EEE302 CONTROL SYSTEMS
LECTURE ASSIGNMENT

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5 QUESTION HOMEWORK:

Assignment

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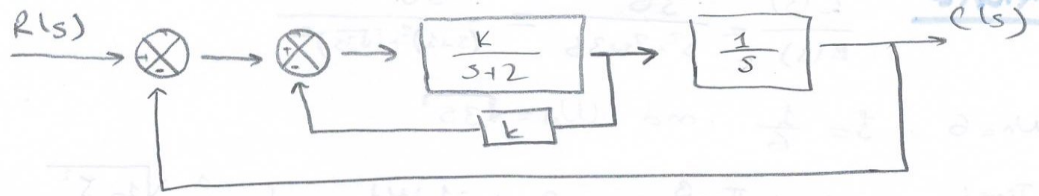
Question-1: Ogata 5th edition, B-5.8

Referring to the system shown in figure, determine the values of K and k such that the system has a damping ratio ζ of 0.7 and undamped natural frequency ω_n of 4 rad/sec.



Solution-

$$G_1 = \frac{\frac{K}{s+2}}{1 + \frac{K.k}{s+2}} = \frac{K}{s+2+K.k}$$

$$G_2 = \frac{K}{s+2+K.k} \cdot \frac{1}{s} = \frac{K}{s^2+2s+Kks}$$

$$G_3 = \frac{\frac{K}{s^2+2s+Kks}}{1+\frac{K}{s^2+2s+Kks}} = \boxed{\frac{K}{s^2+2s+Kks+K} = \frac{C(s)}{R(s)}}$$

* $K = \omega_n^2 = 4^2 = \underline{\underline{16}}$, $2\zeta\omega_n = 2 + K.k$

* $2 \times 0.7 \times 4 = 2 + k.K = 2 + 16k$

* $k = \underline{\underline{0.225}}$

Question-2: Ogata 5th edition B-5.12

Obtain analytically the rise time, peak time, max. overshoot, and settling time in the unit-step response of a closed-loop system given by

$$\boxed{\frac{C(s)}{R(s)} = \frac{36}{s^2+2s+36}}$$

Solution:

$$\frac{C(s)}{R(s)} = \frac{36}{s^2+2s+36} = \frac{36}{(s+1)^2+(\sqrt{35})^2}$$

$$\rightarrow \omega_n = 6 \,, \quad \zeta = \frac{1}{6} \,, \text{ and } \omega_d = \sqrt{35}$$

Rise Time:

$$t_r = \frac{\pi - \beta}{\omega_d} \rightarrow \beta = \tan^{-1}\frac{\omega_d}{\zeta\omega_n} = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\rightarrow \beta = \tan^{-1}\frac{\sqrt{\frac{35}{36}}}{\frac{1}{6}} = 1.4034 \text{ rad}$$

$$\rightarrow t_r = \frac{3.1416-1.4034}{\sqrt{35}} = \underline{\underline{0.2938 \text{ sec}}}$$

Peak time:

$$t_p = \frac{\pi}{\omega_d} = \frac{3.1416}{\sqrt{35}} = \underline{\underline{0.5310 \text{ sec}}}$$

Max. overshoot:

$$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi} = e^{-\frac{\pi}{\sqrt{35}}} = e^{-0.5310}$$

$$\underline{\underline{M_p = 0.5880}}$$

Settling time:

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{\frac{1}{6}\times 6} = \underline{\underline{4 \text{ sec}}}$$

Question-3: Ogata 5th edition, B-5-19

Consider the differential equation system given by

$$\boxed{\ddot{y} + 3\dot{y} + 2 = 0; \quad y(0) = 0.1, \quad \dot{y}(0) = 0.05}$$

Obtain the response $y(t)$, subject to the given initial condition.

Solution:

We obtain,

$$\boxed{s^2 Y(s) - sy(0) - \dot{y}(0) + 3[sY(s) - y(0)] + 2Y(s) = 0}$$

By substituting the given initial condition, we get

$$\boxed{(s^2+3s+2)Y(s) = 0.1s + 0.35}$$

$$\rightarrow Y(s) = \frac{0.1s+0.35}{s^2+3s+2} = \frac{0.1s+0.35}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$\rightarrow A = 0.25, \ B = -0.15 \longrightarrow Y(s) = \frac{0.25}{s+1} - \frac{0.15}{s+2}$$

$$\rightarrow \boxed{L^{-1}\{Y(s)\} = y(t) = 0.25e^{-t} - 0.15e^{-2t}}$$

Question-4: Ogata 5th Edition, B-5.20

Determine the range of K for stability of a unity feedback control system whose open-loop transfer function is.

$$\boxed{G(s) = \frac{K}{s(s+1)(s+2)}}$$

Solution:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+1)(s+2)}}{1+\frac{K}{s(s+1)(s+2)}} = \frac{K}{s(s+1)(s+2)+K}$$

* Characteristic equation is $\Rightarrow s^3 + 3s^2 + 2s + K = 0$
* The Routh array becomes;

$$\begin{array}{lll} s^3 & 1 & 2 \\ s^2 & 3 & K \\ s^1 & \frac{6-K}{3} & \\ s^0 & K & \end{array}$$

For stability we require $6 > K$ and $K > 0$,

$$\boxed{6 > K > 0}$$

Question-5: Ogata 5th edition, B-5.21

Consider the following characteristic equation:

$$\boxed{s^4 + 2s^3 + (4+K)s^2 + 9s + 25 = 0}$$

Using the Routh stability criterion, determine the range of K for stability.

Solution =

$$\begin{array}{llll} s^4 & 1 & 4+K & 25 \\ s^3 & 2 & 9 & 0 \\ s^2 & \frac{2K-1}{2} & 25 & \\ s^1 & \frac{18K-109}{2K-1} & 0 & \\ s^0 & 25 & & \end{array}$$

* for stability, we require

$$\frac{2K-1}{2} > 0 \quad , \quad \frac{18K-109}{2K-1} > 0$$

$$\downarrow \qquad\qquad \downarrow$$

$$K > 0.5 \quad , \quad 18K > 109$$

$$\boxed{K > \frac{109}{18} = 6.056}$$

MATLAB HOMEWORK:

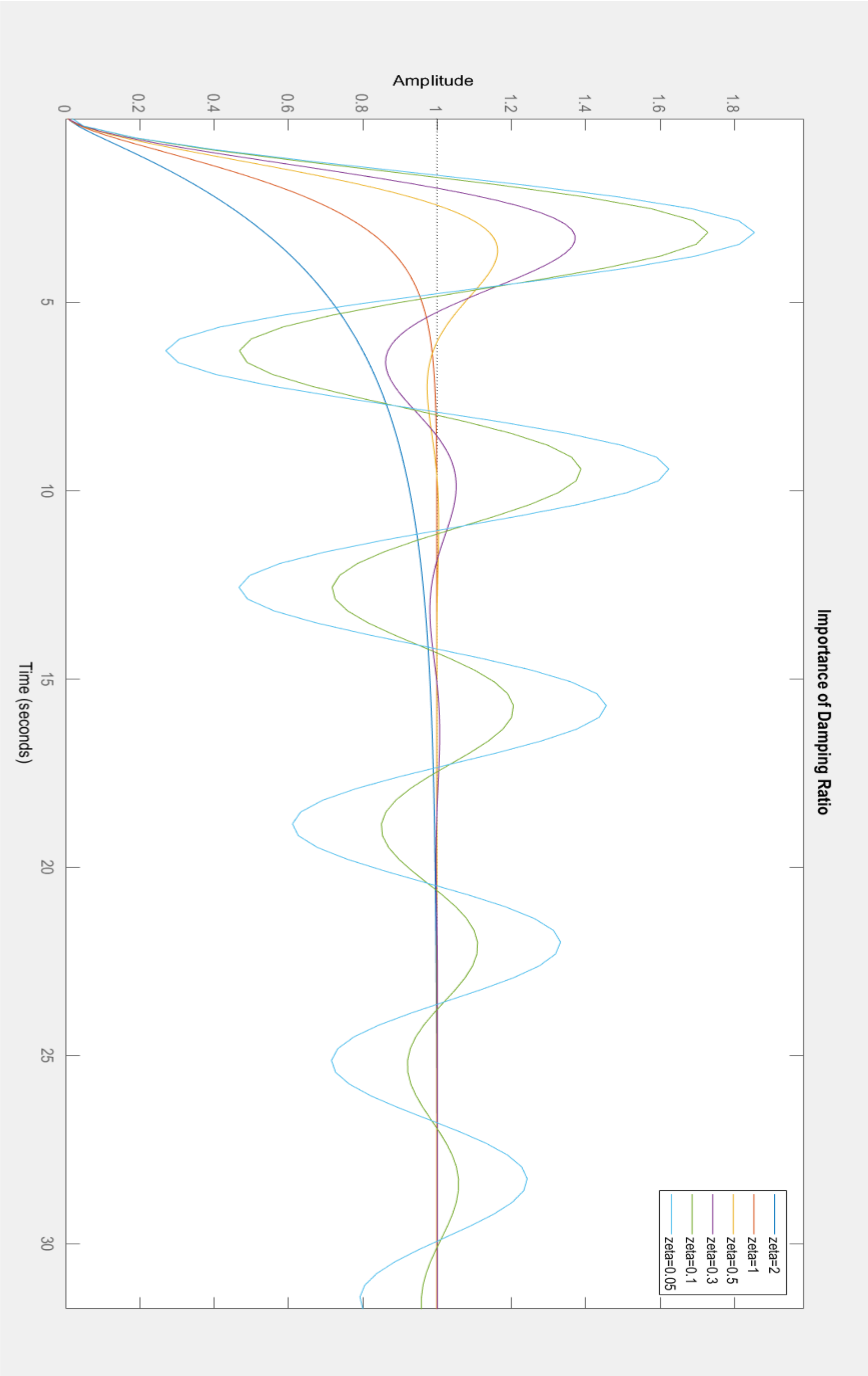
OBSERVING DAMPING RATIO VARIATION

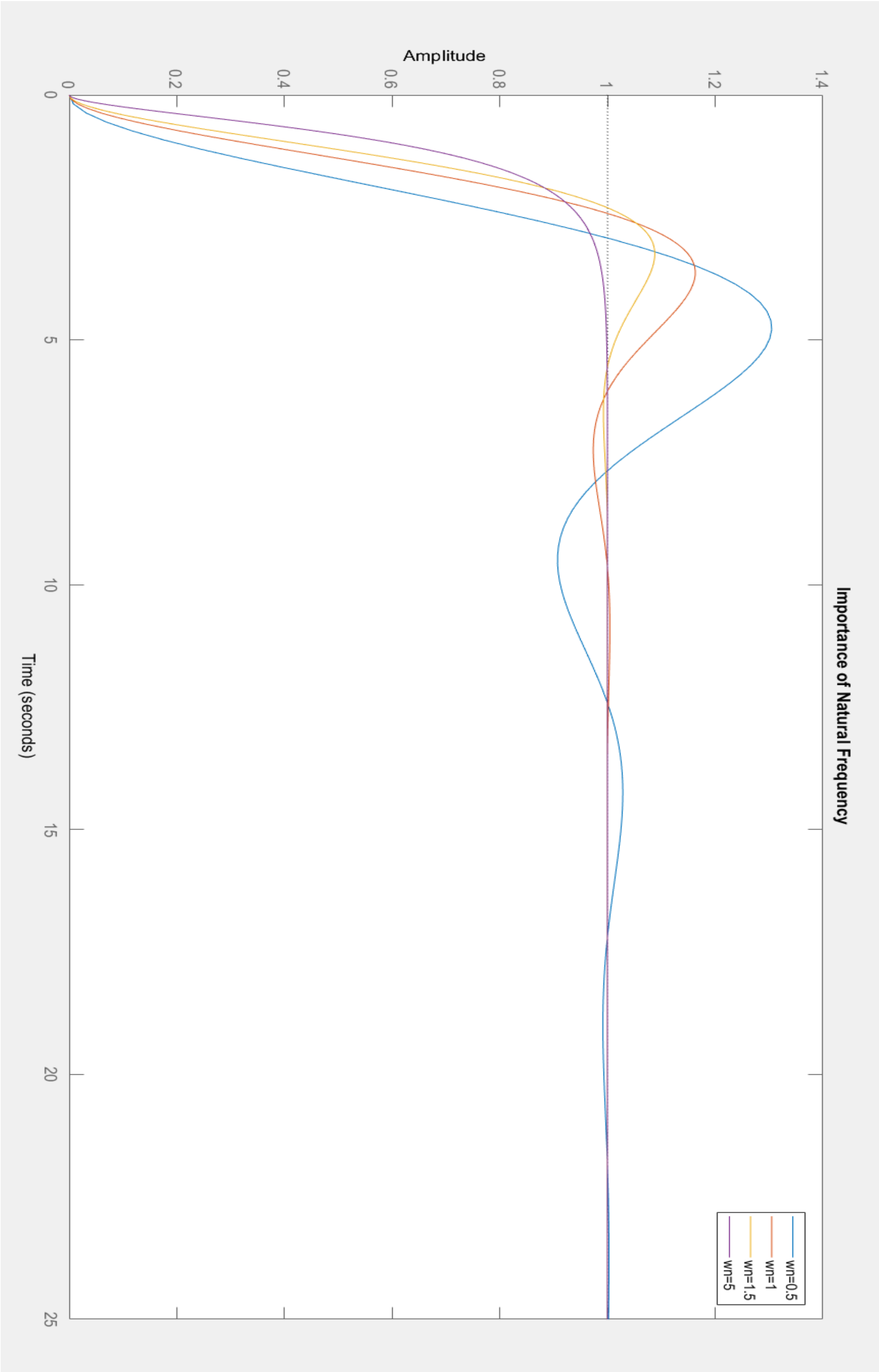
CODES:

```
% Observing Damping Ratio Variation

zeta=2;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
hold on
zeta=1;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
hold on
zeta=0.5;
wn=1;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
hold on
zeta=0.3;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
hold on
zeta=0.1;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
hold on
zeta=0.05;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
legend('zeta=2','zeta=1','zeta=0.5','zeta=0.3','zeta=0.1','zeta=0.05')
title('Importance of Damping Ratio')
```

```
% Observing Natural Frequency Variation

zeta=0.5;
wn=0.5;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
hold on
zeta=0.5;
wn=1;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
hold on
zeta=0.5;
wn=1.5;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
hold on
zeta=0.5;
wn=5;
y=tf([wn],[1 2*zeta*wn wn]);
step(y);
legend('wn=0.5','wn=1','wn=1.5','wn=5')
title('Importance of Natural Frequency')
```

OUTPUTS:


Importance of Damping Ratio
Amplitude
Time (seconds)
zeta=2
zeta=1
zeta=0.5
zeta=0.3
zeta=0.1
zeta=0.05


Importance of Natural Frequency
Amplitude
Time (seconds)
wn=0.5
wn=1
wn=1.5
wn=5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
5
10
15
20
25