Problem 1

a) FALSE (Acme Computer's revenue was larger)

b) TRUE (Both of them true and $\wedge$ between them)

c) TRUE ($F \vee T \equiv T$)

d) TRUE ($F \to T \equiv T$)

e) TRUE ($T \leftrightarrow T \equiv T$)

Problem 2

p: It is below freezing

q: It is snowing

a) $p \wedge q$

b) $p \wedge \neg q$

c) $\neg p \wedge \neg q$

d) $p \vee q$

e) $p \to q$

f) $(p \vee q) \wedge (p \to \neg q)$

g) $p \leftrightarrow q$

Problem 3

a) 1 variable $\Rightarrow 2^1 = 2$ rows

b) 4 variables $\Rightarrow 2^4 = 16$ rows

c) 6 variables $\Rightarrow 2^6 = 64$ rows

d) 4 variables $\Rightarrow 2^4 = 16$ rows

Problem 4

a)

p	$\neg p$	$p \land \neg p$
T	F	F
F	T	F

b)

$p \lor \neg p$
T
T

d)

p	q	$p \lor q$	$p \land q$	$(p \lor q) \to (p \land q)$
T	F	T	F	F
F	T	T	F	F
T	T	T	T	T
F	F	F	F	T

c)

p	q	$\neg q$	$p \lor \neg q$	$(p \lor \neg q) \to q$
T	F	F	F	F
F	T	F	F	T
T	T	F	T	T
F	F	T	T	F

(4) f)

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

e)

p	q	$p \to q$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Problem 5 First professor said „I don't know", because she doesn't know do other professors want a coffee. To remind the question was „Does everyone wants a coffee?" And if she didn't want, she would tell „no". Same about second professor. But third professor said „No, not everyone wants a coffee" which means that she doesn't want.

Problem 6

a) Jan is not rich, ~~and~~ or is not happy.

b) Carlos will not bicycle and will not run tomorrow.

c) Mei doesn't walk and doesn't take the bus to class.

d) Ibrahim is not smart, ~~and~~ or is not hard working.

Problem 7

a)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	F	F	T
F	T	F	T
T	T	T	T
F	F	F	T

b)

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

c)

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

7 d)

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e)

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

f)

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

Problem 8

a) $(p \wedge q) \rightarrow p$

$\neg(p \wedge q) \vee p$

$(\neg p \vee \neg q) \vee p$

$(\neg p \vee p) \vee \neg q$

$T \vee \neg q \equiv T$

b) $p \rightarrow (p \vee q)$

$\neg p \vee (p \vee q)$

$(\neg p \vee p) \vee q$

$T \vee q \equiv T$

c) $\neg p \rightarrow (p \rightarrow q)$

$\neg\neg p \vee (p \vee q)$

$(p \vee p) \vee q$

$T \vee q \equiv T$

d) $(p \wedge q) \rightarrow (p \rightarrow q)$

$\neg(p \wedge q) \vee (p \vee q)$

$(\neg p \vee \neg q) \vee (p \vee q)$

$(\neg p \vee p) \vee (\neg q \vee q)$

$T \vee T \equiv T$

e) $\neg(p \rightarrow q) \rightarrow p$

$\neg(\neg(p \vee q)) \vee p$

$\neg(\neg p \wedge \neg q) \vee p$

$(p \vee q) \vee p$

$(p \vee p) \vee q$

$T \vee q \equiv T$

f) $\neg(p \rightarrow q) \rightarrow \neg q$

$\neg(\neg(p \vee q)) \vee \neg q$

$\neg(\neg p \wedge \neg q) \vee \neg q$

$(p \vee q) \vee \neg q$

$(p \vee (\neg q \vee q))$

$p \vee T \equiv T$

Problem 9

p	q	r	$p \to r$	$q \to r$	$(p \to r) \land (q \to r)$	$p \lor q$	$(p \lor q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	F	T
F	T	F	T	F	F	T	F
T	F	F	F	T	F	T	F
F	F	F	T	T	T	F	T

logically equivalent

Problem 10

✓a) $\exists_x (P(x) \land Q(x))$

✓b) $\exists_x (P(x) \land \neg Q(x))$

✓c) $\forall_x (P(x) \lor Q(x))$

×d) $\neg \forall_x (P(x) \lor Q(x))$ OR $\forall_x (\neg P(x) \lor \neg Q(x))$

$\forall_x \neg (P(x) \lor Q(x))$ OR $\forall_x ((\neg P(x)) \land (\neg Q(x)))$

OR $\neg \exists_x (P(x) \lor Q(x))$

Problem 11

a) TRUE $\forall n (n^2 \geq 0) \rightarrow$ any integers gives us zero or greater than zero

b) FALSE $\exists n (n^2 = 2) \rightarrow$ any integer cannot give us 2 if it is squared

c) TRUE $\forall n (n^2 \geq n) \rightarrow$ any integer gives us equal to or greater than n

d) FALSE $\exists n (n^2 < 0) \rightarrow$ positive $n^2 > 0$, negative $n^2 > 0$, $0^2 \not< 0$.

Problem 12

a) $\exists x P(x)$ $P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4)$

b) $\forall x P(x)$ $P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

c) $\exists x \neg P(x)$ $\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

d) $\forall x \neg P(x)$ $\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

e) $\neg \exists x P(x)$ same answer as in (c)

f) $\neg \forall x P(x)$ same solution as in (d)

Problem 13

✓ a) $\exists x H(x)$

✓ b) $\forall x F(x)$

✓ c) $\exists x \neg C(x)$

✓ d) $\exists x M(x)$

✓ e) $\neg \exists x P(x)$ OR $\forall x \neg P(x)$

Problem 14

✓ a) $\neg M(\text{Chou}, \text{Koko})$

b) $\neg M(\text{Arlene}, \text{Sarah}) \wedge \neg T(\text{Arlene}, \text{Sarah})$

✓ c) $\neg M(\text{Deborah}, \text{Jose})$

✓ d) $\forall x M(x, \text{Ken})$

✓ e) $\forall x \neg T(x, \text{Nina})$

✓ f) $\forall x T(x, \text{Avi}) \vee \forall x M(x, \text{Avi})$ or $\forall x (T(x, \text{Avi}) \vee M(x, \text{Avi}))$

✓ g) $\exists x \forall y (M(x, y))$

✓ h) $\exists x \forall y (M(x, y)) \vee \exists x \forall y (T(x, y))$ or $\exists x \forall y (M(x, y) \vee T(x, y))$

✓ i) $\exists x \exists y (M(x, y)) \wedge \exists x \exists y (M(y, x))$ or $\exists x \exists y (M(x, y) \wedge M(y, x))$

✓ j) $\exists x M(x, x)$

Prob. 14

k) $\exists_x \forall_y (\neg M(y, x) \land \neg T(y, x))$

l) $\forall_x \exists_y (M(y, x) \lor T(y, x))$

m) $\exists_x \exists_y (M(x, y) \land T(y, x))$

n) $\exists_x \exists_y (\forall_z (M(x, z) \lor M(y, z) \lor T(x, z) \lor T(y, z)))$