Assignment-5

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Question-1: Consider the unity-feedback system with the open-transfer function: $\longrightarrow$ $G(s) = \frac{10}{s+1}$

* Obtain the steady-state output of the system when it is subjected to each of the following inputs:

a) $r(t) = \sin(t+30^\circ)$

b) $r(t) = 2\cos(2t-45^\circ)$

c) $r(t) = \sin(t+30^\circ) - 2\cos(2t-45^\circ)$

Solution =

→ The output to the input relation is;

$$\left.\frac{Y(s)}{R(s)}\right|_{s=j\omega} = \frac{10}{j\omega+1}$$

→ the magnitude of the open-loop transfer function is,

$$|G(j\omega)| = \frac{10}{\sqrt{1+\omega^2}},$$

* The phase angle of the open-loop transfer function,

$$\rightarrow \angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{1}\right)$$

a) $r(t) = \sin(t+30^\circ) \Rightarrow r(t) = M\sin(\omega t+\phi)$

→ $\omega = 1$ rad/s

* $y_{ss}(t) = |G(j\omega)|\sin(t+30^\circ + \angle G(j\omega))|_{\omega=1}$

$$= \frac{10}{\sqrt{1+\omega^2}}\sin\left(t+30^\circ - \tan^{-1}(\omega)\right)\Big|_{\omega=1}$$

$$\boxed{= 7.07\sin(t-15^\circ)}$$

b) $r(t) = 2\cos(2t-45^\circ) = M\cos(\omega t+\phi) \longrightarrow \omega = 2$ rad/s

* $y_{ss}(t) = |G(j\omega)|(2)\cos(2t-45^\circ + \angle G(j\omega))|_{\omega=2}$

$$= \frac{10}{\sqrt{1+\omega^2}}(2)\cos\left(2t-45^\circ - \tan^{-1}\omega\right)\Big|_{\omega=2} = \boxed{8.94\cos(2t-108.43^\circ)}$$

c) $r(t) = \sin(t+30^\circ) - 2\cos(2t-45^\circ)$

$$\rightarrow y_{ss} = \left[|G(j\omega)| \sin(t+30^\circ + \angle G(j\omega)) \right]_{\omega=1} - \left[|G(j\omega)|(2)\cos(2t-45^\circ + \angle G(j\omega)) \right]_{\omega=2}$$

$$= \frac{10}{\sqrt{1+1^2}} \sin\left(t+30^\circ - \tan^{-1}\left(\frac{1}{1}\right)\right) - \frac{20}{\sqrt{1+2^2}} \cos\left(2t-45^\circ - \tan^{-1}\left(\frac{2}{1}\right)\right)$$

$$\rightarrow \boxed{y_{ss}(t) = 7.07\sin(t-15)^\circ - 8.94\cos(2t-108.43^\circ)}$$

Question-2 Consider the system whose closed-loop transfer function is

$$\boxed{\frac{C(s)}{R(s)} = \frac{K(T_2 s+1)}{T_1 s+1}}$$

- Obtain the steady-state output of the system when it is subjected to the input $r(t) = R\sin\omega t$.

Solution = $$\frac{C(s)}{R(s)} = \frac{K(j\omega T_2 + 1)}{(j\omega T_1 + 1)}$$

→ The magnitude of the function is,

$$\left| \frac{C(s)}{R(s)} \right| = \boxed{\frac{K\sqrt{1+\omega^2 T_2^2}}{\sqrt{1+\omega^2 T_1^2}}}, \text{ The phase angle, } \boxed{\phi = \tan^{-1}\omega T_2 - \tan^{-1}\omega T_1}$$

- $r(t) = R\sin\omega t \rightarrow y_{ss}(t) = R(\text{magnitude of the function})\sin(\omega t + \text{phase angle})$

$$\boxed{y_{ss}(t) = \frac{RK\sqrt{1+\omega^2 T_2^2}}{\sqrt{1+\omega^2 T_1^2}} \sin(\omega t + \tan^{-1}\omega T_2 - \tan^{-1}\omega T_1)}$$

Question-3: Given $G(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

show that $|G(j\omega_n)| = 1/2\xi$

Solution = $G(j\omega) = \dfrac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2} = \dfrac{1}{-\dfrac{\omega^2}{\omega_n^2} + j2\xi\dfrac{\omega}{\omega_n} + 1}$

$* \; |G(j\omega)|_{\omega=\omega_n} = \dfrac{1}{|-1 + j2\xi + 1|} = \dfrac{1}{|j2\xi|} = \boxed{\dfrac{1}{2\xi}}$

Question-4: Find analytical expression for the magnitude and phase response for each $G(s)$ below.

a. $G(s) = \dfrac{1}{s(s+2)(s+4)}$

b. $G(s) = \dfrac{s+5}{(s+2)(s+4)}$

c. $G(s) = \dfrac{(s+3)(s+5)}{s(s+2)(s+4)}$

Solution =

(a) $G(j\omega) = \dfrac{1}{j\omega(j\omega+2)(j\omega+4)} = \dfrac{1}{-6\omega^2 + j(8\omega - \omega^3)}$

$* \; |G(j\omega)| = \dfrac{1}{\sqrt{(-6\omega^2)^2 + (8\omega - \omega^3)^2}}$

$* \; \phi = -\tan^{-1}\left(\dfrac{8\omega - \omega^3}{-6\omega^2}\right)$

(b) $G(j\omega) = \dfrac{j\omega + 5}{(j\omega+2)(j\omega+4)} = \dfrac{j\omega+5}{(8-\omega^2+6j\omega)}$

$$|G(j\omega)| = \frac{\sqrt{\omega^2+25}}{\sqrt{(8-\omega^2)^2+(6\omega)^2}}, \quad \phi = \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right)$$

(c) $G(j\omega) = \dfrac{(j\omega+3)(j\omega+5)}{j\omega(j\omega+2)(j\omega+4)} = \dfrac{15-\omega^2+j8\omega}{-6\omega^2+j(8\omega-\omega^3)}$

$$|G(j\omega)| = \boxed{\frac{\sqrt{(15-\omega^2)^2+(8\omega)^2}}{\sqrt{(-6\omega^2)^2+(8\omega-\omega^3)^2}}}, \quad \boxed{\phi = \tan^{-1}\left(\frac{8\omega}{15-\omega^2}\right) - \tan^{-1}\left(\frac{8\omega-\omega^3}{-6\omega^2}\right)}$$