recommended the strategy with a lower total impulse.

In the following pictures are shown two graphics of the values of the strategies. In the second graphic isn't shown the bielliptical strategy due to its high time needed


Strategies summary
x 10^5
Manoeuvre time [s]
Impulse cost [Km/s]
Strategy 1a
Strategy 1b
Strategy 1c
Strategy 2
Strategy 3
Strategy 4


Strategies summary
x 10^4
Manoeuvre time [s]
Impulse cost [Km/s]
Strategy 1a
Strategy 1b
Strategy 1c
Strategy 2
Strategy 3
Strategy 4