

EEE391

①

Homework -2

Turgut Alp Edis

21702587

Q1)

a) $x[n] = \underbrace{\delta[n]}_{\text{impulse function}} \Rightarrow y[n] = \frac{1}{4}(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3])$

$$\Rightarrow y[n] = h[n] = \boxed{\frac{1}{4}(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3])}$$

b) $H(w) = \int h[n] e^{-jwn} dn$

\Rightarrow using $h[n]$ from part a,

$$H(w) = \int (\frac{1}{4}(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3])) e^{-jwn} dn$$

$$\Rightarrow \boxed{H(w) = \frac{1}{4}(1 + e^{-jw} + e^{-j2w} + e^{-j3w})}$$

c) Substitute $x[n]$,

$$y[n] = \frac{1}{4}(5 + 4\cos(0.2\pi n) + 3\cos(0.5\pi n) + 5 + 4\cos(0.2\pi(n-1)) + 3\cos(0.5\pi(n-1)) + 5 + 4\cos(0.2\pi(n-2)) + 3\cos(0.5\pi(n-2)) + 5 + 4\cos(0.2\pi(n-3)) + 3\cos(0.5\pi(n-3)))$$

$$y[n] = \frac{1}{4}(4n + 20 + 4\cos(0.2\pi n) + 3\cos(0.5\pi n))$$

(2)

d) $y_1[n] = y[n]$ for all $n \geq 0$

Q2) $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ | $x(n) = \frac{1}{2\pi j} \int x(z) z^{-1} dz$

a) From formula of $X(z)$,

$$x(n) = \delta[n+5]$$

$$\Rightarrow \delta[n] = 1 \text{ at } n=0 \Rightarrow \delta[n+5] = 1 \text{ at } n=-5$$

$$\Rightarrow \sum_{n=-\infty}^{-6} \delta[n+5] z^{-n} + \sum_{n=-5}^{\infty} \delta[n+5] z^{-n} + \sum_{n=6}^{\infty} \delta[n+5] z^{-n}$$

$$= \sum_{n=-5}^{-1} \delta[n+5] z^{-n} \Rightarrow n=-5 \Rightarrow \delta[0] z^{(-5)} = z^5$$

b) Same steps as in part a, $\delta[n-5] = 1$ at $n=5$

$$\text{So, } X(z) = z^{-5}$$

c) Same steps as in part a, $\delta[n-1] = 1$ at $n=1$

$$\text{So, } X(z) = z^{-1}$$

d) Inverse formula $x(n)$ find from $X(z)$,

$$1(2, z^0 + 4, z^{-1} + 6, z^{-2} + 4, z^{-3} + 2, z^{-4}) = X(z)$$

$$\Rightarrow x(0)=2, x(1)=4, x(2)=6, x(3)=4, x(4)=2$$

\Rightarrow inverse is $x(n) = \{ \dots, 2, 4, 6, 4, 2, \dots, \infty \}$

e) same as part d $\Rightarrow x(0)=1, x(1)=-2, x(3)=3, x(5)=-1, x(2)=x(4)=0$

$$x(n) = \{ \dots, -1, -2, 0, 3, 0, -1, \dots \}$$

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Q3)

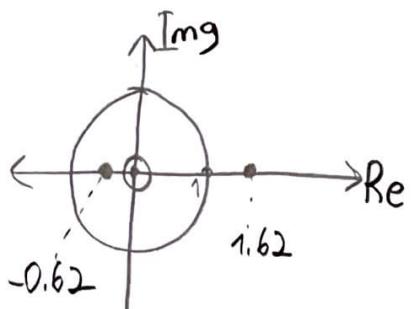
$$y[n] = y[n-1] + y[n-2] + x[n-1] \quad \text{taking } z\text{-transform}$$

$$Y(z) = z^{-1}y(z) + z^{-2}y(z) + z^{-1}X(z)$$

$$z^{-1}X(z) = Y(z)(1 - z^{-1} - z^{-2}) \Rightarrow \frac{Y(z)}{X(z)} = \frac{z^{-1}}{(1 - z^{-1} - z^{-2})} = \frac{z}{z^2 - z - 1} = H(z)$$

$$z^2 - z - 1 = 0 \Rightarrow \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$$

≈ 1.62 ≈ -0.62

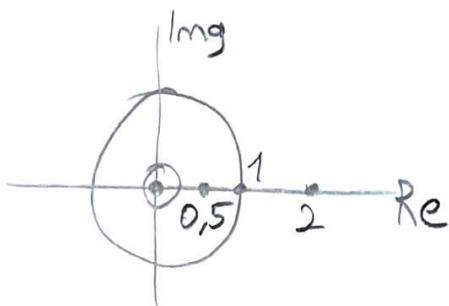


$$y[n-1] - \frac{5}{2}y[n] + y[n+1] = x[n] \quad \text{taking } z\text{-transform}$$

$$z^{-1}Y(z) - \frac{5}{2}Y(z) + zY(z) = X(z)$$

$$X(z) = Y(z)\left(z^{-1} - \frac{5}{2} + z\right) \Rightarrow \frac{Y(z)}{X(z)} = H(z) = \frac{2z}{2z^2 - 5z + 2}$$

$$2z^2 - 5z + 2 = 0 \Rightarrow z=0.5 \text{ or } z=2$$



Q4)

(4)

a) $h[n] = \delta[n-2] \Rightarrow h[-n] = \delta[-n-2] = \delta[n+2]$

Conv. of $h[-n]$ and $y[n]$

$$x[n] = h[-n] * y[n] = \sum \delta[n+2] \times (v[n-3] - v[n-6])$$

apply convolution formula

$$\rightarrow x[n] = \sum_{m=0}^{\infty} v[n-m-3] \times \delta[m+2] - \sum_{m=0}^{\infty} v[n-m-6] \times \delta[m+2]$$

$$\Rightarrow x[n] = \underbrace{\sum_{m=0}^{\infty} v[n-3] \times \delta[2]}_{=1} - \underbrace{\sum_{m=0}^{\infty} v[n-6] \times \delta[2]}_{=1}$$

$$\Rightarrow x[n] = v[n-3] - v[n-6]$$

b) Same formula as part a, $h[-n] = \delta[n] + \delta[n+1]$

$$\Rightarrow x[n] = \sum \delta[n-m] \times (\delta[m] - \delta[m-1]) \Rightarrow \text{eval. for } m=0,1$$

$$\Rightarrow x[n] = \delta[n] - \delta[n-1]$$

c) $h[-n] = \frac{1}{4} \delta[n] + \frac{1}{4} \delta[n+1] + \frac{1}{4} \delta[n+2] + \frac{1}{4} \delta[n+3]$

$$\Rightarrow x[n] = h[-n] * y[n] = \sum (-5\delta[n-m]) \times \left(\frac{1}{4}\delta[m] + \frac{1}{4}\delta[m+1] \right.$$

$$\left. + \frac{1}{4}\delta[m+2] + \frac{1}{4}\delta[m+3] \right) \Rightarrow \text{eval. for } m=0,1,2,3$$

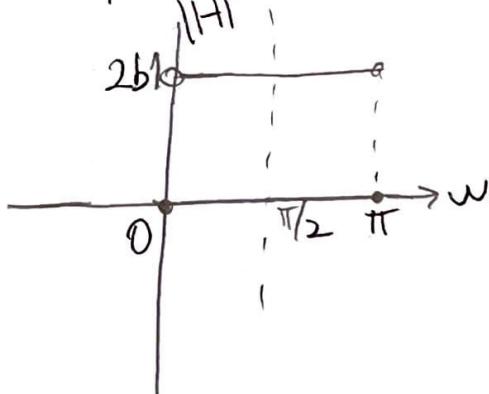
$$\Rightarrow x[n] = -5\delta[n] - 5\delta[n-1] - 5\delta[n-2] - 5\delta[n-3]$$

5

$$Q5) H(e^{j\omega}) = b_0 + b_1 e^{-j\omega} + b_2 e^{-j2\omega} + b_3 e^{-j3\omega}$$

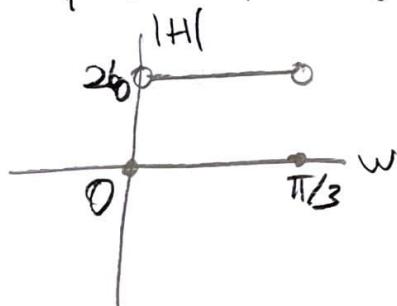
a) $b_0 = b_3 = 0 \Rightarrow H(e^{j\omega}) = b_1 e^{-j\omega} + b_2 e^{-j2\omega}$

$$|H(e^{j\omega})| = \sqrt{b_1^2 + b_2^2 - 2b_1 b_2 \cos(\omega)} \xrightarrow[b_1=b_2]{=} \sqrt{2b_1^2(1-\cos(\omega))}$$



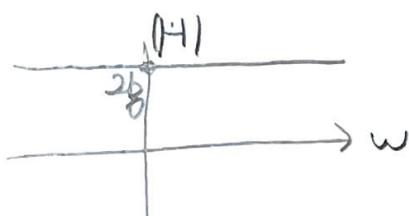
b) $b_1 = b_2 = 0 \Rightarrow H(e^{j\omega}) = b_0 + b_3 e^{-j3\omega}$

$$|H(e^{j\omega})| = \sqrt{b_0^2 + b_3^2 - 2b_0 b_3 \cos(3\omega)} = \sqrt{2b_0^2(1-\cos(3\omega))}$$



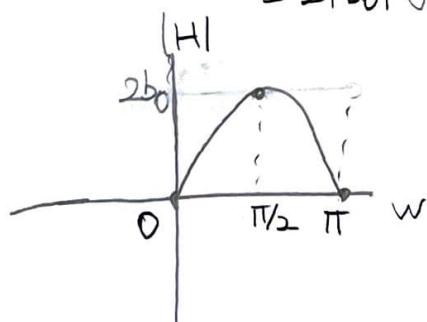
c) $b_0 = b_1 = b_2 = b_3 = 0 \Rightarrow$

$$|H(e^{j\omega})| = \sqrt{4b_0^2}$$



d) $b_0 = -b_1 = b_2 = -b_3$

$$|H(e^{j\omega})| = \sqrt{4b_0^2 - 4b_0^2 \cos(\omega)} = 2|b_0| \sqrt{1-\cos(\omega)}$$



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Q6)

- a) $t < 12$, system is stable, output is bounded
 $t = 2$, output amplitude = 1, system is stable

b) System present at $2 < t < 12$, right sided so system is causal

c) $x(t) = \delta(t-2) \Rightarrow y(t) = x(t) * h(t) \Rightarrow y(t) = \delta(t-2) * h(t)$

$$\Rightarrow \text{time shifting} \Rightarrow y(t) = h(t-2) = \begin{cases} e^{-0.1(t-2-2)} & 2 \leq t < 12 \\ 0 & \text{otherwise} \end{cases}$$

$$y(t) = \begin{cases} e^{-0.1(t-4)} & 2 \leq t < 12 \\ 0 & \text{o.w.} \end{cases}$$

Q7)

a) $h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{jw}) e^{jwn} dw = \frac{1}{2\pi} \left[\int_{-\frac{\pi}{2}-w_c}^{\frac{\pi}{2}+w_c} e^{jwn} dw + \int_{\frac{\pi}{2}-w_c}^{\frac{\pi}{2}+w_c} e^{jwn} dw \right]$

$$= \frac{\sin((w_c - \frac{\pi}{2})n) + \sin((w_c + \frac{\pi}{2})n)}{\pi n}$$

$$w_c = \pi/5$$

$$\hookrightarrow h[n] = \frac{-\sin(\frac{3\pi}{10}n) + \sin(\frac{7\pi}{10}n)}{\pi n}$$

As w_c increases, $h[n]$ is more concentrated about the origin

b) $w_c = \pi/4$

$$h[n] = \frac{-\sin(\frac{\pi}{4}n) + \sin(\frac{3\pi}{4}n)}{\pi n}$$

c) $w_c = \pi/3$

$$h[n] = \frac{-\sin(\frac{\pi}{6}n) + \sin(\frac{5\pi}{6}n)}{\pi n}$$

Q 8)

7

- a) Linear, because satisfies superposition
Time-variant, because it scales with $(-t)$
Stable, since bounded
Not causal, since it scales with $(-t)$
- b) Linear, because satisfies superposition
Time-variant since $y(t-t_0) = \cos 3(t-t_0) \neq \cos 3t x(t-t_0)$
Stable since bounded
Causal since $y(-t) = x(t)$
- c) Linear, integrals always are linear
Scaling, so not causal and time variant
Unstable, integral
- d) Linear, satisfying superposition
Time variant, dependent on time ($y(t-t_0)$)
Stable
- e) Non-linear, time invariant, causal, stable
 $\tilde{v}(x(t))$
- f) Linear, time variant, not causal, stable
(not causal)
- g) Differentiation always linear
time invariant, causal, stable
no future

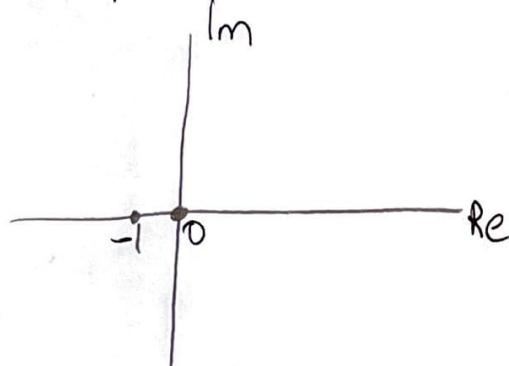
8

Q9)

a) $y[n] + y[n-5] = x[n] \Rightarrow y(z) + z^{-5}y(z) = X(z)$

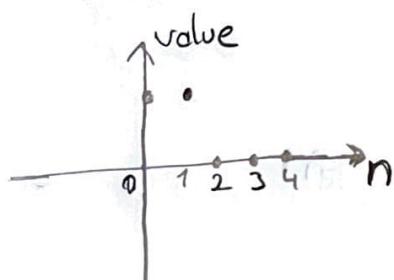
$$\Rightarrow y(z) = \frac{X(z)}{(1+z^{-5})} \Rightarrow H(z) = \frac{1}{(1+z^{-5})}$$

b) One pole at $z=0$,



c) $y(z) = H(z)X(z) \Rightarrow X(z) = 1+z^{-1}$

$$Y(z) = \frac{1+z^{-1}}{1+z^{-5}} \Rightarrow y[n] = \frac{1}{5}(\delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4])$$



d) Period is 5

(9)

Q10)

To sample, Nyquist theorem must be satisfied

$$\frac{1}{T_s} \geq 2f_0 \Rightarrow 2000\pi \cdot 2 = 4000\pi = f_0 \cdot 2$$

$$\Rightarrow T_s = \frac{1}{4000\pi} \approx 0.00008 \Rightarrow \text{recoverable: } t \in (0, 0.00008]$$