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Modeling of a blow-down propulsion system

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Contents

| | |
|--|-----------|
| Contents | i |
| Nomenclature | ii |
| Acronyms | ii |
| Symbols | ii |
| Subscripts | iii |
| 1 Introduction and literature overview | 1 |
| 1.1 Blow-down heritage | 1 |
| 1.2 Analysis of losses | 1 |
| 2 Modelling of propulsion system: DRY-1 | 2 |
| 2.1 Input data | 2 |
| 2.2 Nominal sizing | 3 |
| 2.3 System dynamics | 5 |
| 3 Results analysis | 8 |
| 4 Nozzle losses | 8 |
| 4.1 Losses calculations | 8 |
| 4.2 Effects on the nominal design | 8 |
| 5 Additive manufacturing influences | 10 |
| 6 Cooling analysis | 10 |
| Bibliography | 11 |

Nomenclature

Acronyms

LRE Liquid Rocket Engine
LOX Liquid Oxygen

RP-1 RP-1 fuel
AM Additive Manufacturing

Symbols

| | | | | | |
|------------------|----------------------|--------------------------------------|----------------|----------------------|---|
| a | [m/s] | Speed of sound | p | [Pa] | Pressure |
| A | [m ²] | Area | Δp | [Pa] | Pressure loss / difference |
| B | [-] | Blow-down ratio | Pr | [-] | Prandtl number |
| c_P | [J/kg K] | Specific heat at constant pressure | \dot{q} | [W/m ²] | Heat flux |
| c_T | [-] | Thrust coefficient | \dot{Q} | [W] | Heat transfer rate |
| c^* | [m/s] | Characteristic velocity | r | [-] | Recovery factor |
| C_d | [-] | Discharge coefficient | R | [J/kg K] | Specific gas constant |
| D | [m] | Diameter | \mathcal{R} | [J/mol K] | Universal gas constant |
| E | [m/s] | Erosion rate | Re | [-] | Reynolds number |
| f | [-] | Darcy friction factor | Re' | [-] | Modified Reynolds number |
| h | [W/m ² K] | Convective heat transfer coefficient | t | [s] | Time |
| H | [m] | Height | Δt | [s] | Time step |
| i | [-] | First iteration index | T | [K] | Temperature |
| I_{sp} | [s] | Specific impulse | T | [N] | Thrust |
| I_{tot} | [Ns] | Total impulse | u | [m/s] | Velocity |
| j | [-] | Second iteration index | V | [m ³] | Volume |
| k | [m ⁻¹] | Curvature | ΔV | [m ³] | Volume change |
| K | [-] | Pressure loss coefficient | α_{AM} | [deg] | Deposition angle of AM |
| L | [m] | Length | α_{con} | [deg] | Convergent semi-aperture angle |
| L^* | [m] | Characteristic length | | | Injector pressure drop as percentage of initial combustion chamber pressure |
| m | [kg] | Mass | β | [%] | |
| \dot{m} | [m/s] | Mass flow rate | γ | [-] | Heat capacity ratio |
| M | [-] | Mach number | ε | [-] | Area ratio |
| M | [kg/mol] | Molar mass | λ | [-] | Nozzle losses coefficient |
| N | [-] | Number of | μ | [Pa s] | Dynamic viscosity |
| O/F | [-] | Oxidizer to fuel ratio | ρ | [kg/m ³] | Density |
| $\overline{O/F}$ | [-] | Mean O/F ratio | σ | [-] | Correction factor across boundary layer |

Subscripts

| | | | |
|------------|--------------------|------------|-----------------|
| aw | Adiabatic wall | max | Maximum |
| c | Combustion chamber | min | Minimum |
| cea | From CEAM software | ox | Oxidizer |
| con | Convergent | p | Propellants |
| e | Nozzle exit | pr | Pressurizer gas |
| eff | Effective | r | Real |
| f | Final | t | Nozzle throat |
| fd | Feeding lines | tc | Thrust chamber |
| fu | Fuel | tk | Tank |
| i | Initial | tot | Total |
| id | Ideal | wg | Gas side wall |
| inj | Injector | | |

1 Introduction and literature overview

In this work a preliminary design of a 1 kN semi-cryogenic LRE (LOX/RP-1) equipped with a blow-down feeding system is discussed. In particular, a first literature analysis was done in order to review previous studies regarding this particular architecture. Recent developments in additive manufacture (AM) techniques were analyzed to obtain some knowledge regarding processes and precision of this new frontier. Moreover, due the reduced size of this system, some criticalities regarding boundary layer and erosion losses were researched. The second part of the paper aim at designing the engine with some imposed initial conditions and some assumptions. The whole dimensioning of the system, including the tanks and feeding lines, is carried on including the dynamics of the system. The final sizing will accomplish the maximization of the total impulse, with the initial and final constraints. An off-design analysis is then performed to quantify the performances with nozzle losses and AM uncertainties. Finally, a feasibility analysis of nozzle fuel cooling is discussed.

1.1 Blow-down heritage

The blow-down architecture is the simplest feeding technique for LRE since it does not require additional pressurizing gas tanks with failure-prone pressure regulator valves nor complex turbomachinery. The scheme includes only two liquid propellant tanks filled with helium or nitrogen, eventually separated by a membrane. The major downsides of this simplicity is relative to the non-stationarity of the tank pressures that induce chamber pressure drop, decrease of propellant mass flow rate and as a consequence O/F ratio variation. This chain of events degrades performances overtime and must be carefully evaluated since combustion efficiency relies upon viable domains of injection pressure and correct mass flow ratio. The interest on blow-down is although justified with respect to well-known pressure regulated feed system since this last can also manifest some criticalities in terms of long-term reliability. In particular, propulsion systems play crucial roles for mission success, such as long interplanetary trips, and they must ensure failure-free lifetime. This is a major concern when focusing on pressure regulated feeding lines in which a pressure regulator valve is present. This kind of elements can be quite complex and hence add a weakness for the whole system^[1]. Considering these facts, a blow-down type architecture could be of interest since it decrease system complexity. Moreover, different feasibility analysis for blow-down units are present in the literature in which also an external re-pressurization tank is considered^[2]. This is an upgrade that allows to recover performance of the feeding pressure and hence combustion properties. Although the valve complexity is removed since a pyro valve can be used to discharge the gas with a single shot application, the eventual re-pressurization can be a crucial point as the sudden change in pressure could induce unwanted instabilities. Other configurations could foresee the use of a Venturi valve to maintain constant mass flow rate by cavitating the liquid and choking the flow on the feeding line. However, neither extra tank nor Venturi valves will be considered in this work in order to meet the requirements presented in **REFERENCE**.

The whole evaluation of the dynamics of the examined propulsion system was not based upon previous works, instead a self-made model was developed. subsectionAdditive manufacturing state of art

1.2 Analysis of losses

2 Modelling of propulsion system: DRY-1

The workflow for the sizing of DRY-1 is introduced by Figure 1 and it is divided into three stages:

- **Input data:** the problem is set up.
- **Nominal sizing:** the system is sized according to initial conditions and general assumptions.
- **System dynamics:** an iterative process is set up to model the blow-down dynamics and finalize the sizing.

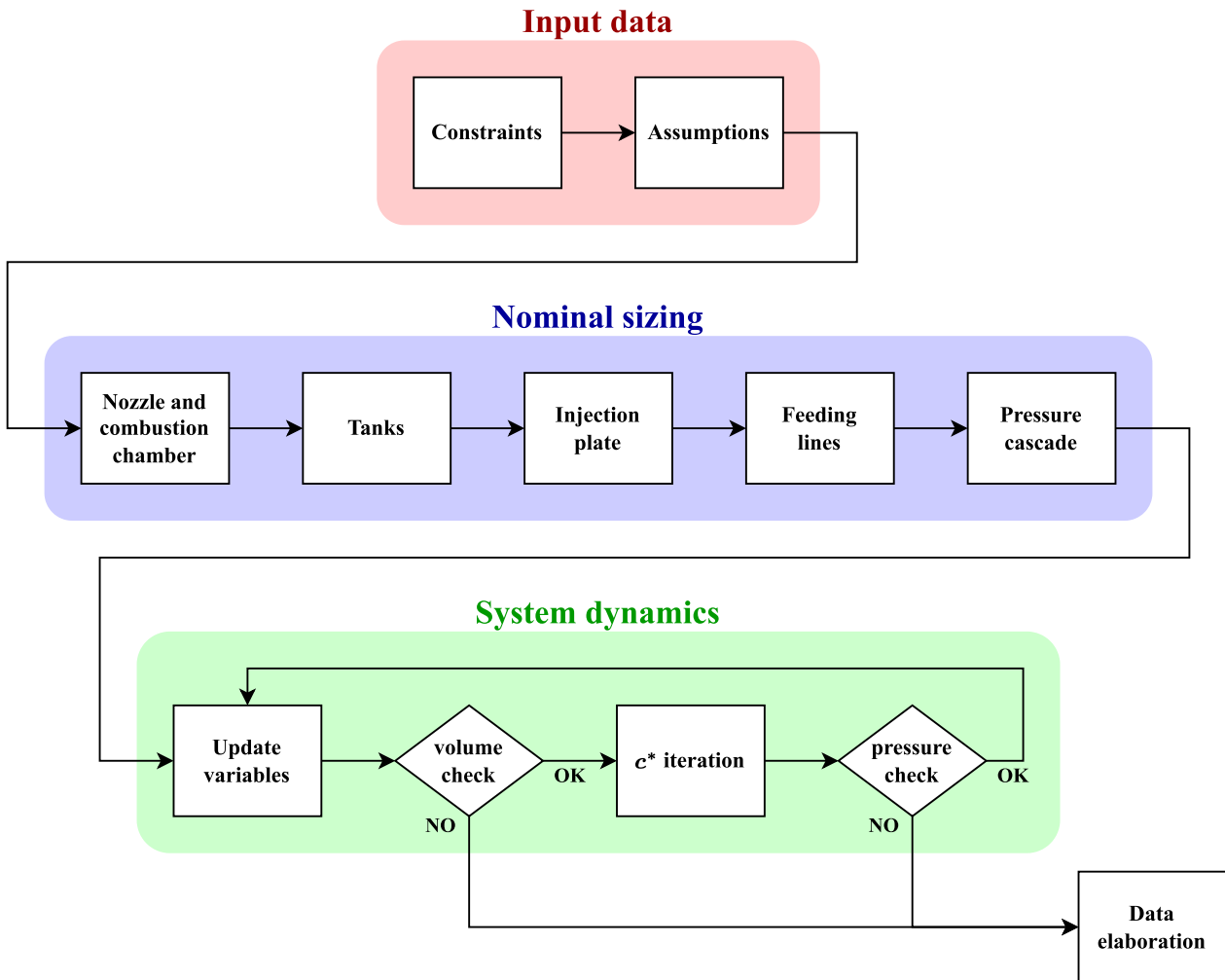


Figure 1: Flowchart of the simulation model

2.1 Input data

The input data defined different kind of requirements, related to operability environment, engine performance, size constraints, chemistry, architecture and manufacturing techniques. Regardless of the development of the engine design, the system shall respect the following pinpoints:

- **Environment:** vacuum for the whole operation.
- **Thrust T :** initial magnitude of 1 kN, no lower boundary.
- **Chamber pressure p_c :** initial value of 50 bar, always above 20 bar throughout the whole mission.
- **Allocated space:** tanks, combustion chamber and convergent nozzle occupancy is exactly 80% of the volume occupied by a cylinder of 1 meter diameter and 2 meter length. No bounds on the extension of the divergent.
- **Propellants:** semi-cryogenic couple of LOX and RP-1.
- **Architecture:** blow-down type.
- **Manufacturing:** all the system is produced in AM, no restriction on material nor techniques.

The nominal sizing refers to the design of the overall system considering the imposed initial constraints as static conditions. This design choice was dictated by the dynamics of the blow-down system, which imposes the maximum flow rate at the beginning of the mission, leading to the oversizing of the engine throughout the rest of the mission.

Various hypothesis were necessary to develop the system, this values are reported in Table 1.

| O/F [-] | ε [-] | ε_{con} [-] | L^* [m] |
|-----------|-------------------|-------------------------|-----------|
| 2.42 | 300 | 10 | 1.143 |

Table 1: Hypothesis from literature and previous design

The choices of L^* and O/F were only dictated by the propellant couple^[3], while ε was chosen as the characteristic of the engine refers to an in-space application^[4]. Regarding the value of ε_{con} a mean value between 5 and 15 was taken. Smaller values entails longer combustion chamber and small cross sectional area, with large pressure drops. Larger values refers to bigger chamber cross sectional area, with limited length for the combustion. From the literature the suggestion for the choice of this value is to refer to previous successful engines design, considering the same application^[5]. Therefore, a 400 N bi-propellant apogee motor was taken as reference and revealed a value of $\varepsilon_{con} \approx 10$ ^[4].

2.2 Nominal sizing

After defining the main input data, the workflow is carried out as shown in Figure 1. All the combustion simulations were performed with Nasa-CEA software, implemented in Matlab (CEAM). In particular the "rocket problem" was considered, imposing frozen equilibrium, infinite combustion chamber ($M_c = 0$), initial injecting temperatures of the propellant equal to the storage temperatures. The chamber pressure was set as $p_c = 50$ bar and the mixture ratio as $O/F = 2.42$. Latter refinement of this last value will be performed. The output values used from the simulation are represented (vacuum value is considered for the c_T):

| c^* [m/s] | c_T [-] | T_c [K] | γ_c [-] |
|-------------|-----------|-----------|----------------|
| 1851 | 1.935 | 3708 | 1.1405 |

Table 2: First run on CEAM

From this results, the propellant mass flow rate and the throat area can be calculated:

$$\dot{m}_p = \frac{T}{c_T c^*} \quad A_t = \frac{\dot{m}_p c^*}{P_c} \quad (1)$$

From the geometry assumption of Table 1, the nozzle exit area and the combustion chamber geometry can be retrieved:

$$A_e = \varepsilon A_t \quad (2)$$

$$A_c = \varepsilon_{con} A_t \quad L_c = \frac{L^*}{\varepsilon_{con}} \quad (3)$$

Nozzle geometry model will be delved in section 4. The results for the previous calculation are the following:

| \dot{m}_p [kg/s] | D_t [cm] | D_e [cm] | D_c [cm] | L_c [cm] |
|--------------------|------------|------------|------------|------------|
| 0.279 | 1.15 | 19.86 | 3.63 | 11.43 |

Table 3: Preliminary DRY-1 geometry

A check on the compliance of the chamber Mach number is done ($M_c < 0.3$):

$$\frac{1}{\varepsilon_{con}} = M_c \left[\frac{1 + \frac{\gamma_c - 1}{2}}{1 + \frac{\gamma_c - 1}{2} M_c^2} \right]^{\frac{\gamma_c + 1}{2(\gamma_c - 1)}} \xrightarrow{\text{fsolve}} M_c = 0.059 \quad (4)$$

From the geometry of the motor and the allocated space constraints, the total height of the tanks can be calculated. The volume around the thrust chamber (combustion chamber + convergent) can be assessed:

$$V_{tc} = \frac{\pi}{4} \left[L_c D_c^2 + \frac{L_{con}}{3} (D_c^2 + D_t^2 + D_c D_t) \right] \quad (5)$$

Also, the volume of the cylinder that covers the length of the thrust chamber and with the total diameter of 1 meter (D_{tot}) can be computed. From there, the empty volume around the thrust chamber can be computed as a difference:

$$V_{lost} = V_{tc} - \frac{\pi}{4} (L_c + L_{con}) D_{tot}^2 \quad (6)$$

This value must be 20% of the total cylinder volume, as cited in [subsection 2.1](#). As the computed value was lower, additional volume had to be removed from the tanks in order to meet the requirement. The height dedicated to the tanks is calculated as follow:

$$H_{tk} = H - \left[L_c + L_{con} + \frac{4}{\pi D_{tot}^2} (0.2 V_{tot} - V_{lost}) \right] \quad (7)$$

The total volume allocated to the tanks is hence fully defined. In order to calculate the masses of pressurizer and propellants, some assumption have to be made:

- adiabatic expansion of the pressurizing gas;
- blow-down ratios can be tuned;
- mean value of the oxidizer to fuel ratio.

A system of equations can be set up:

$$\begin{cases} \frac{m_{ox}}{m_{fu}} = \overline{O/F} \\ m_{ox} = \rho_{ox} V_{ox} \\ m_{fu} = \rho_{fu} V_{fu} \\ V_{ox} = V_{pr,f}^{ox} - V_{pr,i}^{ox} \\ V_{fu} = V_{pr,f}^{fu} - V_{pr,i}^{fu} \\ V_{pr,f}^{ox} = B_{ox}^{\frac{1}{\gamma_{pr,ox}}} V_{pr,i}^{ox} \\ V_{pr,f}^{fu} = B_{fu}^{\frac{1}{\gamma_{pr,fu}}} V_{pr,i}^{fu} \end{cases} \quad (8)$$

In order to solve [System 8](#), the pressurizer and the initial temperature of the propellants have to be set. Different pressurizing gas are available, mainly nitrogen or helium are the most common choices. The main differences for the two are the storage temperature, the molar mass and the specific heat ratio. The latter parameter influences the adiabatic discharge, higher values implies faster pressure discharge. The molar mass affects the amount of gas to be embarked at a given pressure. The storage temperature is a matter of compatibility with the propellant. Nitrogen gas was chosen to pressurize RP-1 since no cryogenic conditions were present also it guarantees lower discharge, with the downside of increasing the mass of the system. On the other side, LOX required a cryogenic compatibility that can be ensured by helium. Even though an efficient insulating bladder is employed, the design choice was dictated by a more conservative approach. The employment of nitrogen also with LOX was discarded since storage temperature and pressure are not compatible with its properties^[6].

| $T_{pr,fu}$ [K] | $T_{pr,ox}$ [K] | γ_{N_2} [-] | γ_{He} [-] |
|-----------------|-----------------|--------------------|-------------------|
| 300 | 90 | 1.40 | 1.67 |

Table 4: Initial values for pressurizer gases and specific heat ratio values

| $\overline{O/F}$ [-] | $B_{pr,ox}$ [-] | $B_{pr,fu}$ [-] | V_{tot} [m ³] |
|----------------------|-----------------|-----------------|-----------------------------|
| 2.42 | 2.5 | 2.5 | 1.27 |

Table 5: Assumed or calculated values as first iteration

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The masses and volumes of oxidizer, fuel and pressurizing gases are computed.

| m_{fu} [kg] | m_{ox} [kg] | $V_{pr,i}^{fu}$ [m ³] | $V_{fu,i}$ [m ³] | $V_{pr,i}^{ox}$ [m ³] | $V_{ox,i}$ [m ³] |
|---------------|---------------|-----------------------------------|------------------------------|-----------------------------------|------------------------------|
| 165.34 | 400.13 | 0.2217 | 0.2049 | 0.4789 | 0.3510 |

Table 6: Propellant and pressurizer quantities as first iteration

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Finally, the feeding lines can be modelled. Considering figure **REFERENCE**, the length of the pipes can be retrieved as a difference:

$$L_{fd,fu} = H - L_c - H_{tk,fu} \quad (9)$$

$$L_{fd,ox} = H - L_c - H_{tk,ox} \quad (10)$$

To completely determine the DRY-1 geometry, the injection plate must be modelled. The pressure drop across the injector Δp_{inj} have to be assumed as a percentage of the initial chamber pressure. Acceptable range of this fraction goes from 5% to 30%. A value of 20% is chosen for both oxidizer and fuel lines. The fuel and oxidizer injector area can be computed assuming reasonable values for the discharge coefficient C_d . A reasonable assumption has been made according to the superficial roughness quality of AM.

$$\dot{m}_{fu} = \frac{1}{1 + O/F} \dot{m}_p \quad \dot{m}_{ox} = \frac{O/F}{1 + O/F} \dot{m}_p \quad (11)$$

$$A_{inj,tot} = \frac{\dot{m}_p}{C_d \sqrt{2 \Delta p_{inj} \rho_p}} \quad (12)$$

$$K_p = 1 + f \frac{L_{p,fd}}{D_{p,fd}} \left(\frac{A_{p,fd}}{A_{p,inj,tot} C_d} \right)^2 \quad (13)$$

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2.3 System dynamics

From the nominal sizing of the engine, it is necessary to simulate the real dynamics of the system in order to:

- retrieve the performance of the designed system in time;
- check the compliance of the system with the constraints;
- test other designs through iteration to select the best one based on the simulated data of interest.

For this reasons, a numerical method was implemented. An high level explanation for the functioning of the algorithm can be appreciated in Figure 2.

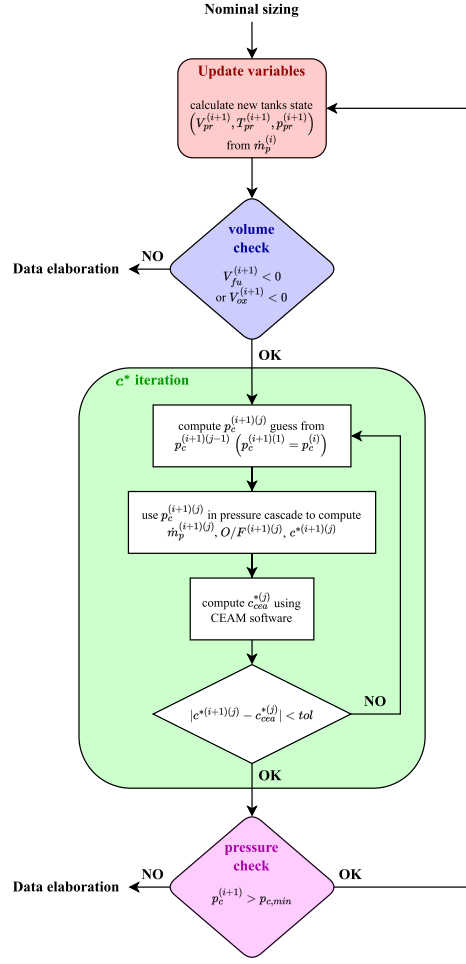


Figure 2: Flowchart of dynamics of the model

The first step of the time cycle is to update the state of the two tanks based on the previous iteration. Assuming a constant propellants flow rate during the time step Δt , the volume of the remaining liquid is decreased by a quantity $\Delta V^{(i+1)}$. Accordingly, the volume of the pressurizer will increase by the same amount.

$$\Delta V^{(i+1)} = \frac{\dot{m}_p^{(i)} \Delta t}{\rho_p} \quad (14)$$

$$V_p^{(i+1)} = V_p^{(i)} - \Delta V^{(i+1)} \quad (15)$$

$$V_{pr}^{(i+1)} = V_{pr}^{(i)} + \Delta V^{(i+1)} \quad (16)$$

From the change of volume, the new pressure and temperature of the pressurizer gas are computed assuming an adiabatic expansion in the tank:

$$p_{pr}^{(i+1)} = p_{pr}^{(i)} \left(\frac{V_{pr}^{(i)}}{V_{pr}^{(i+1)}} \right)^{\gamma_{pr}} \quad (17)$$

$$T_{pr}^{(i+1)} = T_{pr}^{(i)} \left(\frac{V_{pr}^{(i)}}{V_{pr}^{(i+1)}} \right)^{\gamma_{pr}-1} \quad (18)$$

A check must be performed on the remaining volume of propellants in the tanks at current iteration: if the volume of fuel is negative it means that the combustion is over so the simulation stops (the same for oxidizer). If there is some more propellant to use, the iteration goes on with the calculation of the new chamber pressure. This step is complex because it introduces another cycle of iterations inside each time step. The mass flow rate of the propellants depends on the pressure cascade in the feeding lines, hence on the chamber pressure. These

two variables are bounded and both unknown, but there's only one configuration that can match the boundary condition imposed by the critical condition in the throat, so the problem is well-posed.

From literature, the c^* of the chamber has the following expression:

$$c^* = \frac{p_c A_t}{\dot{m}_{fu} + \dot{m}_{ox}} \quad (19)$$

It correlates the chamber pressure with the propellants flow rate in the throat. Moreover, it only depends on the thermodynamics of the combustion process in the chamber, which changes over time due to the architecture of blow-down system. By imposing the critical conditions in the throat, it can be rewritten as:

$$c^* = \sqrt{\frac{\mathcal{R}}{\mathcal{M}}} \frac{T_c}{\gamma} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{\gamma-1}} \quad (20)$$

The system is coherent only if the two expressions give the same result. A system of equations could be created and numerically solved to match both the pressure cascade and c^* .

A reasonable initial guess for chamber pressure $p_c^{(i+1)(1)}$ is taken as the pressure at previous time step $p_c^{(i)}$. The next steps $p_c^{(i+1)(j)}$ will converge progressively towards the real current pressure $p_c^{(i+1)}$ (for increasing j). From $p_c^{(i+1)(j)}$ the $c^{*(i+1)(j)}$ is computed from the pressure cascade as described in Equation 19:

$$u_{fd,p}^{(i+1)(j)} = \sqrt{\frac{2(p_{pr}^{(i+1)} - p_c^{(i+1)(j)})}{\rho_p K_p}} \quad (21)$$

$$m_p^{(i+1)(j)} = \rho_p A_{fd,p} u_{fd,p}^{(i+1)(j)} \quad (22)$$

$$O/F^{(i+1)(j)} = \frac{m_{ox}^{(i+1)(j)}}{m_{fu}^{(i+1)(j)}} \quad (23)$$

$$c^{*(i+1)(j)} = \frac{A_t p_c^{(i+1)(j)}}{\dot{m}_{fu}^{(i+1)(j)} + \dot{m}_{ox}^{(i+1)(j)}} \quad (24)$$

Equation 20 is solved directly using CEAM software, which takes as input $p_c^{(i+1)(j)}$ and $O/F^{(i+1)(j)}$ to return $c_{cea}^{*(j)}$.

The two computed c^* are then compared: if their difference satisfies a certain tolerance, then the cycle stops and returns the new values for the current time step. Else, the inner cycle continues the refinement by guessing a new $p_c^{(i+1)(j+1)}$ from $p_c^{(i+1)(j)}$.

Finally, a check on the new combustion pressure $p_c^{(i+1)}$ is performed in order to stay above the minimum design pressure $p_{c,min}$, as mentioned in **REFERENCE**. Similarly to the previous check, if the pressure drops below the limit the simulation stops and returns the results, else it continues with the next time step.

The same general algorithm is used to refine the initial assumptions of O/F and B , which influence the nominal design of the whole engine (as described in subsection 2.2). In this case, the initial c^* value from design is assumed constant to reduce the computational burden, since the algorithm is applied over a wide range of combinations of O/F and B . This assumption is reasonable because the O/F (and as consequence the thermodynamics of the combustion) hardly varies during the whole burn, as can be noticed in **REFERENCE**.

3 Results analysis

4 Nozzle losses

In order to calculate and evaluate the nozzle losses, further modifications and calculations were added to the model previously presented in [section 2](#). In particular 2D, throat erosion and boundary layer losses were considered.

4.1 Losses calculations

Each loss term has been calculated as follows:

- **2D losses:** for a parabolic Rao nozzle this loss can be computed in a similar way as a conical nozzle by applying [Equation 25](#)

$$\lambda = \frac{1}{2} \left[1 + \cos \left(\frac{\delta + \theta_e}{2} \right) \right] \quad (25)$$

where δ is the cone angle of an fictitious conical nozzle with the same divergent length and area ratio^{Sutton}.

- **Throat erosion losses:** this effect is due to the increasing throat area over time whose behavior can be obtained by considering a constant erosion rate for simplicity. Since this loss is time dependent it needs to be considered inside the dynamic model of the system ([subsection 2.3](#)).

$$D_t^{(i+1)} = D_t^{(i)} + 2E_t \Delta t \quad (26)$$

Usually the erosion rate is calculated through experimental measurements of the propulsion system, in this case a suitable erosion rate has been searched for in literature. Due to the smallness of the system, no acceptable rates were found, therefore, an increase of 2% of the initial throat radius over the entire burn was assumed. This gives an erosion rate of about $1.540 \cdot 10^{-2} \mu\text{m/s}$.^{Suttontesi_malesia}

- **Boundary layer losses:** to determine this contribution the effect of the boundary layer in the throat of the nozzle must be estimated. To achieve this the thermophysical properties of the exhaust gasses at the throat must be recovered from the CEAM outputs of the nominal design ([subsection 2.2](#)). From them the throat Reynolds number can be obtained.

$$Re_t = \frac{\rho_t D_t u_t}{\mu_t} \quad (27)$$

Introducing the curvature of the throat, recovered from the Rao nozzle geometry, a modified Reynolds number is derived as follows:

$$k_t = 0.382 \frac{D_t}{2} \quad (28)$$

$$Re' = \sqrt{\frac{D_t}{2k_t}} Re_t \quad (29)$$

Now it is possible to calculate the throat discharge coefficient from which the real mass flow and effective throat area can be calculated.

$$C_{d,t} = 1 - \left(\frac{\gamma_t + 1}{2} \right)^{\frac{3}{4}} \left[3.266 - \frac{2.128}{\gamma_t + 1} \right] \frac{1}{\sqrt{Re'}} + 0.9428 \frac{(\gamma_t - 1)(\gamma_t + 2)}{Re' \sqrt{\gamma_t + 1}} \quad (30)$$

$$\dot{m}_r = C_{d,t} \dot{m}_{id} \quad (31)$$

$$A_{t,eff} = \frac{\dot{m}_r c^*}{p_c} \quad (32)$$

4.2 Effects on the nominal design

Including all the losses in the dynamic model reduces the overall performance of the system as expected, however, they only have a minor effect on the evolution of the throat area and its discharge coefficient as can be seen both in [Figure 3](#) and [Figure 4](#). This is mainly due to the fact that the erosion rate was recovered by imposing a rather small variation of the throat area over time.

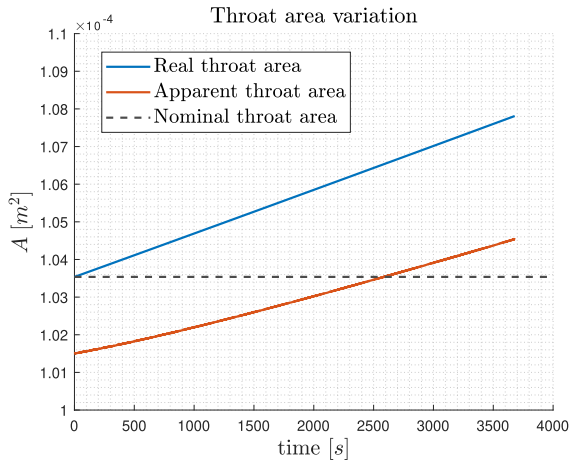


Figure 3: Throat area evolution

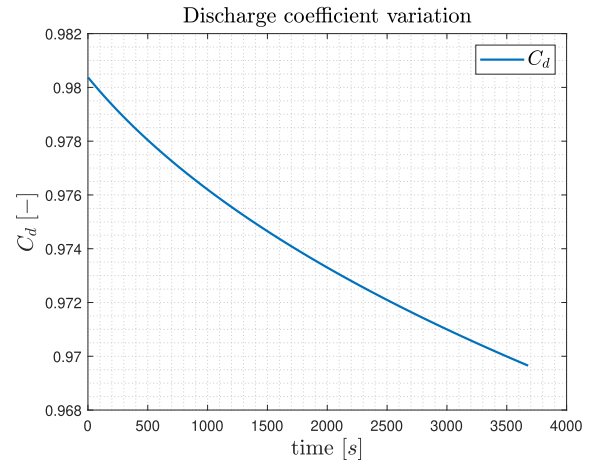


Figure 4: Throat discharge coefficient evolution

The real throat area describes its physical evolution due to erosion while the apparent one is the effective area as seen by the gas flow due also to the presence of the boundary layer. The values found for the discharge coefficients, instead, are compatible with values found in literature together with the range of the modified Reynolds number used to compute them. **REFERENCE** These variations have repercussions over the entire system as the chamber pressure evolution is influenced, so are the pressure cascade, the propellants mass flow rates and all consequent physical quantities as can be seen in the following figures.

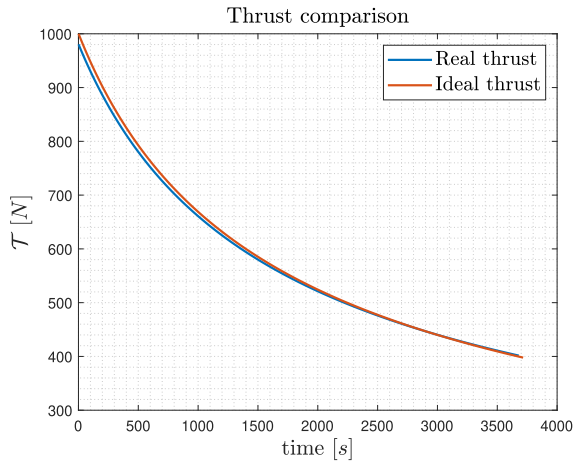


Figure 5: Thrust comparison

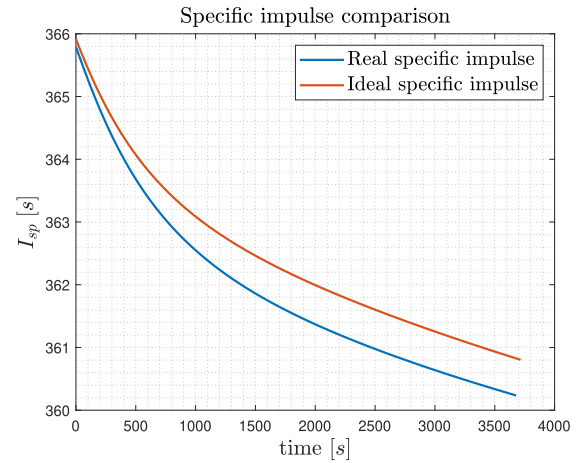


Figure 6: Specific impulse comparison

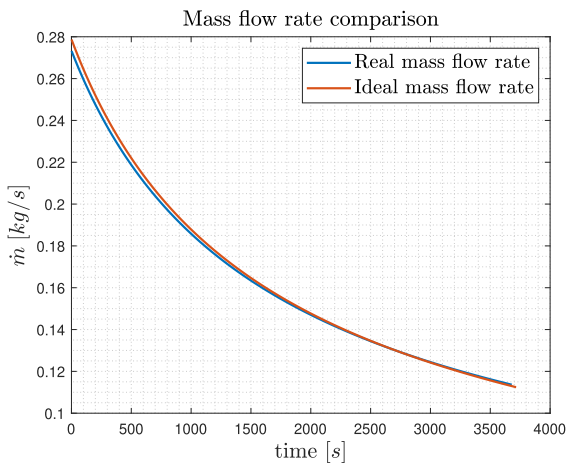


Figure 7: Propellants mass flow rate comparison

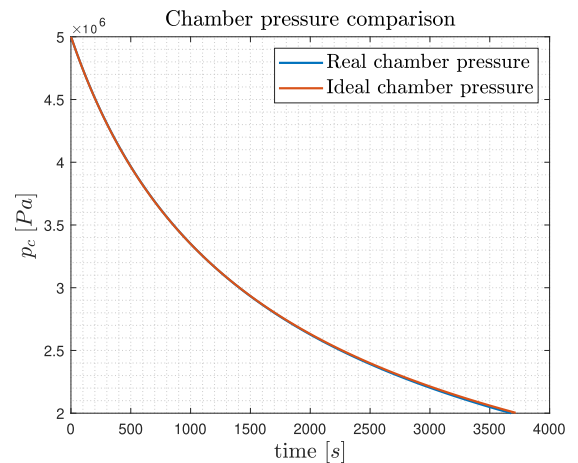


Figure 8: Combustion chamber pressure comparison

Again it is possible to appreciate the fairly small repercussions of these losses as all quantities exhibit basically the same behavior except the specific impulse, which shows a larger separation from the ideal case. The total impulse and the burn time also decrease slightly as reported in [Table 7](#).

| | I_{tot} [Ns] | t_b [s] |
|--------------|--------------------|-----------|
| Ideal | $2.173 \cdot 10^6$ | 3716 |
| Real | $2.140 \cdot 10^6$ | 3679 |

[Table 7](#): Total impulse and burn time comparison

5 Additive manufacturing influences

6 Cooling analysis

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