EEE 391 HW #1

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Q1)

a) $3e^{j\pi/3} + 4e^{-j\pi/6}$ (Polar form) $\Rightarrow \theta_1 = \frac{\pi}{3}$, $\theta_2 = \frac{-\pi}{6}$, $r_1 = 3$, $r_2 = 4$ $\Big\}$ $r_1\cos(\theta_1) + j r_1 \sin(\theta_1) + r_2\cos(\theta_2) + j r_2 \sin(\theta_2)$

$= 3.\cos(\frac{\pi}{3}) + j.3.\sin(\frac{\pi}{3}) + 4.\cos(\frac{-\pi}{6}) + j.4.\sin(\frac{-\pi}{6})$

$= \frac{3}{2} + \frac{3\sqrt{3}}{2}j + 2\sqrt{3} - 2j = \frac{3}{2} + 2\sqrt{3} + j(\frac{3\sqrt{3}}{2} - 2)$ //

b) $(1-j)^2$ (Cartesian Form) $= -2j \Rightarrow x = 0$, $y = -2$ $\Big\}$ $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\frac{y}{x}$

$\Rightarrow r = 2 \quad \theta = \frac{\pi}{2} \Rightarrow 2e^{j\frac{\pi}{2}}$

c) $(\sqrt{3} - 3j)^{10}$ (Cartesian Form) $= -124416 + 124416\sqrt{3}j \Rightarrow x = -124416$, $y = 124416\sqrt{3}$

$\Rightarrow r = 248832 \quad \theta = \frac{2\pi}{3} \Rightarrow 248832\, e^{j\frac{2\pi}{3}}$

d) $(\sqrt{2} + j\sqrt{2})/(1 + j\sqrt{3})$ (Cartesian form) $\Rightarrow x_1 = \sqrt{2}$, $y_1 = \sqrt{2}$, $x_2 = 1$, $y_2 = \sqrt{3}$ $\Big\}$ $r_1 = \sqrt{2+2} = 2$, $\theta_1 = \frac{\pi}{4}$, $r_2 = 2$, $\theta_2 = \frac{\pi}{3}$

$\Rightarrow \frac{2e^{j\frac{\pi}{4}}}{2e^{j\frac{\pi}{3}}} = e^{-j\frac{\pi}{12}}$

e) $Re\{je^{-j\pi/3}\}$ (Cartesian form) $= Re\{e^{j\frac{\pi}{2}} e^{-j\pi/3}\} = Re\{e^{j\frac{\pi}{6}}\} = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{\sqrt{3}}{2} \quad y = 0 \Rightarrow r = \frac{\sqrt{3}}{2} \quad \theta = \pi \Rightarrow \frac{\sqrt{3}}{2}e^{j\pi}$

f) $j(1-j)$ (Cartesian form) $= j + 1 \Rightarrow x = 1 \quad y = 1 \Rightarrow r = 1 \quad \theta = \frac{\pi}{4} \Rightarrow e^{j\frac{\pi}{4}}$

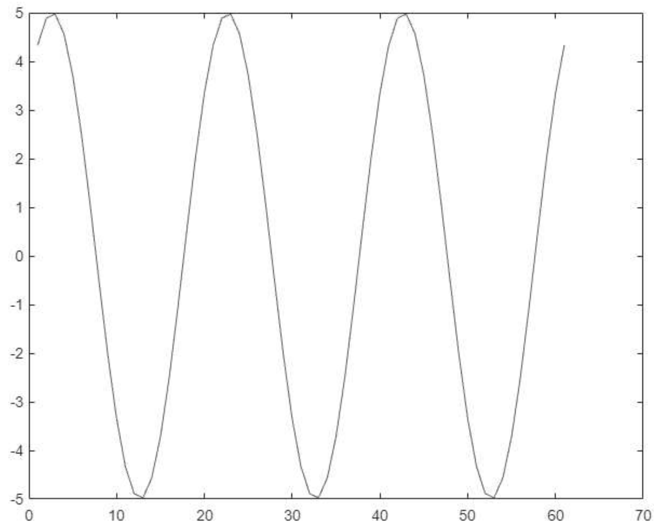
g) $(\sqrt{3} - j3)^{-1}$ (Cartesian form) $\Rightarrow x = \sqrt{3} \quad y = -3 \Rightarrow r = 2\sqrt{3} \quad \theta = \frac{2\pi}{3} \Rightarrow (2\sqrt{3}e^{j\frac{2\pi}{3}})^{-1}$

$= \frac{e^{-j\frac{2\pi}{3}}}{2\sqrt{3}} = \frac{\sqrt{3}e^{-j\frac{2\pi}{3}}}{6}$

2) In this question, I used Matlab top lot the graphs.

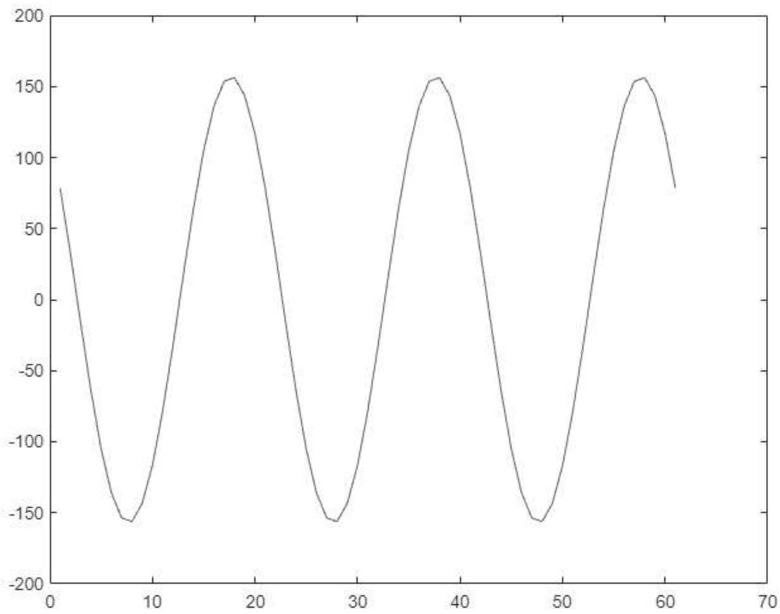
a) The code :

```
t=0:0.01:0.6;
si=5*sin(10*pi*t + (pi/3));

plot(si);
```



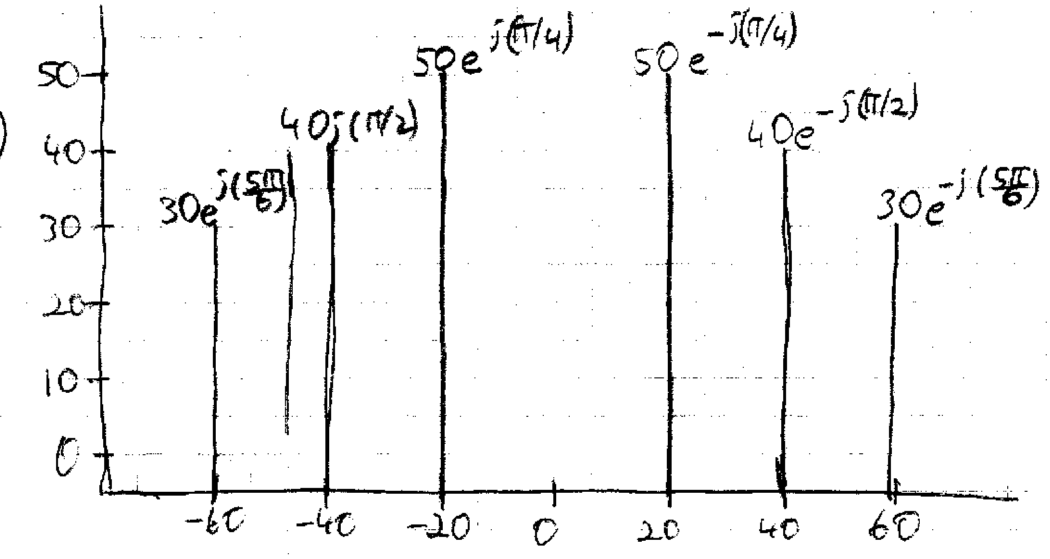
b) The code:

```
t=0:0.01:0.6;
q=50*pi*cos(10*pi*t + (pi/3));

plot(q);
```



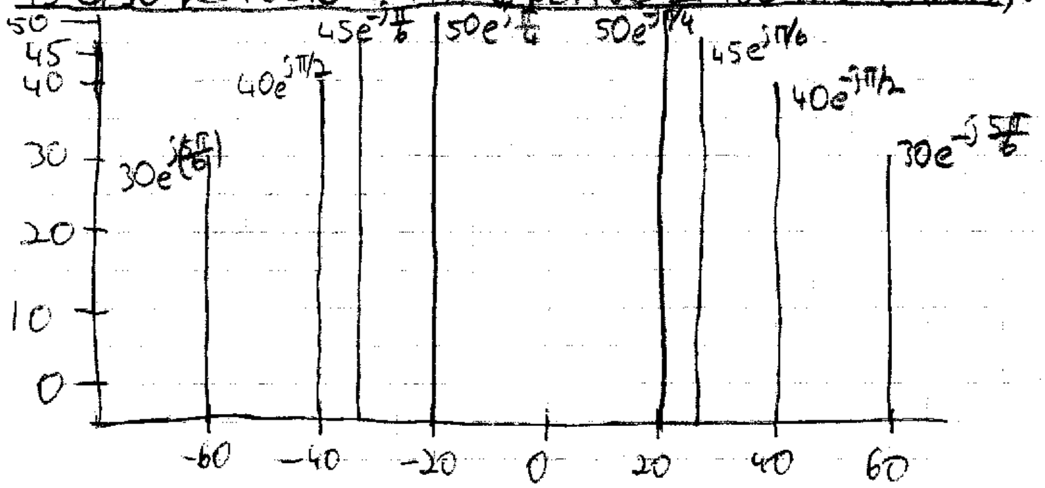
3)

a)



b) This signal is periodic with a period = 50ms (20 Hz).

c) y(t) is periodic with a period 1/30 s (30 Hz). And the signal is also periodic with a period = 100 ms (10 Hz).



d) the new signal is not periodic becouse of the frequency of $140/\pi$ Hz.

So the signal is not periodic

4)

a) $$a_0 = \frac{1}{2}\int_0^1 t\,dt + \frac{1}{2}\int_1^2 (2-t)\,dt = \frac{1}{2}$$

b) $$g(t) = \frac{dx(t)}{dt} = \begin{cases} 1, & 0 \le t \le 1 \\ -1, & 1 \le t \le 2 \end{cases}$$

$$b_0 = \frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_1^2 dt = 0 \quad \text{and} \quad b_k = \frac{1}{2}\int_0^1 g(t) e^{-j(2\pi/T)kt}\,dt - \frac{1}{2}\int_1^2 g(t) e^{-j(2\pi/T)kt}\,dt$$

$$= \frac{1}{j\pi k}\left[1 - e^{-j\pi k}\right]$$

c) $$y_n = jnw X_n \Rightarrow \frac{\left[1-e^{-j\pi k}\right]}{j\pi k} = jkwX_k$$

$$\Rightarrow X_k = \frac{\left[e^{-j\pi k} - 1\right]}{2\pi^2 f k^2}$$

5) $x[n] = 2.2\cos(0.3\pi n - \pi/3)$ $f_s = 6000$ samples/sec

$f = (0.3\pi\, 6000)/2\pi = 900$ Hz

$\boxed{X(t) = 2.2\cos(1800\pi t - \pi/3)}$

$X[n] = X_1[n] = X_2[n] = 2.2\cos(0.3\pi n - \pi/3)$

$X_1[n] = 2.2\cos(-0.3\pi n + 2\pi n + \pi/3)$

$F_1 = 5.1$ KHz

$\boxed{X_1(t) = 2.2\cos(10200\pi t + \pi/3)}$

$X_2[n] = 2.2\cos(2.3\pi n - \pi/3)$

$f_2 = 6.9$ KHz

$\boxed{X_2(t) = 2.2\cos(13800\pi t - \pi/3)}$

6)

a) $y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$y_1[n] = h[-1]\, x[n+1] + h[1]\, x[n-1] = 2x[n+1] + 2x[n-1]$

$y_1[n] = 2\delta[n+1] + 4\delta[n] - 2\delta[n-2] + 2\delta[n-1] + 4\delta[n-2] - 2\delta[n-4]$

$y_1[n] = -2\delta[n-4] + 2\delta[n-2] + 2\delta[n-1] + 4\delta[n] + 2\delta[n+1]$

b) $y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[(n+2)-k] \Rightarrow y_2[n] = y_1[n+2]$

so, $y_2[n] = -2\delta[n-2] + 2\delta[n] + 2\delta[n+1] + 4\delta[n+2] + 2\delta[n+3]$

c) $y_3[n] = x[n] * h[n+2] = \sum_{k=-\infty}^{\infty} h[k+2]\, x[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, x[n+2-k]$

$\Rightarrow y_3[n] = y_1[n+2]$

$y_3[n] = -2\delta[n-2] + 2\delta[n] + 2\delta[n+1] + 4\delta[n+2] + 2\delta[n+3]$