Assignment - 6

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Question - 1) • Consider the closed-loop system whose open-loop transfer function is

$$G(s)\,H(s) = \frac{K e^{-2s}}{s}$$

Find the maximum value of K for which system is stable.

Solution =

$\rightarrow G(s)\,H(s) = \dfrac{K e^{-2s}}{s}$

$\rightarrow G(s)\,H(s)\big|_{s=j\omega} = \dfrac{K e^{-2j\omega}}{j\omega}$

$\rightarrow \angle G(j\omega)H(j\omega) = -90 + \angle \cos 2\omega - j\sin 2\omega$

$= -90 - 2\omega$

* At $2\omega = \frac{\pi}{2}$ rad/sec, the phase angle becomes $-180°$ for stability $|G(j\omega)H(j\omega)|_{\omega=\frac{\pi}{4}} < 1$

* $|G(j\omega)H(j\omega)| = \dfrac{K}{\omega} \rightarrow |e^{-2s}| = 1$

* At $\omega = \frac{\pi}{4}$, we need $\dfrac{K}{\omega}\Big|_{\omega=\frac{\pi}{4}} < 1$

* $\dfrac{K}{\frac{\pi}{4}} < 1 \longrightarrow \boxed{K < 0.785\text{, system is stable}}$

Question - 2) • A system with the open-loop transfer function

$$G(s)\,H(s) = \frac{K}{s^2(T_1 s + 1)}$$

is inherently unstable. This system can be stabilized by adding derivative control. Sketch the polar plots for the open-loop transfer function with and without derivative control.

Solution = $G(s)H(s) = \dfrac{K}{s^2(T_1 s+1)} \longrightarrow G(j\omega)H(j\omega) = \dfrac{K}{-\omega^2(1+j\omega T_1)}$

* It is Type-2 system

$M = |G(j\omega)H(j\omega)| = \dfrac{K}{\omega^2\sqrt{1+\omega^2 T_1^2}}$

$\phi = \angle G(j\omega)H(j\omega) = -180° - \tan^{-1}\omega T_1$

* To draw the polar plot, we calculate M and ϕ for different values of ω when it varies from 0 to ∞.

$\omega \to 0$ $\quad M = \infty \quad \phi = -180^\circ$

$\omega \to \infty$ $\quad M = 0 \quad \phi = -270^\circ$

* As we compare from general expression $G(j\omega) = \dfrac{b_0 (j\omega)^m + ---}{a_0 (j\omega)^n + ----}$

Here, $n = 3 \quad m = 0 \quad \underline{n - m = 3}$

* when a derivative controller is added to the system, the open loop transfer function becomes,

$$G(s)H(s).G_c(s) = \frac{K(T_2 s + 1)}{s^2 (T_1 s + 1)} \quad \text{with} \quad T_2 > T_1$$

* replacing s by $j\omega$ $\longrightarrow G(j\omega)\,H(j\omega) = \dfrac{K(1 + j\omega T_2)}{-\omega^2 (1 + j\omega T_1)}$

* It is Type-2 system $\rightarrow M = |G(j\omega)H(j\omega)| = \dfrac{K\sqrt{1+\omega^2 T_2^2}}{\omega^2 \sqrt{1+\omega^2 T_1^2}}$

$$\phi = \angle G(j\omega)H(j\omega) = -180 + \tan^{-1}\omega T_2 - \tan^{-1}\omega T_1$$

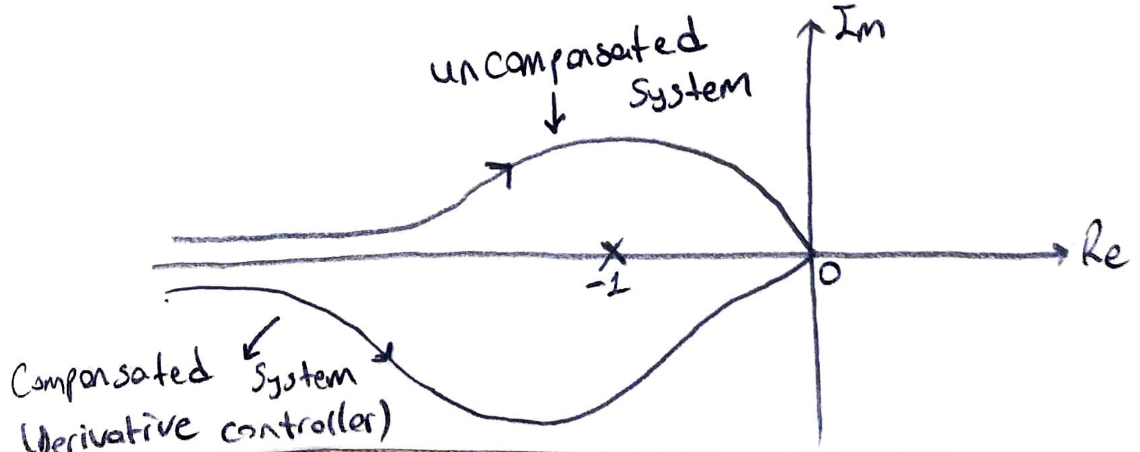
→ for this system when,

* $\omega \to 0 \quad M = \infty \quad \phi = -180^\circ$
* $\omega \to \infty \quad M = 0 \quad \phi = -180^\circ$

→ As we compare from general expression $G(j\omega) = \dfrac{b_0 (j\omega)^m + ---}{a_0 (j\omega)^n + ---}$

→ Here, $n = 3 \quad m = 1 \quad \underline{n - m = 2}$

* The polar plots for both systems are,



Question-3) Consider a unity feedback control system with the following open-loop transfer function:

$$G(s) = \frac{s^2+2s+1}{s^3+0.2s^2+s+1}$$

Draw a Nyquist plot of $G(s)$ and examine the stability of the closed-loop system.

Solution: $G(s) = \dfrac{s^2+2s+1}{s^3+0.2s^2+s+1}$, $H(s) = 1$ $\Rightarrow$ The open-loop system has two zeros and three poles

+ The poles are at $s = -0.72, 0.26 \mp j1.14$
+ The number of the poles in the right-half s plane $P = 2$.
+ The number of zeros of $1 + G(s)H(s)$ in the right-half s plane is $Z = 0$

$\longrightarrow N = Z - P = 0 - 2 = \underline{\underline{-2}}$

+ $1 + G(s)H(s) = 0 \longrightarrow 1 + \dfrac{s^2+2s+1}{s^3+0.2s^2+s+1} = 0 \rightarrow \boxed{s^3+1.2s^2+3s+2=0}$

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 1.2 & 2 \\ s^1 & 1.3 & \\ s^0 & 2 & \end{array}$$

There is no sign changes in the Routh Table. Therefore, there are no poles in the right half of the s-plane. Hence, the system is stable

$\rightarrow$ Determine $GH(j\omega)$. $\rightarrow GH(j\omega) = \dfrac{-\omega^2+2\omega+1}{-j\omega^3-0.2\omega^2+j\omega+1} = \dfrac{(1-\omega^2)+j2\omega}{(1-0.2\omega^2)+j(\omega-\omega^3)}$

+ $|GH(j\omega)| = \sqrt{\dfrac{(1-\omega^2)^2+4\omega^2}{(1-0.2\omega^2)^2+(\omega-\omega^3)^2}}$, $\angle GH(j\omega) = \tan^{-1}\left(\dfrac{2\omega}{(1-\omega^2)}\right) - \tan^{-1}\left(\dfrac{\omega-\omega^3}{1-0.2\omega^2}\right)$

$\rightarrow$ Determine the magnitude and phase of $GH(j\omega)$ from 0 to infinity.

ω	0	0.1	0.76	1	2	10	20	100	∞
$\|GH(j\omega)\|$	1	1.007	1.677	2.5	0.833	0.102	0.05	0.01	0
$\angle GH(j\omega)$	0	0.1	0.95	∞	0.61	-1.75	-1.60	-1.59	0

Question - 4) Consider the unity feedback control system whose open-loop transfer function is

$$G(s) = \frac{as+1}{s^2}$$

Determine the value of a so that the phase margin is $45°$.

Solution = Given $G(s) = \frac{as+1}{s^2}$, $G(j\omega) = \frac{j\omega a+1}{(j\omega)^2}$

$G(j\omega) = \frac{1+j\omega a}{-\omega^2}$, $|G(j\omega)| = M = \frac{\sqrt{1+\omega^2 a^2}}{\omega^2}$, $\angle G(j\omega) = \phi = \tan^{-1}\omega a - 180°$

→ The phase margin is calculated corresponding to the gain crossover frequency ω_1 at which $M=1$ or $0\,dB$.

→ At $\omega = \omega_1$, $M = \frac{\sqrt{1+\omega_1^2 a^2}}{\omega_1^2} = 1$

→ $\boxed{\sqrt{1+\omega_1^2 a^2} = \omega_1^2}$ (Phase margin of system is $45°$)

* $P.M = 180° + \phi_g = 45$, $\phi_g = -135°$

* The phase angle of the system at $\omega = \omega_1$ is,

$\phi_g = \tan^{-1}\omega_1 a - 180$, $\boxed{\omega_1 a = 1}$ → $\omega_1 = 1.189$ rad/sec

Then $\boxed{a = 0.8408}$

Question - 5) Consider a unity-feedback control system whose open-loop transfer function is $G(s) = \frac{K}{s(s^2+s+0.5)}$

Determine the value of the gain K such that the resonant peak magnitude in the frequency response is $2\,dB$, or $M_r = 2\,dB$.

Solution = $G(j\omega) = \frac{K}{j\omega((j\omega)^2+j\omega+0.5)} = \frac{-jK}{\omega(0.5-\omega^2)+j\omega} = \frac{-jK[(0.5-\omega^2)-j\omega]}{\omega[(0.5-\omega^2)+\omega^2]}$

* At resonant frequency imaginary term is zero

* Imaginary Term $= \frac{-jK(0.5-\omega^2)}{\omega[(0.5-\omega^2)^2+\omega^2]}$ → $K(0.5-\omega^2) = 0$, $\boxed{\omega_r = 0.707}$

* $|G(j\omega)| = \left|\frac{-jK}{\omega(0.5-\omega^2)+j\omega}\right| = \frac{K}{\sqrt{(0.5-\omega^2)+\omega^2}}$

* $|G(j\omega)|_{dB} = 20\log\left[\dfrac{K}{\omega\sqrt{(0.5-\omega^2)^2+\omega^2}}\right]$

$= 20\log K - 20\log\omega - 10\log\left[(0.5-\omega^2)^2+\omega^2\right]$

* Given magnitude at rezonance frequency is 2 dB

$\Rightarrow 20\log K - 20\log\omega_r - 10\log\left[(0.5-\omega_r^2)^2+\omega_r^2\right] = 2$

$\Rightarrow 20\log K - 20\log(0.707) - 10\log\left[(0.5-0.707^2)^2+0.707^2\right] = 2$

$\boxed{\Rightarrow K = 0.629}$