EEE 391 – Matlab Assignment #1

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Part A)

A.1)

1)

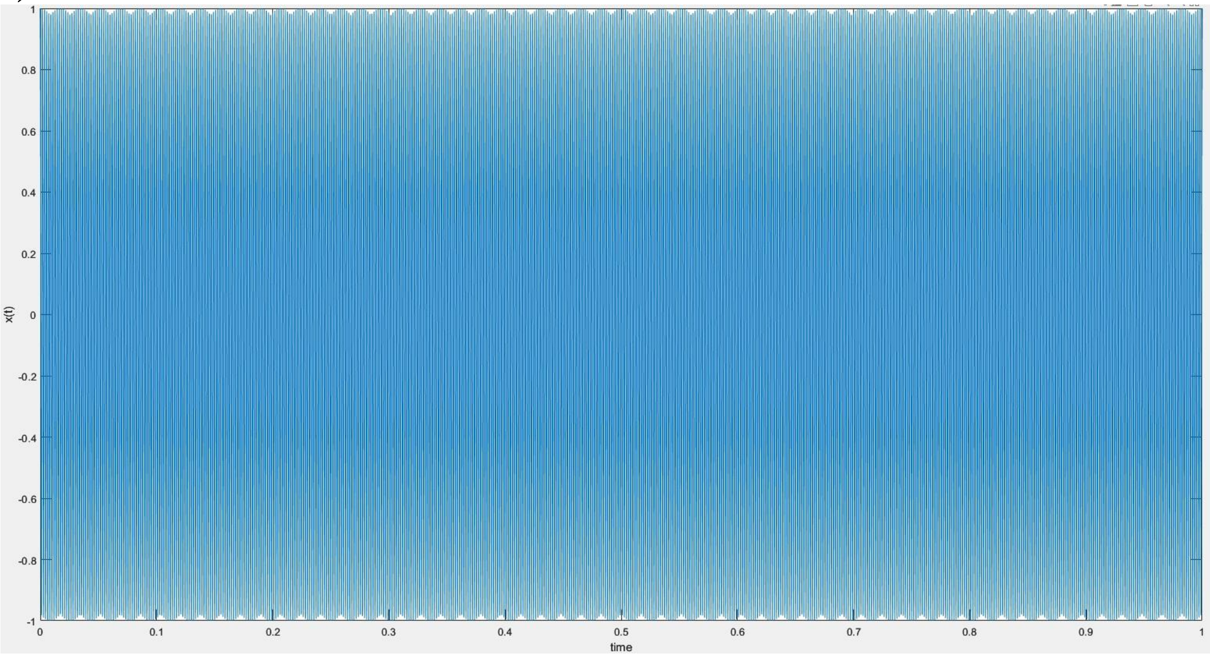


Figure 1. The plot of cos function with f=550 Hz

2) Since the frequency decreases, the sound is thicker than the first sound.

3) Since the frequency increses, the sound is thinner than the first and second sounds.

4)

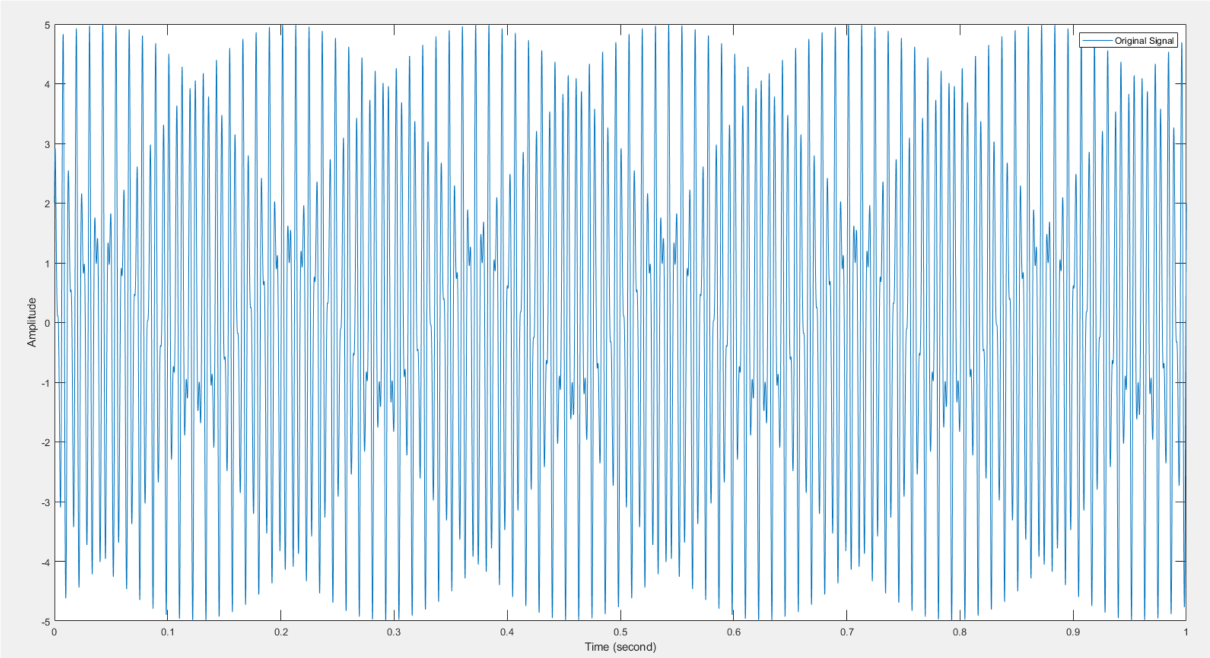


Figure 2. Original plot of y(t)

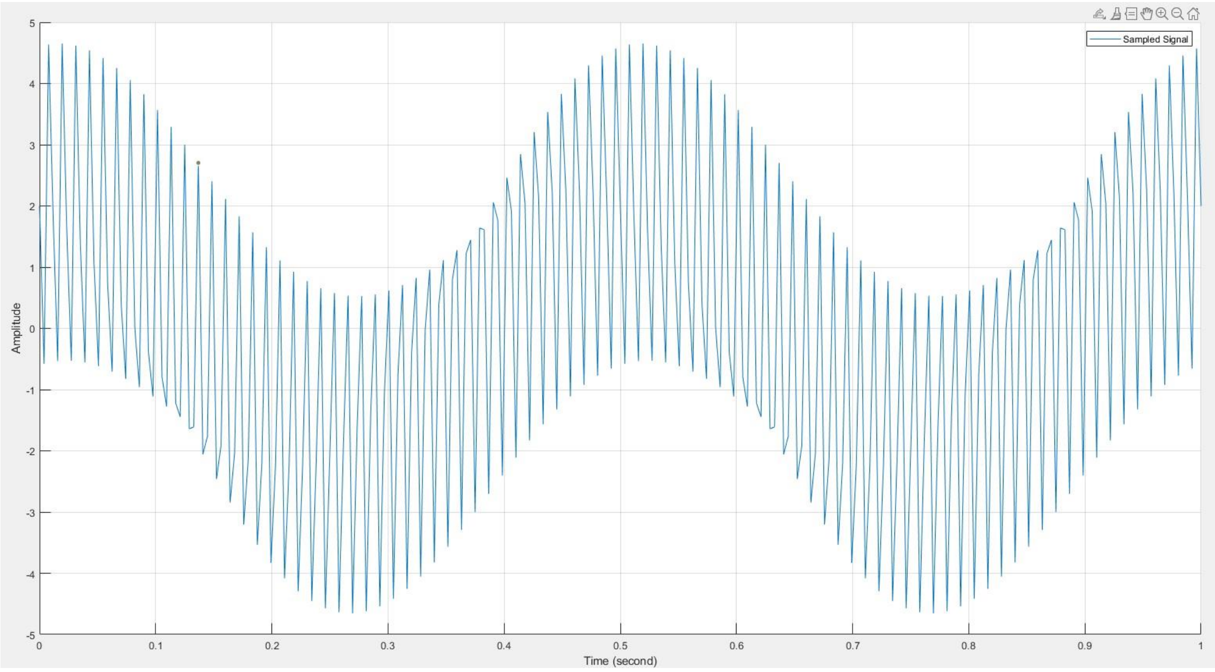


Figure 3. Sampled signal with the rate of fmax = 258 Hz

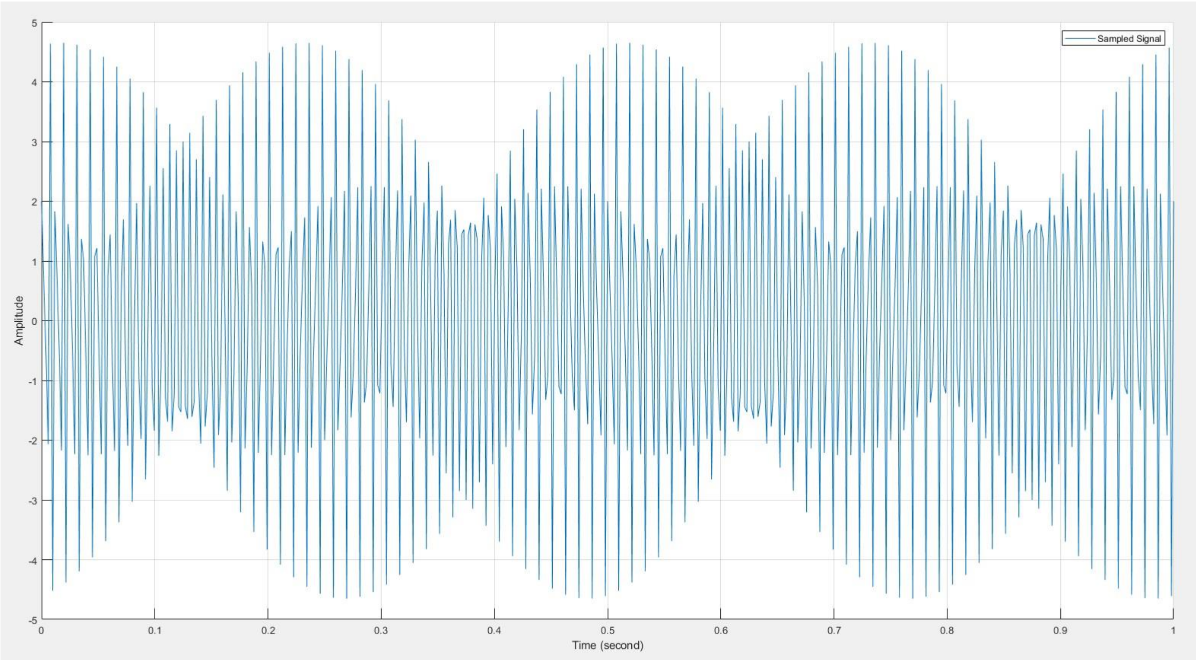


Figure 4. Sampled signal with the rate of 2fmax = 516 Hz

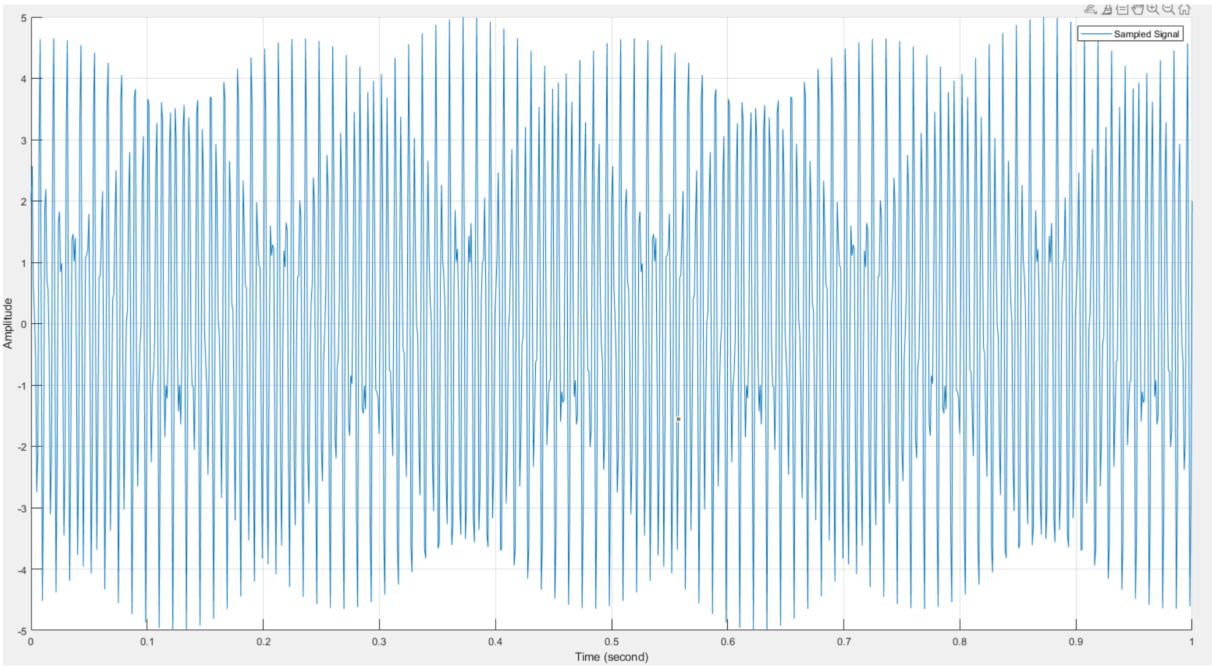


Figure 5. Sampled signal with the rate of 4fmax = 1032 Hz

5) (Code is in the Appendix)

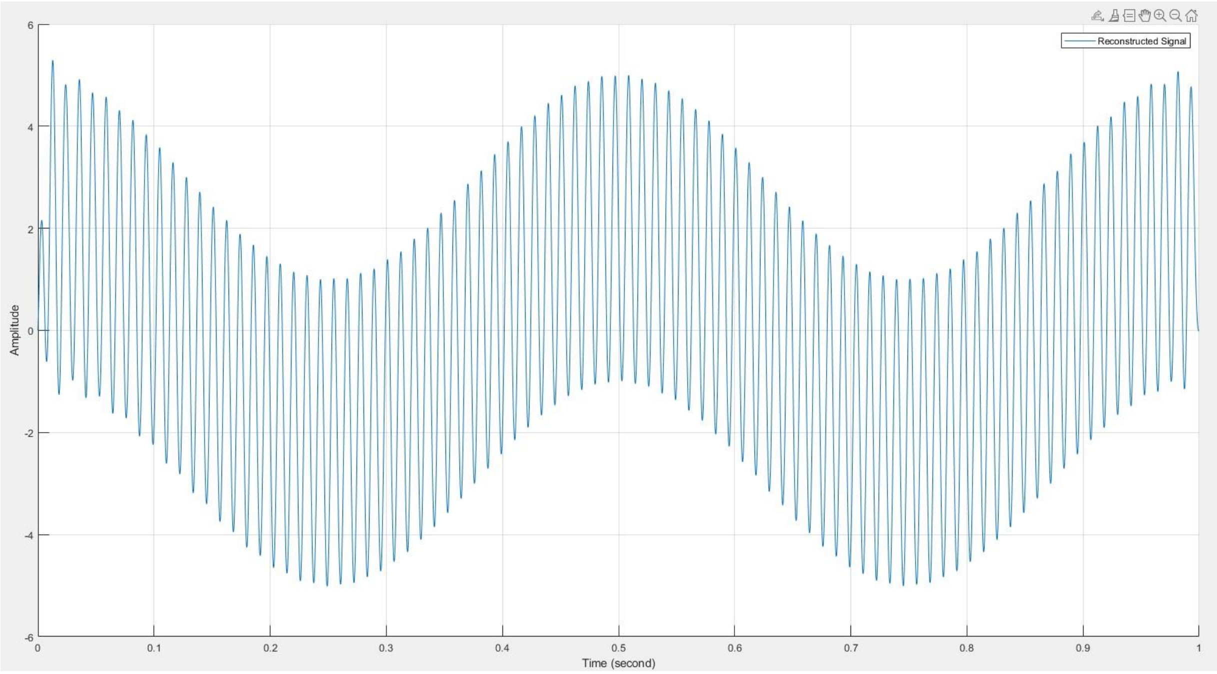


Figure 6. Reconstructed Signal from the sample in Figure 3 (f=258 Hz)

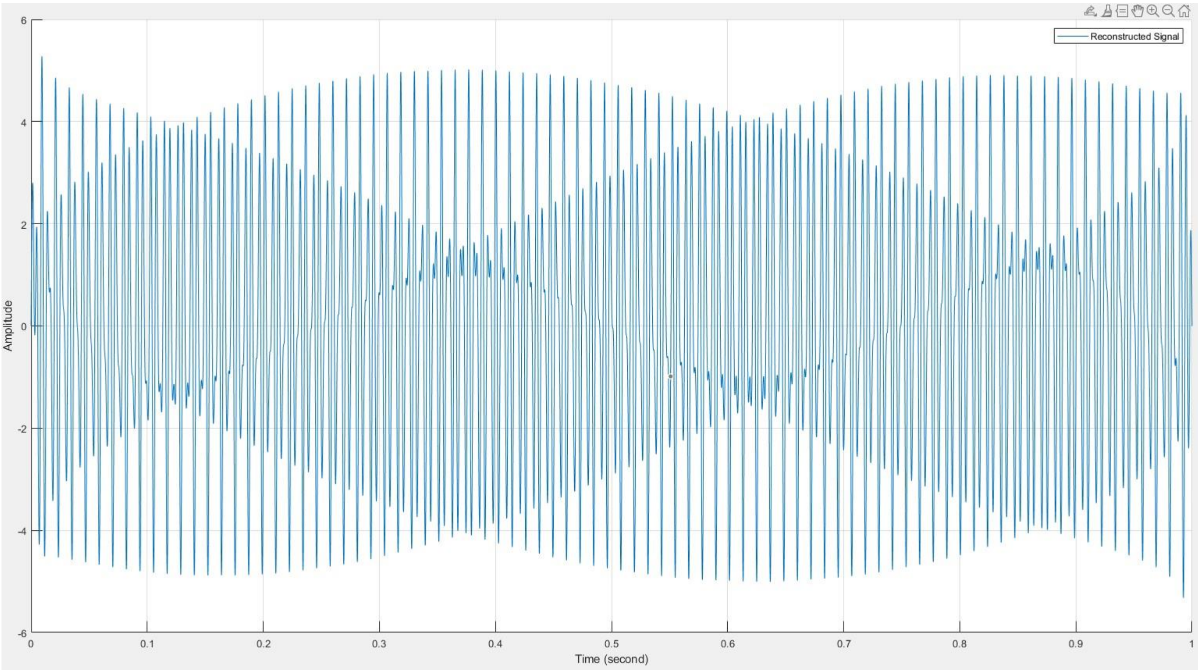


Figure 7. Reconstructed Signal from the sample in Figure 4 (f = 516 Hz)

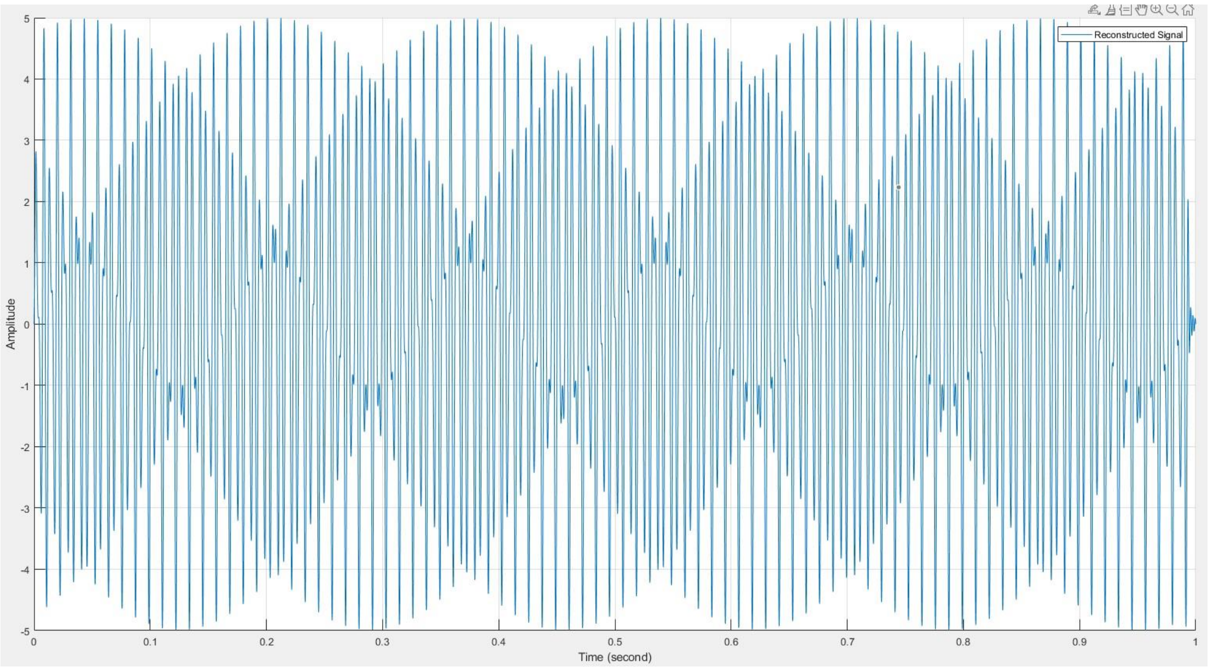
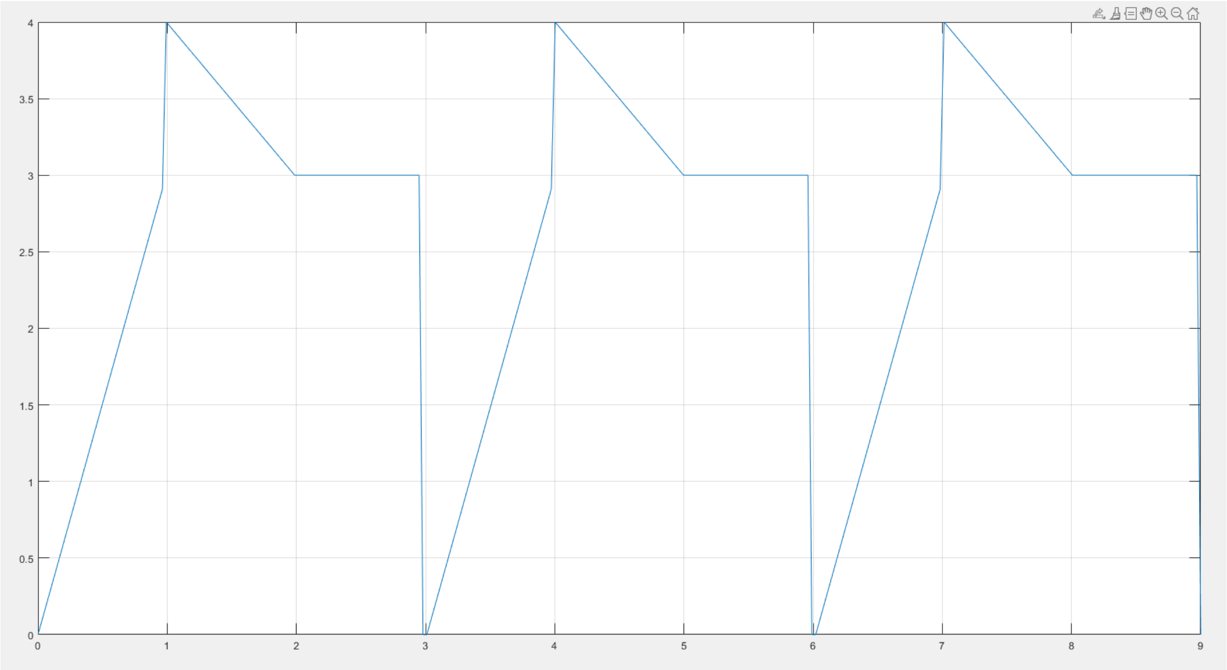


Figure 8. Reconstructed Signal from the sample in Figure 5 (f = 1032 Hz)

The diffences between the figure 6, 7 and 8 is when the frequency is bigger than the 2fmax (as in the Nyquist theorem) the reconstructed signals begin to similar to the original signal. Eventually, in the 4fmax case (Figure 8), the reconstructed signal is almost the same as the original signal (Figure 2).

A.2)

1)



Three Periods of the function x(t)

A.2.2)

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j\left(\frac{2\pi}{T_0}\right)kt}\, dt \Rightarrow T_0 = 3$$

$$a_0 = \frac{1}{3}\int_0^{T_0} x(t)\, e^{\overset{k=0}{0}}\, dt = \frac{1}{3}\left[\int_0^1 3t\,dt + \int_1^2 (5-t)\,dt + \int_2^3 3\,dt\right]$$

$$= \frac{1}{3}\left[\left[\frac{3t^2}{2}\right]_0^1 + \left.\frac{-(5-t)^2}{2}\right]_1^2 + \left.3t\right]_2^3\right] = \frac{1}{3}\left[\frac{3}{2} + \frac{7}{2} + 3\right] = \frac{8}{3}$$

$$a_0 = \frac{8}{3}$$

$$a_1 = \frac{1}{3}\int_0^3 x(t)\, e^{-j\left(\frac{2\pi}{3}\right)t}\, dt = \frac{1}{3}\left(\int_0^1 3t\, e^{-j\left(\frac{2\pi}{3}\right)t} dt + \int_1^2 (5-t)\, e^{-j\left(\frac{2\pi}{3}\right)t} dt\right.$$

$$\left. + \int_2^3 e^{-j\left(\frac{2\pi}{3}\right)t}\, dt\right) = \int_0^1 t\, e^{-j\left(\frac{2\pi}{3}\right)t} dt + \frac{1}{3}\int_1^2 (5-t)\, e^{-j\left(\frac{2\pi}{3}\right)t} dt + \int_2^3 e^{-j\left(\frac{2\pi}{3}\right)t} dt$$

$$\Rightarrow e^{-j\left(\frac{2\pi}{3}\right)t} = a^t \Rightarrow \int_0^1 t\, a^t\, dt + \frac{1}{3}\int_1^2 (5-t)\, a^t\, dt + \underbrace{\frac{e^{-j\left(\frac{2\pi}{3}\right)\cdot 3} - e^{-j\left(\frac{2\pi}{3}\right)\cdot 2}}{-j\left(\frac{2\pi}{3}\right)}}_{I}$$

$$= \left[t\int a^t - \int_0^1 \int a^t\, dt\right]_0^1 + \frac{1}{3}\left[(5-t)\int a^t + \int \int a^t\, dt\right]_1^2 + I \qquad \int_0^1 a^t = \frac{e^{-j\left(\frac{2\pi}{3}\right)} - 1}{-j\left(\frac{2\pi}{3}\right)}$$

$$= t \cdot \frac{e^{-j\left(\frac{2\pi}{3}\right)} - 1}{-j\left(\frac{2\pi}{3}\right)} - \left.\frac{e^{-j\left(\frac{2\pi}{3}\right)}}{-j\left(\frac{2\pi}{3}\right)}\right]_0^1 + \frac{1}{3}\left[\frac{(5-t)\, e^{-j\left(\frac{2\pi}{3}\right)}\left(e^{-j\left(\frac{2\pi}{3}\right)} - 1\right)}{-j\left(\frac{2\pi}{3}\right)} + \frac{e^{-j\left(\frac{2\pi}{3}\right)}\left(e^{-j\left(\frac{2\pi}{3}\right)} - 1\right)}{-j\left(\frac{2\pi}{3}\right)}\right]_1^2$$

$$+ I = \frac{e^{-j\left(\frac{2\pi}{3}\right)} - 1}{-j\left(\frac{2\pi}{3}\right)} + \frac{1}{3}\left(-\left[\frac{e^{-j\left(\frac{2\pi}{3}\right)}\left(e^{-j\left(\frac{2\pi}{3}\right)} - 1\right)}{-j\left(\frac{2\pi}{3}\right)}\right]\right) + \frac{e^{-j\left(\frac{2\pi}{3}\right)\cdot 3} - e^{-j\left(\frac{2\pi}{3}\right)\cdot 2}}{-j\left(\frac{2\pi}{3}\right)}$$

$$= \frac{e^{-j\left(\frac{2\pi}{3}\right)} \cdot \frac{4}{3}}{-j\left(\frac{2\pi}{3}\right)} + \frac{\frac{2}{3}\cdot e^{-j\left(\frac{2\pi}{3}\right)\cdot 2}}{-j\left(\frac{2\pi}{3}\right)} + \frac{e^{-j\left(\frac{2\pi}{3}\right)\cdot 3}}{-j\left(\frac{2\pi}{3}\right)}$$

A.2.3

$$\sum_{-\infty}^{\infty} |X_k|^2 = \frac{1}{T_0} \int_0^{T_0} (x(t))^2 dt = \frac{1}{3}\left[\int_0^1 9t^2 dt + \int_1^2 \underbrace{(5-t)^2}_{25-10t+t^2} dt + \int_2^3 9 dt \right]$$

$$= \int_0^1 3t^2 dt + \int_1^2 \frac{25}{3} dt + \int_1^2 \frac{-10t}{3} dt + \int_1^2 \frac{t^2}{3} dt + \int_2^3 3 dt$$

$$= t^3 \Big|_0^1 + \frac{25t}{3}\Big|_1^2 + \frac{-5t^2}{3}\Big|_1^2 + \frac{t^3}{9}\Big|_1^2 + 3t\Big|_2^3$$

$$= 1 + \frac{25}{3} + \frac{-15}{3} + \frac{7}{9} + 3 = 4 + \frac{37}{9} = \frac{73}{9}$$

$$0.95 \sim E_r = \frac{\sum_{-N}^{N} |X_k|^2}{\sum_{-\infty}^{\infty} |X_k|^2} \Rightarrow \sum_{-N}^{N} |X_k|^2 = 0.95 \times \frac{73}{9} \approx 7.70$$

$N = 8$

Appendix:

```
f0 = 8192;
t0 = [0:1/f0:1];
%This part is only for A.1.1
%{
x1 = cos(2 * pi * 550 * t0);
figure;
plot(t0,x1);
xlabel('time');
ylabel('x(t)');
%soundsc(x1);
%}

%21702587
f1 = 258;
f2 = 170;
y = 2 * cos(2 * pi * f1 * t0) + 3 * cos(2 * pi * f2 * t0 - pi/2);

figure;
plot(t0,y);
legend('Original Signal');
xlabel('Time (second)');
ylabel('Amplitude');

%Sampling values
s1 = 258;
s2 = 2 * s1;
s3 = 4 * s1;
t1 = 1/s1;
t2 = 1/s2;
t3 = 1/s3;

%Sampling Part
ratio = round(t1/(1/8192));
tn = t0(1:ratio:end);
y1 = y(1:ratio:end);

figure
hold on
grid on
plot(tn,y1);
legend('Sampled Signal');
xlabel('Time (second)');
ylabel('Amplitude');

%Reconstruction Part
sincc = zeros(length(t0), length(y1));
nind = 1;

for n = 1:length(y1)
    sincc(:,nind) = y1(nind)*sinc((t0- n* t1)/t1);
    nind = nind + 1;
end
xr = sum(sincc, 2);
figure;
hold on
grid on
plot(t0, xr);
legend('Reconstructed Signal');
xlabel('Time (second)');
```

```
ylabel('Amplitude');

%Sampling Part
ratio = round(t2/(1/8192));
tn = t0(1:ratio:end);
y2 = y(1:ratio:end);

figure
hold on
grid on
plot(tn,y2);
legend('Sampled Signal');
xlabel('Time (second)');
ylabel('Amplitude');

%Reconstruction Part
sincc = zeros(length(t0), length(y2));
nind = 1;

for n = 1:length(y2)
    sincc(:,nind) = y2(nind)*sinc((t0- n* t2)/t2);
    nind = nind + 1;
end
xr = sum(sincc, 2);
figure;
hold on
grid on
plot(t0, xr);
legend('Reconstructed Signal');
xlabel('Time (second)');
ylabel('Amplitude');

%Sampling Part
ratio = round(t3/(1/8192));
tn = t0(1:ratio:end);
y3 = y(1:ratio:end);

figure
hold on
grid on
plot(tn,y3);
legend('Sampled Signal');
xlabel('Time (second)');
ylabel('Amplitude');

%Reconstruction Part
sincc = zeros(length(t0), length(y3));
nind = 1;

for n = 1:length(y3)
    sincc(:,nind) = y3(nind)*sinc((t0- n* t3)/t3);
    nind = nind + 1;
end
xr = sum(sincc, 2);
figure;
hold on
grid on
plot(t0, xr);
legend('Reconstructed Signal');
xlabel('Time (second)');
ylabel('Amplitude');
```