EEE302 CONTROL SYSTEMS
LECTURE ASSIGNMENT

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5 QUESTION HOMEWORK-4: COMPENSATION

Assignment-4

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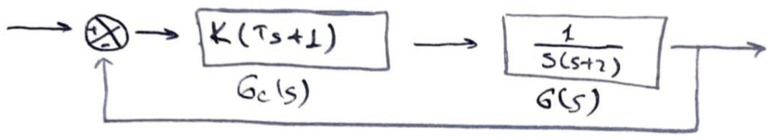
Question-1: Consider the control system shown in Figure. Determine the gain K and time constant T of the controller $G_c(s)$ such that the closed-loop poles are located at $s = -2 \mp j2$



Solution: Calculate the closed-loop transfer function of the system.

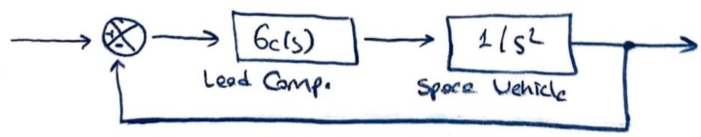
$$* \; C(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} = \frac{K(Ts+1)\frac{1}{s(s+2)}}{1+\frac{K(Ts+1)}{s(s+2)}} = \boxed{\frac{K(sT+1)}{s^2+(2+KT)s+K}}$$

* The closed-loop poles are located at $s=-2+j2$ and $s=-2-j2$

$$\rightarrow s^2+(2+KT)s+K = (s+2+j2)(s+2-j2)$$
$$= s^2+4s+8$$

* So, $\boxed{K=8}$ and $(2+KT)=4$, $\boxed{T=0.25s}$

Question-2: Consider the system shown in Figure. Design a compensator such that the dominator closed-loop poles are located at $s = -1 \mp j1$.



Solution: The open-loop plant is, $G(s) = 1/s^2$

The lead compensation transfer function is, $D_c(s) = K\left(\frac{s+z}{s+p}\right)$

Therefore, the open-loop transfer function is, $L(s) = G(s)D(s)$

$$L(s) = \left(\frac{K}{s^2}\right)\left(\frac{s+z}{s+p}\right) = \frac{K(s+z)}{s^2(s+p)}$$

* The characteristic equation,

$$1+G(s)\Delta(s)=0 \rightarrow 1+\frac{K(s+z)}{s^2(s+p)}=0$$

$\rightarrow \boxed{s^3+ps^2+Ks+Kz=0}$ (charac. eq.)

* Poles are $s=-1\mp j1$

$\rightarrow (s+1-j)(s+1+j)=0 \rightarrow (s^2+2s+2)=0$

* Assume another pole which is a real pole to balance the system.

$s=-a$, so $(s+a)(s^2+2s+2)=0 \rightarrow \boxed{s^3+(2+a)s^2+(2+2a)s+2a=0}$

+ $\boxed{2+a=p}$, $\boxed{2+2a=K}$, $\boxed{2a=Kz}$

$\rightarrow$ we can assume that $p=3.5$.

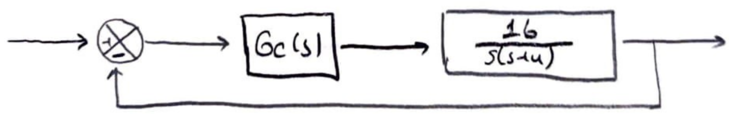
$\rightarrow 2+a=p=3.5,\ a=1.5$

$\rightarrow K=2+2a,\ K=5$

$\rightarrow 2a=Kz,\ z=0.6$

$$\rightarrow \boxed{D_c(s)=K\left(\frac{s+z}{s+p}\right)=5\left(\frac{s+0.6}{s+3.5}\right)}$$

Question-3: Reffering to the system in figure, design a compensator such that the static velocity error constant K_v is $20\ sec^{-1}$ without appreciably changing the original location ($s=-2\mp j2\sqrt{3}$) of a pair of the complex-conjugate closed-loop poles.



Solution= * Calculate the T.F. for uncompensated system.

$$\frac{C(s)}{R(s)}=\frac{\frac{16}{s(s+4)}}{1+\frac{16}{s(s+4)}}=\frac{16}{s^2+4s+16}$$

* write the general form of a lag compensator.

$$\rightarrow G_c(s)=K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}},\quad \beta>1$$

* Calculate the static velocity error constant Kv.

$\longrightarrow K_v = \lim_{s\to 0} s G_c(s) G(s)$

$\longrightarrow 20 = \lim_{s\to 0} s K_c \dfrac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \cdot \dfrac{16}{s(s+4)}$, $\nearrow$ $4\beta K_c = 20$

$\beta K_c = 5$ Assumption

$\beta = 5$, $K_c = 1$

* For a lag compensator, the pole and zero must be located close to the origin.

* Let's assume $T = 20$

$\longrightarrow G_c(s) = \dfrac{s+\frac{1}{20}}{s+\frac{1}{100}} = \boxed{\dfrac{s+0.05}{s+0.01}}$

* The closed-loop poles are located at $s = -2 \mp j2\sqrt{3}$

* Calculate the magnitude of the lag compensator at $s = -2 \mp j2\sqrt{3}$

$\longrightarrow \left| \dfrac{-2+j2\sqrt{3}+0.05}{-2+j2\sqrt{3}+0.01} \right| = \underline{\underline{0.995}}$

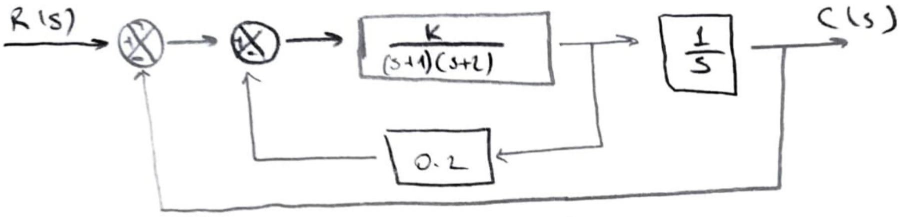
* Calculate the phase angle of lag compensator at $s = -2 \mp j2\sqrt{3}$

$\longrightarrow \angle G_c(s) = \angle -1.95+2\sqrt{3} - \angle -1.99+j2\sqrt{3} = \underline{\underline{0.4999^\circ}}$

* The angle contribution of this lag network is very small and the magnitude of $G_c(s)$ is approximately unity at the desired closed-loop pole. Hence, the designed Lag compensator is satisfactory.

$$\boxed{G_c(s) = \frac{s+0.05}{s+0.01}}$$

Question 4: Consider the system shown in figure. The system involves velocity feedback. Determine the value of gain K such that the dominant closed-loop poles have a damping ratio of 0.5. Using the gain K thus determined, obtain the unit-step response of the system



Solution = $G(s) = \left(\dfrac{\dfrac{K}{(s+1)(s+2)}}{1 + \dfrac{0.2K}{(s+1)(s+2)}} \right) \dfrac{1}{s}$

* $G(s) = \dfrac{K}{s[(s+1)(s+2) + (0.2K)]}$

→ The closed-loop T.F. is → $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)} = \dfrac{K}{s^3+3s^2+(2+0.2K)s+K}$

* The dominant closed pole → $s = -x + j\sqrt{3}x$ → Assumption

→ $(-x + j\sqrt{3}x)^3 + 3(-x + j\sqrt{3}x)^2 + (2+0.2K)(-x + j\sqrt{3}x) + K = 0$

→ $-8x^3 - 6x^2 - 2x + 0.2Kx + K + j\sqrt{3}(3x^2 + x + 0.1Kx) = 0$

* Real part → $-8x^3 - 6x^2 + 2x + 0.2Kx + K = 0$
* Imaginary part → $3x^2 + x + 0.1Kx = 0$ → $\boxed{K = -10(3x+1)}$

→ Real part now → $8x^3 + 12x^2 + 30x + 10 = 0$

$x = -0.562\cancel{2} \mp j1.7354$, $\boxed{x = -0.3756}$

* The dominant closed loop poles are $s = -0.3756 \mp j0.6506$
* $K = -10(3 \times (-0.3756) + 1) = \underline{1.268}$
* $\boxed{\dfrac{C(s)}{R(s)} = \dfrac{1.268}{s^3 + 3s^2 + 2.2536s + 1.268}}$

Question 5 = Plot the root loci for a closed-loop control system with

$$G(s) = \frac{K(s+9)}{s(s^2+4s+11)}, \quad H(s) = 1$$

→ locate the closed-loop poles on the root loci such that the dominant closed loop poles have a damping ratio equal to 0.5. Determine the corresponding value of gain K.

Solution =

→ The poles are, $s_1 = 0$, $s_{2,3} = -2 \mp j2.645$

→ The zeros are, $s = -9$

→ n = Number of poles = 3, m = Number of zeros = 1

→ Therefore, the number of root locus branches is 2.

* Angle of asymptotes = $\frac{\mp(2k+1)180°}{n-m}$ → $k=0 \rightarrow \mp\frac{180}{2} = \mp 90°$; $k=1 = \frac{\mp 180 \times 3}{2} = \mp 270°$

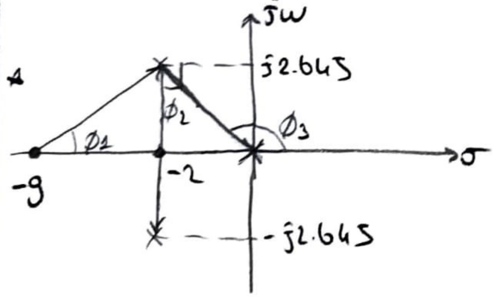
* Centroid $= \sigma_A = \frac{0-2-2+9}{2} = 2.5$

* The characteristic equation => $G(s) = \frac{C(s)}{R(s)} = \frac{\frac{K(s+9)}{s(s^2+4s+11)}}{1+\frac{K(s+9)}{s^2+4s+11}} = \frac{K(s+9)}{\boxed{s(s^2+4s+11)+K(s+9)}}$ ↳ charac. eq.

* $K = -\frac{s(s^2+4s+11)}{s+9} \rightarrow \frac{dK}{ds} = \boxed{2s^3+31s^2+72s+99=0}$

↳ $s = -13.028, \; -1.235 \mp j1.507$

↳ For these values of root, K is negative so, there is no break away point.



→ $\phi_1 = \tan^{-1}\left(\frac{2.645}{7}\right)$

$\phi_1 = 20.7°$

→ $\phi_2 = 90°$

→ $\phi_3 = 180° - \tan^{-1}\left(\frac{2.645}{2}\right) = 127.09°$

→ $\phi_d = 180° + \phi = 180° + \phi_1 - (\phi_2 + \phi_3)$

$= 180 + 20.7° - (90° + 127.09°) = -16.39°$

→ The char. eq $= s^3 + 4s^2 + s(11+k) + 9k = 0$

Routh. Table

s^3	1	$11+k$	0
s^2	4	$9k$	0
s	$\frac{44-5k}{4}$	0	
1	$9k$		

→ $\frac{44-5k}{4} = 0$ → $\underline{k = 8.8}$

→ $4s^2 + 9k = 0$

$s = \mp j\,4.49$

↳ Root locus crosses the imaginary axis