Assignment-7

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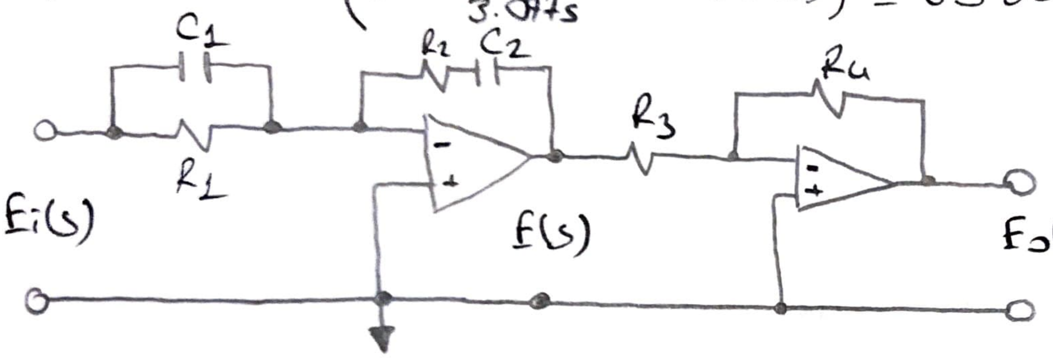
PID Controllers and Modified PID Controllers:

Question-1 = Consider the electronic PID controller shown in figure. Determine the values of R_1, R_2, R_3, R_4, C_1 and C_2 of the controller such that the transfer function $G_c(s) = E_o(s)/E_i(s)$ is

$$G_c(s) = 39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 30.3215\,\frac{(s+0.65)^2}{s}$$



Solution = For the given PID controller

* $$Z_1 = \frac{1}{\frac{1}{R_1} + C_1 s} = \frac{R_1}{R_1 C_1 s + 1}, \quad Z_2 = R_2 + \frac{1}{C_2 s} = \frac{R_2 C_2 s + 1}{C_2 s}$$

* $$\frac{E(s)}{E_i(s)} = -\frac{Z_2}{Z_1} = -\left[\frac{R_2 C_2 s + 1}{C_2 s}\right]\left[\frac{R_1 C_1 s + 1}{R_1}\right] = -\frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_1 C_2 s}$$

* $$\frac{E_o(s)}{E(s)} = -\frac{R_4}{R_3}, \quad G_c(s) = \frac{E_o(s)}{E_i(s)} = \frac{E_o(s)}{E(s)} \times \frac{E(s)}{E_i(s)}$$

* $$G_c(s) = \left[-\frac{R_4}{R_3}\right]\left[\frac{-(R_1 C_1 s)(R_2 C_2 s + 1)}{R_1 C_2 s}\right] = \frac{R_4 R_2}{R_3} C_1 \frac{\left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)}{s}$$

* $$G_c(s) = 30.3215\,\frac{(s+0.65)^2}{s} = 39.42\left(1 + \frac{1}{3.007s} + 0.7692s\right)$$

$$\rightarrow \frac{1}{R_1 C_1} = \frac{1}{R_2 C_2} = 0.65 \Rightarrow R_1 C_1 = R_2 C_2 = 1.538$$

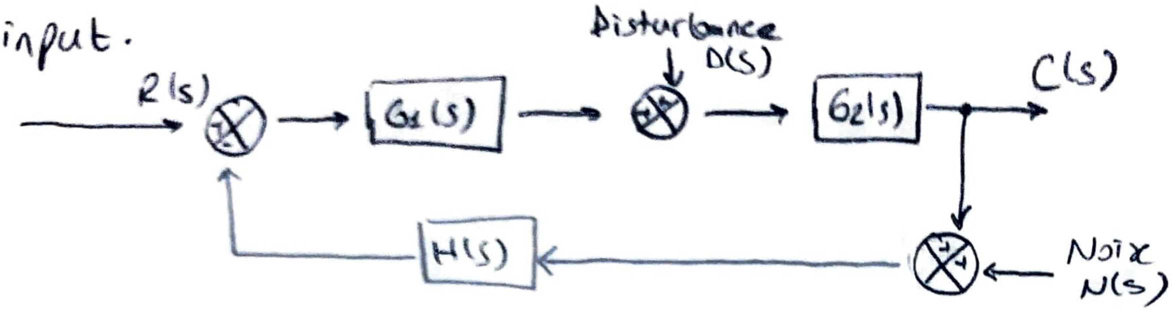
* $$R_1 = \frac{1}{0.65 C_1}, \quad R_2 = \frac{1}{0.65 C_2}, \quad R_4 = \frac{30.321}{R_2 C_2} R_3$$

* Assume $C_1 = C_2 = 10\,\mu F$, $R_3 = 10\,k\Omega$

$R_1 = 15.84\,k\Omega$
$R_2 = 153.84\,k\Omega$
$R_4 = 197.5\,K\Omega$

Question 2 =

Consider the system shown in figure. This system is subjected to three input signal: the reference input, disturbance input, and noise input. Show that the characteristic equation of this system is the same regardless of which input signal is chosen as input.



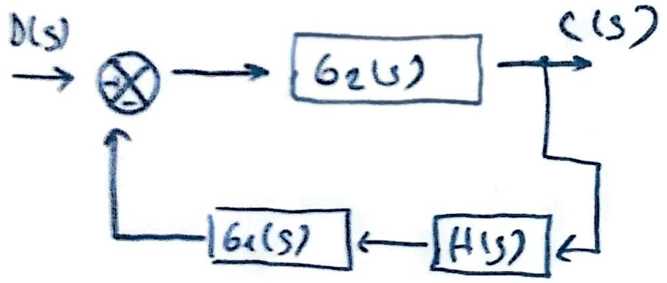
Solution = Consider R(s) as an input.

$$\frac{C(s)}{R(s)} = \frac{G_1(s)\,G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

the characteristic equation is

$$\boxed{1+G_1(s)\,G_2(s)H(s)=0}$$

* Consider D(s) as input,

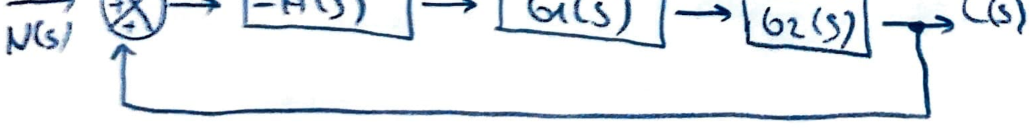


$$\frac{C(s)}{R(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

the characteristic equation is

$$\boxed{1+G_1(s)\,G_2(s)\,H(s)=0}$$

* Consider N(s) as input,



$$\frac{C(s)}{N(s)} = \frac{-G_1(s)\,G_2(s)\,H(s)}{1+G_1(s)G_2(s)\,H(s)}$$

the characteristic equation is

$$\boxed{1+G_1(s)G_2(s)H(s)=0}$$

* Thus the characteristic equation of the system is the same regardless of which input signal is chosen as input

Question - 3

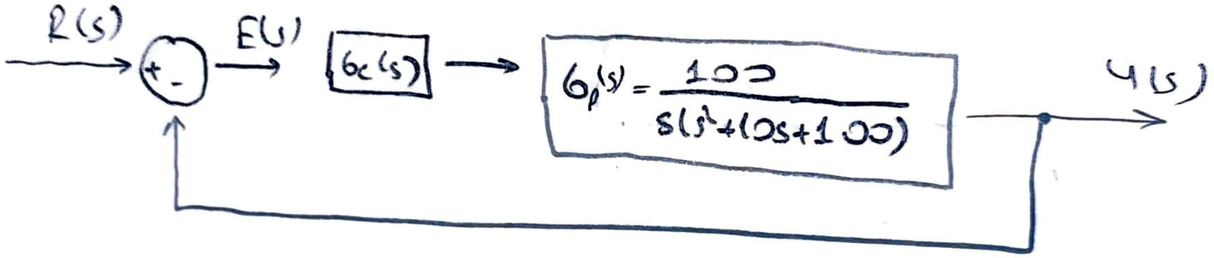
The block diagram of a control system with a series controller is shown in Fig. find the transfer function of the controller $G_c(s)$ so that the following specifications are satisfied:

The ramp error constant $K_v = 5$

The closed-loop transfer function is of the form

$$M(s) = \frac{Y(s)}{R(s)} = \frac{K}{(s^2+20s+100)(s+a)}$$

where K and a are real constant. find the values of K and a.



Solution = $M(s) = \frac{Y(s)}{R(s)} = \frac{K}{(s^2+20s+100)(s+a)} = \frac{K}{s^3+(20+a)s^2+(20a+100)s+100a}$

$\rightarrow G(s) = \frac{M(s)}{1-M(s)} = \frac{K}{s^3+(20+a)s^2+(200+20a)s+200a-K}$

* For the type 1 system, the constant term, $200a-K$ is equal to zero

$200a - K = 0 \rightarrow K = 200a$

$\rightarrow$ Determine the ramp error constant for type 1 system and equal to 5.

* $K_v = \lim_{s\to 0} s\,G(s) = \frac{sK}{s^3+(20+a)s^2+(200+20a)s+0}$

$\rightarrow K_v = \frac{K}{200+20a} = 5 \rightarrow \frac{200a}{200+20a} = 5$, $\boxed{\begin{cases} a = 10 \\ K = 2000 \end{cases}}$

Question-4)

The forward path transfer function of a unity-feedback control system is

$$G(s) = \frac{K}{s(\tau s+1)}$$

Find the value of K an τ so that the overshoot = 25.4% at $\xi = 0.4$

Solution = $\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{\tau}}{\boxed{s^2 + \frac{1}{\tau}s + \frac{K}{\tau}}}$ characteristic eq.

→ Standard form of second order charac. eq = $s^2 + 2\xi w_n s + w_n^2 = 0$

→ $2\xi w_n = 1/\tau$ → $0.8 w_n = 1/\tau$ → $\tau = 1/0.8 w_n$

→ $w_n^2 = K/\tau$ → $w_n = 0.8K$

→ $\tau = 1/0.8(0.8K) = 1.5625/K$

→ let's assume $K = 1$, then $\tau = 1.5625$

Question-5) The loop transfer function of a system is

$$G(s)H(s) = \frac{60}{s(0.4s+1)(s+1)(s+6)}$$

Design a PD controller to satisfy the following specification

→ $K_v = 10$, the phase margin is 45 degrees. Find K_p.

Solution = The general form transfer function of PD = $K_p + K_d s$

→ $G(s)H(s) = \frac{60(K_p + K_d s)}{s(0.4s+1)(s+1)(s+6)}$ → $K_v = 10$, → phase margin $= 45°$

→ $K_v = \lim_{s \to 0} \frac{60(K_p + K_d s)}{s(0.4s+1)(s+1)(s+6)} = 10 = \frac{60 K_p}{6}$

→ $K_p = 1$