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## Title

**MSc in Space Engineering**

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## Abstract

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# 1. Interplanetary mission

## 1.1. Introduction

### 1.1.1. Description of the problem

The first part of the assignment aims at designing an interplanetary transfer from Mars to asteroid 1036 Ganymed exploiting a powered gravity assist on Earth. The problem is analyzed through the patched conics method, without considering the injection and arrival hyperbolae. The initial and final velocity vector of the satellite are assumed to be the same of the respective celestial body. The two heliocentric legs are calculated through the Lambert problem. The trajectory is selected with the only criteria of minimizing the cost of the mission, assessed through the total  $\Delta V$ . The latter is computed by summing different contributions:

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 + \Delta V_3 \quad (1.1)$$

Where the three terms are defined as:

- $\Delta V_1$  related to the injection in the first heliocentric leg;
- $\Delta V_2$  related to the exit from the second heliocentric leg;
- $\Delta V_3$  related to the impulse given by the engine at pericentre of hyperbola fly-by;

All the manoeuvres are assumed to be impulsive, i.e. they change only the velocity vector of the spacecraft, maintaining invariated the position vector. Note that  $\Delta V_1$  and  $\Delta V_2$  are related to heliocentric velocities, while  $\Delta V_3$  is calculated through relative geocentric velocities.

For each manoeuvre the cost is computed as:

$$\Delta V_i = \|V_{+,i} - V_{-,i}\| \quad (1.2)$$

where  $V_{-,i}$  and  $V_{+,i}$  are the velocity vectors before and after the  $i$ -th manoeuvre respectively.

### 1.1.2. Assigned data and constraints

A few constraints were considered:

- earliest departure date  $t_{min,dep} = [01/01/2028 00 : 00 : 00]$  in Gregorian calendar
- latest arrival date  $t_{max,arr} = [01/01/2058 00 : 00 : 00]$  in Gregorian calendar
- time of flights of the two heliocentric arcs must be greater than the associated parabolic time
- minimum pericentre radius of the fly-by hyperbola  $r_p = r_E + 500 \text{ km}$
- single-revolution Lambert problem was considered
- reasonable total time of the mission

Note that the constraint on the pericentre radius of the fly-by is considered both for avoid impact on Earth and to prevent undesired atmospheric drag effects.

## 1.2. Algorithms description

The targeting problem previously defined can be seen as an optimization problem with three degrees of freedom (DOFs). Indeed, once the departing date and the two times of flight of the heliocentric legs are chosen, both the Lambert's arcs and the fly-by hyperbola are fully defined. Regarding the formulation of the Lambert's problem, it requires the knowledge of the initial and final position and also the imposed time of flight between them. For two Lambert's arcs it would be needed a total of six DOFs, but this quantities are dependant one to each other:

- the final position vector of the first arc corresponds to the initial position of the second one;
- the initial date for the departing on the second arc corresponds the arrival date on the first arc;
- fixing the first Lambert's arc, the final position and arrival date for the second Lambert's arc are related through the analytical ephemerides.

Once the two heliocentric legs are determined, the powered gravity assist follows as the geocentric velocity vectors are known.

The method implemented to solve the optimization problem is the **brute force algorithm** refined with the **gradient descent algorithm**. Then, to validate the results, the **brute force algorithm** has been used solely with a more dense search grid in a reasonable domain.

### 1.2.1. Refined brute force algorithm

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#### Algorithm 1 Bruteforce algorithm

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```

Require:  $T_{dep}, \Delta T_1, \Delta T_2$ 
 $\Delta V_{min} = 10^{10}$ 
for  $i$  in  $T_{dep}$  do
    for  $j$  in  $\Delta T_1$  do
        Calculate  $T_{fly-by}$ 
        Calculate first Lambert's arc
        Calculate Mars' velocity at  $T_{dep}$ 
        Calculate  $\Delta V_1$ 
        for  $k$  in  $\Delta T_2$  do
            Calculate  $T_{arr}$ 
            Calculate second Lambert's arc
            Calculate Asteroid's velocity at  $T_{arr}$ 
            Calculate  $\Delta V_2, \Delta V_3, \Delta V_{tot}$ 
            if  $\Delta v_{tot} < \Delta v_{min}$  and  $r_p > r_{Earth} + 500\ km$  then
                 $\Delta v_{min} = \Delta v_{tot}$ 
                 $T_{dep,min} = T_{dep}$ 
                 $T_{fb,min} = T_{fb}$ 
                 $T_{arr,min} = T_{arr}$ 
            end if
        end for
    end for
end for

```

Minimize cost using  $fminunc$  with initial guess  $(T_{dep,min}; T_{fb,min}; T_{arr,min})$   
**return**  $\Delta V_{min}; T_{dep,min}; T_{fb,min}; T_{arr,min}$

---

The presented Algorithm 1 is reliable, yet computationally demanding since the research of the minimum is performed through a triple-nested *for* loop. The time of the execution highly depends on the refinement chosen for the three selected periods of time. From this considerations and also noticing that the function to minimize presents high irregularities, it is clear that narrow time windows can help in the overall research. Moreover, with this last observation, it is possible to use a less fine grid of research, since it is reasonable to think that the function will present less irregularities.

As a consequence, it was decided to perform a physical analysis (section 1.3) in order to wisely reduce the time domains for departure, fly-by and arrival. The idea of Algorithm 1 is that, once this reduction is performed, the brute-force search can be carried out on a small but refined grid. To speed up the process and refine the solution, *fminunc* of MATLAB was used. The selected initial guess is the outcome of the brute force research.

In order to completely validate the results, the robustness of the brute force algorithm has been exploited. Referring to Algorithm 1, *fminunc* was removed after the for-loop search and the research grid was refined. Moreover, with this computation it is possible to validate the time reduction analyzed in section 1.3.

## 1.3. Reduction of the time windows

### 1.3.1. Resonance period analysis

The first natural reduction on the domain of interest that could come to mind is to search the frequency on which the three celestial bodies repeat the relative positions on their orbits. On the approximation of circular orbits, this particular time period would be the synodic period generalized for the case of three bodies. However, this path is unviable because the orbit of the asteroid has a relevant eccentricity. To better comprehend the problem of having that eccentricity, particular attention have to be paid on the definitions of phasing and synodic period for two bodies:

- **Phasing**  $\phi \rightarrow$  the angle between two celestial bodies, calculated as the difference in their **true anomalies**:

$$\phi(t) = \theta^{(2)}(t) - \theta^{(1)}(t) \quad (1.3)$$

- **Synodic period**  $T_{syn} \rightarrow$  if two celestial bodies have initial phasing  $\phi_0$ , they will return to the same phasing after a synodic period  $T_{syn}$ :

$$\phi(t_0) = \phi_0 \rightarrow \phi(t_0 + T_{syn}) = \phi_0 \quad (1.4)$$

As the definitions rely on the **true anomalies** of celestial bodies, non-circular orbits mean that the same phasing does NOT imply the same relative positions between them. In other words, once the synodic period has passed, the phasing of the three considered bodies could result in a completely different relative positions with respect to the initial condition.

The problem needs to be reformulated. The goal is to find the period of time that elapses between a state of orbital positions of the bodies and the next occurrence of the same state. In literature, this particular period is called **period of orbital resonance** and it will be here indicated as  $T_{res}$ . Supposing that the orbits keep the other Keplerian elements unchanged during the revolution, the relation on true anomalies can be expressed as:

$$\begin{cases} \theta^{(1)}(t_0) = \theta_0^{(1)} \\ \theta^{(2)}(t_0) = \theta_0^{(2)} \\ \theta^{(3)}(t_0) = \theta_0^{(3)} \end{cases} \rightarrow \begin{cases} \theta^{(1)}(t_0 + T_{res}) = \theta_0^{(1)} \\ \theta^{(2)}(t_0 + T_{res}) = \theta_0^{(2)} \\ \theta^{(3)}(t_0 + T_{res}) = \theta_0^{(3)} \end{cases} \quad (1.5)$$

Since the true anomaly for an orbit repeats itself every orbital period  $T$ , it results:

$$\begin{cases} \theta_0^{(1)} = \theta^{(1)}(t_0 + iT^{(1)}) = \theta^{(1)}(t_0 + T_{res}) \\ \theta_0^{(2)} = \theta^{(2)}(t_0 + jT^{(2)}) = \theta^{(2)}(t_0 + T_{res}) \\ \theta_0^{(3)} = \theta^{(3)}(t_0 + kT^{(3)}) = \theta^{(3)}(t_0 + T_{res}) \end{cases} \rightarrow T_{res} = iT^{(1)} = jT^{(2)} = kT^{(3)} \quad (i, j, k \in \mathbb{N}) \quad (1.6)$$

As obtained in [Equation 1.6](#), the resonance period  $T_{res}$  must be a multiple of all the three orbital periods. To find three compatible natural numbers for  $i, j, k$ , the following procedure can be followed:

```

i = 1; j = i · T(1)/T(2); k = i · T(1)/T(3)
while j  $\notin \mathbb{N}$  or k  $\notin \mathbb{N}$  do            $\triangleright$  a tolerance tol must be implemented
    i = i + 1
    j = i ·  $T^{(1)}$ / $T^{(2)}$ 
    k = i ·  $T^{(1)}$ / $T^{(3)}$ 
end while
return i, j, k

```

Note that, since a perfect resonance of three celestial bodies is realistically impossible, a certain tolerance *tol* must be introduced when evaluating  $j, k \in \mathbb{N}$  in order to compute a reasonable  $T_{res}$ . In the specific case of this report, the execution of the above algorithm returned the following results:

<i>tol</i>	<i>i (Earth)</i>	<i>j (Mars)</i>	<i>k (1036 Ganymed)</i>
0.1159	13	6.9119	4.8841

[Table 1.1: Results of the resonance analysis](#)

$T_{res}$  results to be 13 Earth's sidereal years, so the time domain can be restricted accordingly. It is important to keep in mind that this is an approximation, but since the mission has to depart in a reasonable date, it is acceptable to restrain the time window to the first 13 years. In any case, the cost of the mission will repeat similarly after 13 years.

### 1.3.2. Cost-plot analysis

The reduction of the departure time window in [subsection 1.3.1](#) is a powerful but not sufficient restriction for the domain to analyze with [Algorithm 1](#). Hence, a more detailed analysis has to be performed, including some restrictions for the first and second time of flight. The present paragraph presents and studies in detail two cost-plots. The first is related to the injection cost on the first Lambert's arc while the second exit cost is referred to the second Lambert's arc. Notice that the pork-chop plot of each leg can't be plotted, since the the two arcs are linked at the fly-by position. Given this consideration, the two cost plot obtained with the 13-year constrained departure window are following.

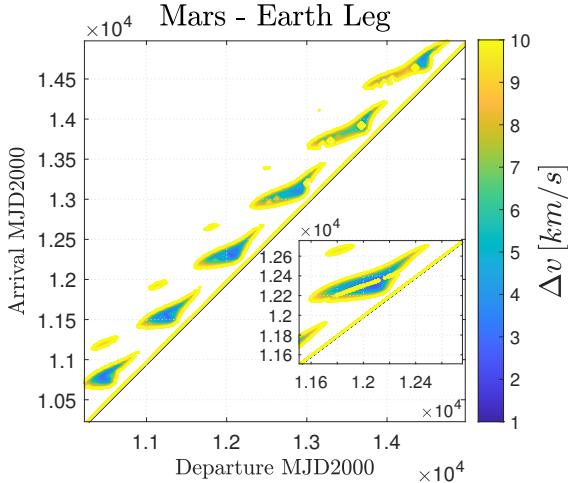


Figure 1.1: Cost plot for 1st leg

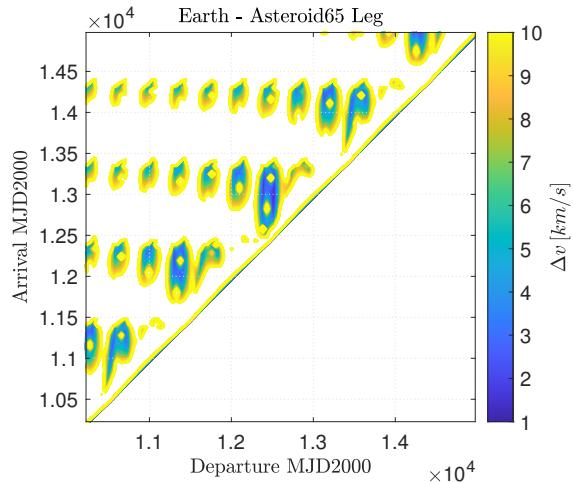


Figure 1.2: Cost plot for 2nd leg

The contour plots are set to a maximum value for the  $\Delta V$  of 10 km/s, greater values are excluded since they wouldn't be relevant for the present study. Moreover, it was decided to use symmetrical time domains for departure and arrival, expressed in *mjd2000* units. Some important information can be retrieved from the contour plot of Figure 1.1:

- all the local minima that reach the lowest values are located along the diagonal, indicating that there is a constraint on the time of flight for the first arc. This behaviour is present also in the second arc Figure 1.2. Above the diagonal of both the plots the minima are higher Figure 1.2, or not existent Figure 1.1 for the set of contour lines considered. This is related to the fact that the Lambert's arc were calculated without considering the multi-revolution solutions subsection 1.1.2. Indeed, if the time of flight imposed is higher, the Lambert's arc solution will have a higher semi-major axis. This results in a higher  $\Delta V$  to inject or exit the transfer arc since the energy difference between orbits becomes considerable. The solution that embraces multi-revolution arcs would benefit in this sense and could open new scenarios for the interplanetary arcs, as cited in<sup>[1]</sup>.
- It can be noticed that a repetition pattern of the minima is present in both the figures. With particular emphasis on the Mars-Earth leg, the minimum shown as the blue-coloured zone, repeats every synodic period Table 1.2. This is due to the quasi-circular orbits of both the planets.
- A repetition pattern is noticeable also in Figure 1.2. To better analyze the plot, the values of the last three columns of Table 1.2 can be exploited. It can be noticed that the synodic period  $T_{syn}^{E,G}$  can not explain the reoccurrence of the minima while the resonance periods are both meaningful. The high tolerance resonance period  $T_{res,1}^{E,G}$  fits all the repeated minima along the diagonal. The low tolerance resonance period  $T_{res,2}^{E,G}$  can catch a lower frequency pattern, appreciable by focusing on the first and second-last *P-shaped* minima along the diagonal.
- Regarding the cost-plot of Earth-to-Mars transfer Figure 1.1, a particular behaviour of the minimum can be noticed on the zoomed box. The focus allows to appreciate a yellow ridge that separates the minimum zone in to two sub-area that contains mainly two minimum zones. As mentioned in<sup>[1]</sup>, the two zones correspond to two solutions for the Lambert's problem, in particular to the value of the transfer angle  $\Delta\theta$ . The upper zone is related to  $\Delta\theta > \pi$  while the lower zone corresponds to  $\Delta\theta < \pi$ . When  $\Delta\theta = \pi$ , since the orbital planes of Earth and Mars are not exactly coincident, the plane of the heliocentric transfer leg becomes almost perpendicular to the ecliptic, increasing the injection cost of the mission<sup>[2]</sup>. When the co-planar and the circular assumptions on Earth and Mars orbits are made, the central part of the minimum lobe in the zoomed box coincides with the Hohmann transfer<sup>[1]</sup>.

$T_{syn}^{M,E}$ [mjd2000]	$T_{syn}^{E,G}$ [mjd2000]	$T_{res,1}^{E,G}$ (high tol) [mjd2000]	$T_{res,2}^{E,G}$ (low tol) [mjd2000]
779.9418	585.0697	1095.7711	2922.0562

Table 1.2: Results of the resonance analysis

From the observations presented above, it was decided to analyze the same cost graph but plotting the time of flight on the y-axis. In order to consider reasonable time of flight domains, some coarse reduction was adopted. In particular, long times of flight are excluded for the already mentioned reasons, while very short times of flight are not taken into the domain since they are associated to high transfer-costs. Infact, this last situation corresponds to the yellow diagonal line of the plots in Figure 1.1 and Figure 1.2.

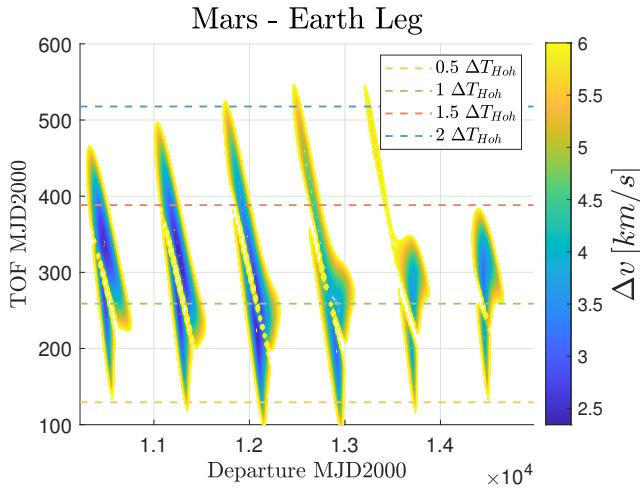


Figure 1.3: Cost plot for 1st leg

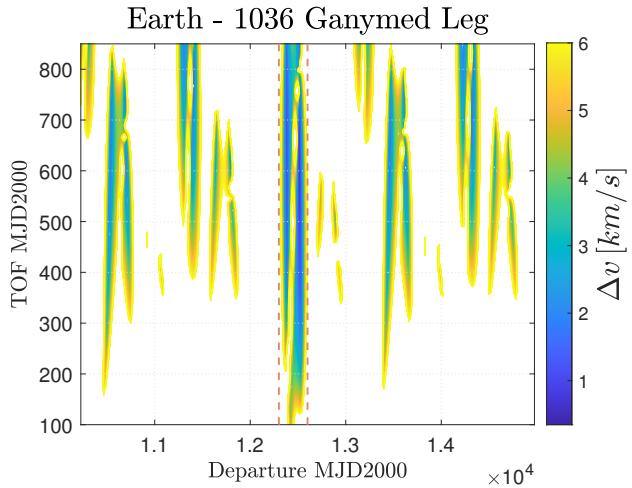


Figure 1.4: Cost plot for 2nd leg

For the Earth-Mars transfer, multiples and sub-multiples of the Hohmann transfer time were considered. The hypothesis is reasonable since Mars and Earth have almost circular and co-planar orbits<sup>[3]</sup>. The same hypothesis can't be applied on the second heliocentric leg because of Ganymed's eccentric orbit and Hohmann's hypothesis doesn't hold. A relevant area of lowest minima can be noticed in Figure 1.4, in particular the vertical bar in the narrow of the fly-by date  $1.25 \cdot 10^4 \text{ mjd2000}$ . This means that by choosing this flyby date, the choice of the second time of flight is less relevant since the minima develops along all the plotted vertical. In addition to this, the Hohmann's time of flight multiples and submultiples can be considered for the first time of flight by subtracting them to the considered fly-by date. This operation generates a resized domain for the departure dates. Summing up all the considerations, the following restricted time domain was selected:

- $T_{dep} = 1.25 \cdot 10^4 - [0.65, 2] \cdot \Delta T_{Hoh} \text{ mjd2000}$
- $\Delta T_1 = [0.65, 2] \cdot \Delta T_{Hoh} \text{ mjd2000}$
- $\Delta T_2 = [200, 800] \text{ mjd2000}$

## 1.4. Conclusion and results

It was then used Algorithm 1 in the time domain selected at the end of subsection 1.3.2, using 40 elements for each array. The obtained results in terms of cost and dates are reported in the following tables:

$\Delta V_1 [\text{km/s}]$	$\Delta V_2 [\text{km/s}]$	$\Delta V_3 [\text{km/s}]$	$\Delta V_{tot} [\text{km/s}]$
4.0813	3.3130	$2.7715 \cdot 10^{-6}$	7.3942

 Table 1.3: Costs  $\Delta V$  of the optimal mission

Dep. Date	Fb. Date	Arr. Date
[03/06/2033]	[10/02/2034]	[27/03/2036]

Table 1.4: Optimal mission dates in Gregorian calendar

The obtained results were validated through the pure brute-force algorithm, with a more refined grid of search in the same domain. In particular 100 elements where considered, the data obtained were:  
 ... put the two tables of brute force algorithm

### 1.4.1. Heliocentric legs

The two heliocentric legs associated with the solution computed through Algorithm 1 can be characterized through the keplerian elements shown in Table 1.5:

Arc	$a [\text{AU}]$	$e [-]$	$i [\text{deg}]$	$\Omega [\text{deg}]$	$\omega [\text{deg}]$	$\theta_{dep} [\text{deg}]$	$\theta_{arr} [\text{deg}]$
M→E	1.1602	0.2870	1.18	321.55	105.97	195.38	74.03
E→G	1.9445	0.5029	4.17	141.55	339.91	20.09	231.37

Table 1.5: Heliocentric legs

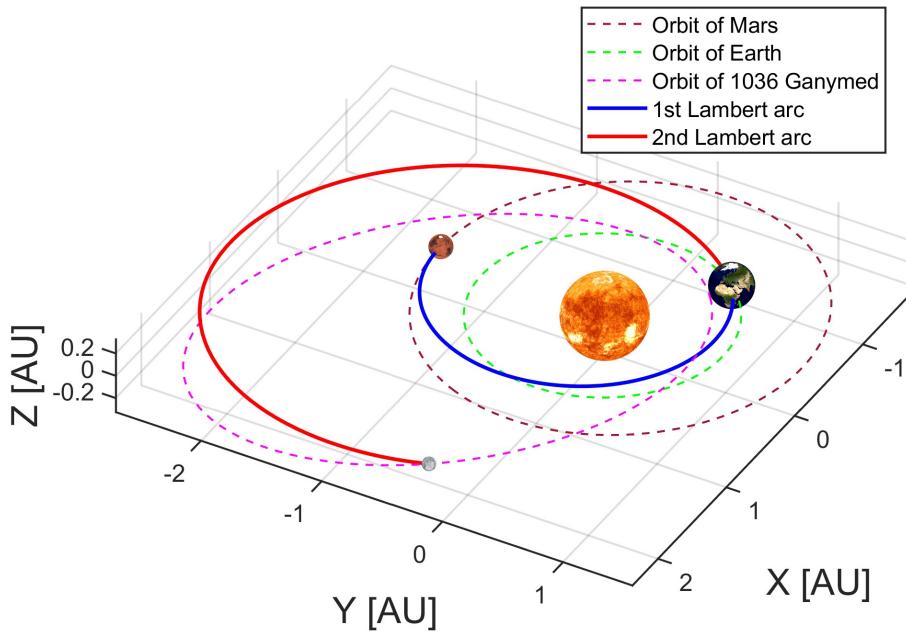


Figure 1.5: Optimal transfer trajectory - planet's dimension not in scale

The two heliocentric legs are displayed in Figure 1.5. The represented positions of Mars, Earth and 1036 Ganymed are respectively at departure, fly-by and arrival. All the orbits are plotted in the Heliocentric ecliptic inertial frame.

#### 1.4.2. Gravity assist

Known the incoming and outcoming heliocentric velocities of the spacecraft and the Earth's velocity at the fly-by date, it was possible to completely characterize the two branches of hyperbola in an Earth-centred ecliptic frame. The main parameters of the hyperbola are reported in Table 1.6:

$r_p$ [km]	$\delta_{tot}$ [deg]	$v_\infty^-$ [km/s]	$v_\infty^+$ [km/s]	$e_-$ [-]	$e_+$ [-]	$\Delta T_{soi}$ [days]	$\Delta V_{helio}$ [km/s]
7763.39	54.8918	7.743439	7.743435	2.167834	2.167833	2.6871	7.1381

Table 1.6: Gravity assist data

It can be noticed that the fly-by hyperbola respect the constraint on the pericentre radius given in subsection 1.1.2. Moreover, the impulse given by the engine at the pericentre (Table 1.3) is negligible with respect to the heliocentric fly-by manoeuvre cost, calculated as  $\Delta V_{helio} = \|V_\infty^+ - V_\infty^-\|$ . This confirms the efficiency effect of the examined fly-by. The permanence time passed inside the sphere of influence has been computed using the hyperbolic time law. In particular, considering the intersection between the two hyperbolic arcs and the radius of the Earth's sphere of influence (Equation 1.7), the true anomalies at entry and exit of SOI are computed.

$$r_{SOI,E} = r_{S \rightarrow E} \left( \frac{M_E}{M_S} \right)^{2/5} = 9.24647 \cdot 10^5 \text{ km} \quad (1.7)$$

## 2. Planetary mission

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