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SCUOLA DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Title

MSc in Space Engineering

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Abstract

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1.1. Symbols

Analisi della missione

$\boldsymbol{A_e}$	$[m^2]$	area di efflusso totale
$\boldsymbol{\phi}$	$[rad]$	angolo di traiettoria del razzo

Analisi della missione 2

$\boldsymbol{A_e}$	$[m^2]$	area di efflusso totale
$\boldsymbol{\phi}$	$[rad]$	angolo di traiettoria del razzo

1.2. Introduction

1.2.1. Description of the problem

The first part of the assignment aims at designing an interplanetary transfer from Mars to asteroid 1036 Ganymed exploiting a powered gravity assist on Earth. The problem is analyzed through the patched conics method, without considering the injection and arrival hyperbolae. The initial and final velocity vector of the satellite are assumed to be the same of the respective celestial body. The two heliocentric legs are calculated through the Lambert problem. The trajectory is selected with the only criteria of minimizing the cost of the mission, assessed through the total ΔV. The latter is computed by summing different contributions:

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 + \Delta V_3 \tag{1.1}$$

where $\boldsymbol{V_{-,i}}$ and $\boldsymbol{V_{+,i}}$ are the velocity vectors before and after the i-th manoeuvre respectively.
Where the three terms are defined as:

- ΔV_1 related to the injection in the first heliocentric leg;
- ΔV_2 related to the exit from the second heliocentric leg;
- ΔV_3 related to the impulse given by the engine at pericentre of hyperbola fly-by;

All the manoeuvres are assumed to be impulsive, i.e. they change only the velocity vector of the spacecraft, mantaining invariated the position vector. Note that ΔV_1 and ΔV_2 are related to heliocentric velocities, while ΔV_3 is calculated through relative geocentric velocities.
For each manoeuvre the cost is computed as:

$$\Delta V_i = \|\boldsymbol{V_{+,i}} - \boldsymbol{V_{-,i}}\| \tag{1.2}$$

1.2.2. Assigned data and constraints

A few constraints were considered:

- earliest departure date $t_{min,dep} = [01/01/2028\ \ 00:00:00]$ in Gregorian calendar
- latest arrival date $t_{max,arr} = [01/01/2058\ \ 00:00:00]$ in Gregorian calendar
- time of flights of the two heliocentric arcs must be greater than the associated parabolic time
- minimum pericentre radius of the fly-by hyperbola $r_p = r_E + 500\ km$
- single-revolution Lambert problem was considered
- reasonable total time of the mission

Note that the constraint on the pericentre radius of the fly-by is considered both for avoid impact on Earth and to prevent undesired atmospheric drag effects.

1.3. Algorithms description

The targeting problem previously defined can be seen as an optimization problem with three degrees of freedom (DOFs). Indeed, once the departing date and the two times of flight of the heliocentric legs are chosen, both the Lambert's arcs and the fly-by hyperbola are fully defined. Regarding the formulation of the Lambert's problem, it requires the knowledge of the initial and final position and also the imposed time of flight between them. For two Lambert's arcs it would be needed a total of six DOFs, but this quantities are dependant one to each other:

- the final position vector of the first arc corresponds to the initial position of the second one;
- the initial date for the departing on the second arc corresponds the arrival date on the first arc;
- fixing the first Lambert's arc, the final position and arrival date for the second Lambert's arc are related through the analytical ephemerides.

Once the two heliocentric legs are determined, the powered gravity assist follows as the geocentric velocity vectors are known.
Two methods were implemented to solve the optimization problem: **brute force algorithm** and the **gradient descent algorithm**.

1.3.1. Brute force algorithm

Algorithm 1 Bruteforce algorithm

Require: $T_{dep}, T_{flyby}, T_{arr}$
Ensure: ΔV_{min}
  $\Delta V_{min} = 10^{10}$
  for i in T_{dep} **do**
    for j in T_{flyby} **do**
      for k in T_{arr} **do**

      end for
    end for
  end for

1.3.2. Gradient descent algorithm

Algorithm 1

1.4. Reduction of the time windows

1.4.1. Resonance period analysis

The first natural reduction on the domain of interest that could come to mind is to search the frequency on which the three celestial bodies repeat the relative positions on their orbits. On the approximation of circular orbits, this particular time period would be the synodic period generalized for the case of three bodies. However, this path is unviable because the orbit of the asteroid has a relevant eccentricity. To better comprehend the problem of having that eccentricity, particular attention have to be paid on the definitions of phasing and synodic period for two bodies:

- **Phasing ϕ** $\rightarrow$ the angle between two celestial bodies, calculated as the difference in their **true anomalies**:

$$\phi(t) = \theta^{(2)}(t) - \theta^{(1)}(t) \tag{1.3}$$

- **Synodic period T_{syn}** $\rightarrow$ if two celestial bodies have initial phasing ϕ_0, they will return to the same phasing after a synodic period T_{syn}:

$$\phi(t_0) = \phi_0 \quad \rightarrow \quad \phi(t_0 + T_{syn}) = \phi_0 \tag{1.4}$$

As the definitions rely on the **true anomalies** of celestial bodies, non-circular orbits mean that the same phasing does NOT imply the same relative positions between them. In other words, once the synodic period has passed, the phasing of the three considered bodies could result in a completely different relative positions with respect to the initial condition.
The problem needs to be reformulated. The goal is to find the period of time that elapses between a state of orbital positions of the bodies and the next occurrence of the same state. In literature, this particular period is called **period of orbital resonance** and it will be here indicated as T_{res}. Supposing that the orbits keep the other Keplerian elements unchanged during the revolution, the relation on true anomalies can be expressed as:

$$\begin{cases} \theta^{(1)}(t_0) = \theta_0^{(1)} \\ \theta^{(2)}(t_0) = \theta_0^{(2)} \\ \theta^{(3)}(t_0) = \theta_0^{(3)} \end{cases} \rightarrow \begin{cases} \theta^{(1)}(t_0 + T_{res}) = \theta_0^{(1)} \\ \theta^{(2)}(t_0 + T_{res}) = \theta_0^{(2)} \\ \theta^{(3)}(t_0 + T_{res}) = \theta_0^{(3)} \end{cases} \tag{1.5}$$

Since the true anomaly for an orbit repeats itself every orbital period T, it results:

$$\begin{cases} \theta_0^{(1)} = \theta^{(1)}(t_0 + iT^{(1)}) = \theta^{(1)}(t_0 + T_{res}) \\ \theta_0^{(2)} = \theta^{(2)}(t_0 + jT^{(2)}) = \theta^{(2)}(t_0 + T_{res}) \\ \theta_0^{(3)} = \theta^{(3)}(t_0 + kT^{(3)}) = \theta^{(3)}(t_0 + T_{res}) \end{cases} \rightarrow \quad T_{res} = iT^{(1)} = jT^{(2)} = kT^{(3)} \qquad (i, j, k \in \mathbb{N}) \tag{1.6}$$

As obtained in Equation 1.6, the resonance period T_{res} must be a multiple of all the three orbital periods. To find three compatible natural numbers for i, j, k, the following procedure can be followed:

$i = 1$; $j = i \cdot T^{(1)}/T^{(2)}$; $k = i \cdot T^{(1)}/T^{(3)}$
while $j \notin \mathbb{N}$ or $k \notin \mathbb{N}$ **do** ▷ a tolerance *tol* must be implemented
  $i = i + 1$
  $j = i \cdot T^{(1)}/T^{(2)}$
  $k = i \cdot T^{(1)}/T^{(3)}$
end while
return i, j, k

Note that, since a perfect resonance of three celestial bodies is realistically impossible, a certain tolerance *tol* must be introduced when evaluating $j, k \in \mathbb{N}$ in order to compute a reasonable T_{res}. In the specific case of this report, the execution of the above algorithm returned the following results:

tol	$i\ (Earth)$	$j\ (Mars)$	$k\ (1036\,Ganymed)$
0.1159	13	6.9119	4.8841

Table 1.1: Results of the resonance analysis

T_{res} results to be 13 Earth's sidereal years, so the time domain can be restricted accordingly. It is important to keep in mind that this is an approximation, but since the mission has to departure in a reasonable date, it is acceptable to restrain the time window to the first 13 years. In any case, the cost of the mission will repeat similarly after 13 years.

1.4.2. Cost-plot analysis

1.4.3. Final time window selection

1.5. Conclusion and results

[1]

2. Planetary mission

Bibliography

[1] Howard D. Curtis. *Orbital Mechanics for Engineering Students*. Elsevier, 2014.