EEE391

Homework-2

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Q1)

a) $x[n]=\underbrace{\delta[n]}_{\text{impulse function}} \Rightarrow y[n]=\frac{1}{4}(\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3])$

$\Rightarrow y[n]=\boxed{h[n]=\frac{1}{4}(\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3])}$ //

b) $H(w)=\int h[n]e^{-jwn}dn$

$\Rightarrow$ using $h[n]$ from part a,

$H(w)=\int(\frac{1}{4}(\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3]))e^{-jwn}dn$

$\Rightarrow \boxed{H(w)=\frac{1}{4}(1+e^{-jw}+e^{-j2w}+e^{-j3w})}$ //

c) Substitute $x[n]$,

$$y[n]=\frac{1}{4}(5+4\cos(0.2\pi n)+3\cos(0.5\pi n)+5+4\cos(0.2\pi(n-1)) +3\cos(0.5\pi(n-1))+5+4\cos(0.2\pi(n-2))+3\cos(0.5\pi(n-2)) +5+4\cos(0.2\pi(n-3))+3\cos(0.5\pi(n-3)))$$

$$y[n]=\frac{1}{4}(4n+20+4\cos(0.2\pi n)+3\cos(0.5\pi n))$$

d) $y_1[n] = y[n]$ for all $n \geq 0$

Q2) $X(z) = \sum_{n=-\infty}^{\infty} x(n) . z^{-n}$ | $x(n) = \frac{1}{2\pi j} \int X(z) . z^{-1} dz$

a) From formula of $X(z)$,

$x(n) = \delta[n+5]$

$\Rightarrow \delta[n] = 1$ at $n=0 \Rightarrow \delta[n+5] = 1$ at $n=-5$

$\Rightarrow \sum_{n=-\infty}^{-6} \delta[n+5] z^{-n} + \sum_{n=-5} \delta[n+5] z^{-n} + \sum_{n=-4}^{\infty} \delta[n+5] z^{-n}$

$= \sum_{n=-5} \delta[n+5] z^{-n} \Rightarrow n=-5 \Rightarrow \delta[0] . z^{-(-5)} = z^5$

b) Same steps as in part a, $\delta[n-5] = 1$ at $n=5$

So, $X(z) = z^{-5}$

c) Same steps as in part a, $\delta[n-1] = 1$ at $n=1$

So, $X(z) = z^{-1}$

d) Inverse formula $x(n)$ and from $X(z)$,

$2 . z^0 + 4 . z^{-1} + 6 . z^{-2} + 4 . z^{-3} + 2 . z^{-4} = X(z)$

$\Rightarrow x(0) = 2, x(1) = 4, x(2) = 6, x(3) = 4, x(4) = 2$

$\Rightarrow$ inverse is $x(n) = \{-\infty \ldots, 2, 4, 6, 4, 2, \ldots, \infty\}$

e) Same as part d $\Rightarrow x(0) = 1, x(1) = -2, x(3) = 3, x(5) = -1, x(2) = x(4) = 0$

$x(n) = \{-\infty \ldots 1, -2, 0, 3, 0, -1, \ldots \infty\}$

Q3)

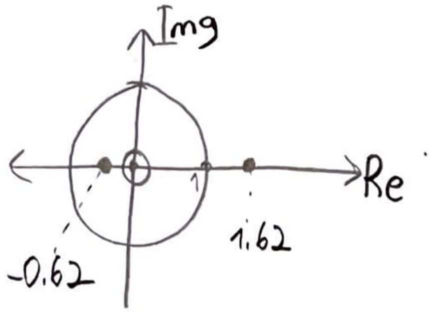
$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

taking z-transform

$$Y(z) = z^{-1}Y(z) + z^{-2}Y(z) + z^{-1}X(z)$$

$$z^{-1}X(z) = Y(z)\left(1 - z^{-1} - z^{-2}\right) \Rightarrow \frac{Y(z)}{X(z)} = \frac{z^{-1}}{(1-z^{-1}-z^{-2})} = \frac{z}{z^2-z-1} = H(z)$$

$$z^2 - z - 1 = 0 \Rightarrow \underbrace{\frac{1+\sqrt{5}}{2}}_{\sim 1.62}, \underbrace{\frac{1-\sqrt{5}}{2}}_{\sim -0.62}$$



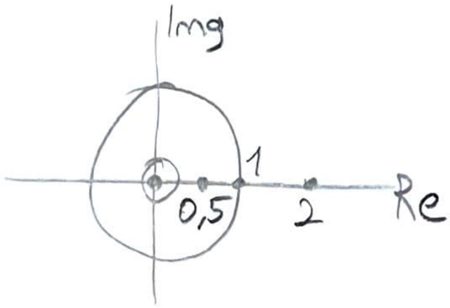
$$y[n-1] - 5/2\, y[n] + y[n+1] = x[n]$$

taking z transform

$$z^{-1}Y(z) - \frac{5}{2}Y(z) + zY(z) = X(z)$$

$$X(z) = Y(z)\left(z^{-1} - \frac{5}{2} + z\right) \Rightarrow \frac{Y(z)}{X(z)} = H(z) = \frac{2z}{2z^2-5z+2}$$

$$2z^2 - 5z + 2 = 0 \Rightarrow z = 0,5 \text{ or } z = 2$$



Q4)

a) $h[n] = \delta[n-2] \Rightarrow h[-n] = \delta[-n-2] = \delta[n+2]$

Conv. of $h[-n]$ and $y[n]$

$x[n] = h[-n] \times y[n] = \sum \delta[n+2] \times (u[n-3] - u[n-6])$

apply convolution formula

$x[n] = \sum u[n-m-3] \times \delta[m+2] - \sum u[n-m-6] \times \delta[m+2]$

$\xRightarrow[m=0]{} x[n] = \sum u[n-3] \times \underbrace{\delta[2]}_{=1} - \sum u[n-6] \times \underbrace{\delta[2]}_{=1}$

$\Rightarrow \boxed{x[n] = u[n-3] - u[n-6]}$

b) Same formula as part a, $h[-n] = \delta[n] + \delta[n+1]$

$\Rightarrow x[n] = \sum \delta[n-m] \times (\delta[m] - \delta[m-1]) \Rightarrow$ eval. for $m=0,1$

$\Rightarrow \boxed{x[n] = \delta[n] - \delta[n-1]}$

c) $h[-n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n+1] + \frac{1}{4}\delta[n+2] + \frac{1}{4}\delta[n+3]$

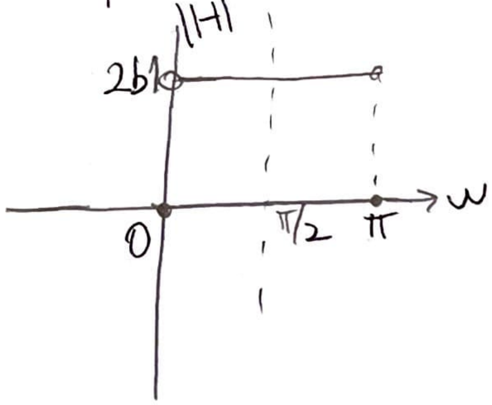
$\Rightarrow x[n] = h[-n] \times y[n] = \sum(-5\delta[n-m]) \times (\frac{1}{4}\delta[m] + \frac{1}{4}\delta[m+1]$
$+ \frac{1}{4}\delta[m+2] + \frac{1}{4}\delta[m+3]) \Rightarrow$ eval. for $m=0,1,2,3$

$\Rightarrow x[n] = -5\delta[n] - 5\delta[n-1] - 5\delta[n-2] - 5\delta[n-3]$

Q5) $H(e^{jw}) = b_0 + b_1 e^{-jw} + b_2 e^{-j2w} + b_3 e^{-j3w}$

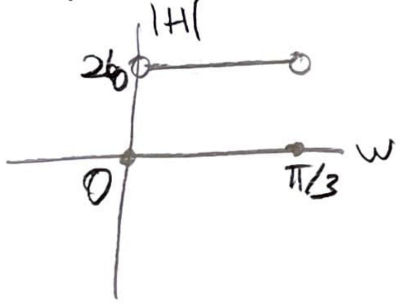
a) $b_0 = b_3 = 0 \Rightarrow H(e^{jw}) = b_1 e^{-jw} + b_2 e^{-j2w}$

$|H(e^{jw})| = \sqrt{b_1^2 + b_2^2 - 2 b_1 b_2 \cos(w)} \xRightarrow[b_1 = b_2]{} \sqrt{2b_1^2(1-\cos(w))}$



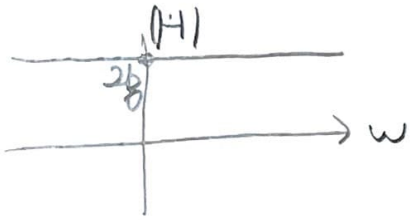
b) $b_1 = b_2 = 0 \Rightarrow H(e^{jw}) = b_0 + b_3 e^{-j3w}$

$|H(e^{jw})| = \sqrt{b_0^2 + b_3^2 - 2b_0 b_3 \cos(3w)} = \sqrt{2b_0^2(1-\cos(3w))}$



c) $b_0 = b_1 = b_2 = b_3 \Rightarrow$

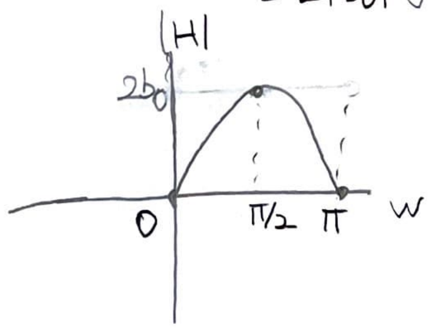
$|H(e^{jw})| = \sqrt{4 b_0^2}$



d) $b_0 = -b_1 = b_2 = -b_3$

$|H(e^{jw})| = \sqrt{4b_0^2 - 4b_0^2 \cos(w)}$

$= 2|b_0| \sqrt{1-\cos(w)}$



Q6)

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a) $t<12$, system is stable, output is bounded
$t=2$, output amplitude $=1$, system is stable

b) System present at $2<t<12$, right sided so system is casual

c) $x(t)=\delta(t-2) \Rightarrow y(t)=x(t)\times h(t) \Rightarrow y(t)=\delta(t-2)\times h(t)$

$\Rightarrow$ time shifting $\Rightarrow y(t)=h(t-2)=\begin{cases} e^{-0.1(t-2-2)} & 2\leq t<12 \\ 0 & \text{otherwise} \end{cases}$

$$\boxed{y(t)=\begin{cases} e^{-0.1(t-4)} & 2\leq t<12 \\ 0 & \text{o.w.} \end{cases}}$$

//

Q7)

a) $h[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{jw})e^{jwn}dw=\frac{1}{2\pi}\left[\int_{-\frac{\pi}{2}-w_c}^{-\pi/2+w_c} e^{jwn}dw+\int_{\frac{\pi}{2}-w_c}^{\frac{\pi}{2}+w_c} e^{jwn}dw\right]$

$=\dfrac{\sin((w_c-\frac{\pi}{2})n)+\sin((w_c+\frac{\pi}{2})n)}{\pi n}$

As w_c increases, $h[n]$ is more concentrated about the origin

$w_c=\pi/5$

$\hookrightarrow h[n]=\dfrac{-\sin(\frac{3\pi}{10}n)+\sin(\frac{7\pi}{10}n)}{\pi n}$

b) $w_c=\pi/4$

$h[n]=\dfrac{-\sin(\frac{\pi}{4}n)+\sin(\frac{3\pi}{4}n)}{\pi n}$

c) $w_c=\pi/3$

$h[n]=\dfrac{-\sin(\frac{\pi}{6})+\sin(\frac{5\pi}{6})}{\pi n}$

Q8)

a) Linear, becouse satisfies superposition

Time—variant, because it scales with $(-t)$

Stable, since bounded

Not causal, since it scales with $(-t)$

b) Linear, becouse satisfies superposition

Time-varrant since $y(t-t_0) = \cos 3(t-t_0)\, x(t-t_0) \neq \cos 3t\, x(t-t_0)$

Stable since bounded

Causal since $y(-t) = x(-t)$

c) Linear, integrals always are linear

Scaling, so not causal and time variant

Unstable, integral

d) Linear, satisfying superposition

Time varrant, depend on time $(y(t-t_0)$

Stable

e) Non-linear, time invariant, causal, stable

$u(x(t))$

f) Linear, time variant, not causal, stable

g) Differentiation always linear

time invariant, causal, stable

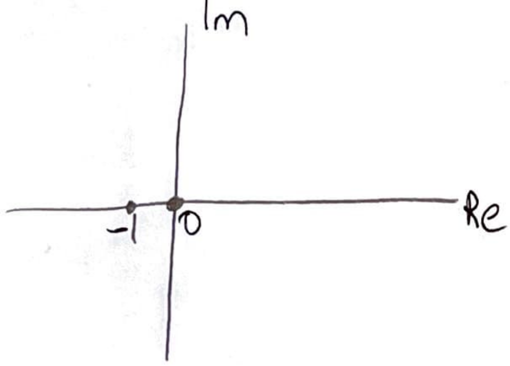
no future

Q9) 8

a) $y[n] + y[n-5] = x[n] \Rightarrow Y(z) + z^{-5} Y(z) = X(z)$

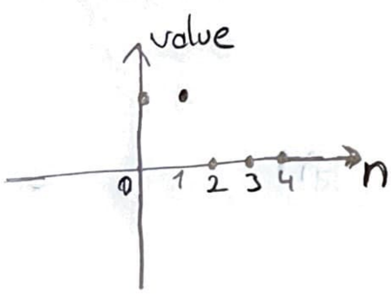
$$\Rightarrow Y(z) = \frac{X(z)}{(1+z^{-5})} \Rightarrow H(z) = \frac{1}{(1+z^{-5})}$$

b) One pole at $z=0$,



c) $Y(z) = H(z)X(z) \Rightarrow X(z) = 1 + z^{-1}$

$$Y(z) = \frac{1+z^{-1}}{1+z^{-5}} \Rightarrow y[n] = \frac{1}{5}(\delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4]$$



d) Period is 5

Q10)

To sample, Nyquist theorem must be satisfied

$$\frac{1}{T_s} \geqslant 2f_0 \Rightarrow 2000\pi \cdot 2 = 4000\pi = f_0 \cdot 2$$

$$\Rightarrow T_s = \frac{1}{4000\pi} \approx 0.00008 \Rightarrow \text{recoverable} : T \in (0, 0.00008]$$