Control LAB-4

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Prelab (On Paper)

* Unity feedback system which has a $G(s) = \frac{K(s+2)}{s^2-4s+13}$ transfer function.

i) Draw root locus.

ii) Determine the points where the root loci cross the imaginary axis and find the gain on these points.

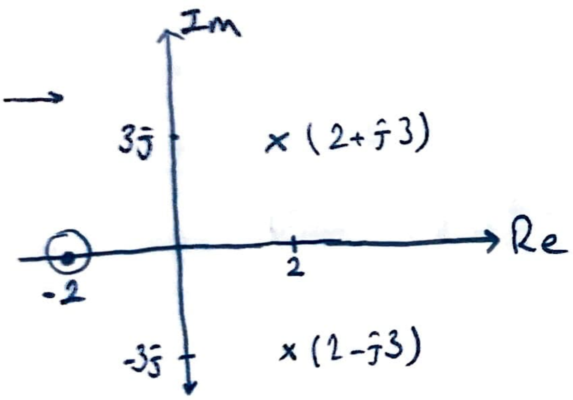
iii) Determine the angle of departure from the complex-conjugate open-loop poles.

iv) Determine the points where the root loci cross the real axis.

Solution=

→ Given, $G(s) = \frac{K(s+2)}{s^2-4s+13}$ → $\boxed{\text{zero} \Rightarrow s=-2}$ → $\boxed{\text{poles} \Rightarrow s = 2 \mp 3j}$

→



* Centroid, $\sigma = \frac{(\Sigma \text{ Real part of pole} - \Sigma \text{ real part of zero})}{\text{pole number} - \text{zero number}}$

* $\sigma = \frac{2+2-(-2)}{2-1} = \underline{\underline{6}}$

* Angle of asymptotes, $\theta_1 = \frac{(2\ell+1)}{p-z} \times 180°$ $\ell=0$

$p \to 2$, $z \to 1$

→ $\boxed{\theta = 180°}$

* Point of intersection of Root locus with respect to imaginary axis.

→ Routh's Stability

* Closed-loop character equation ⇒ $1 + G(s) = 1 + \frac{K(s+2)}{s^2-4s+13}$

* $s^2 + s(K-4) + (13+2K) = 0$

s^2	1	13+2K
s	K-4	0
1	13+2K	

→ $K-4=0$

$\underline{\underline{K=4}}$

s^2 row = auxiliary eq.

$s^2 + (13+2K) = 0$

$s^2 = -21$ $\boxed{w = \mp j4.58}$

ii) Root locus will cross the imaginary axis at $\underline{\underline{w = \mp j4.58}}$

Gain at $j4.58$

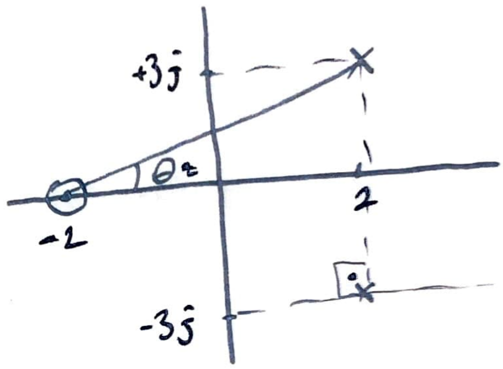
$$K = \left| \frac{-(s^2 - 4s + 13)}{s+2} \right| ,$$

when $s = +4.58$, $\underline{\underline{K = 2.38}}$

when $s = -4.58$, $\underline{\underline{K = 20.27}}$

iii) Angle of departure from complex conjugate pole.

$$\theta_D = 180^\circ - \angle Pole + \angle Zero$$



$$\theta_z = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$\theta_p = 90^\circ$$

$\rightarrow \theta_D = 180 - 90^\circ + 36.87^\circ = \underline{\underline{126.87^\circ}}$

iv) Point where root loci cross the real axis,

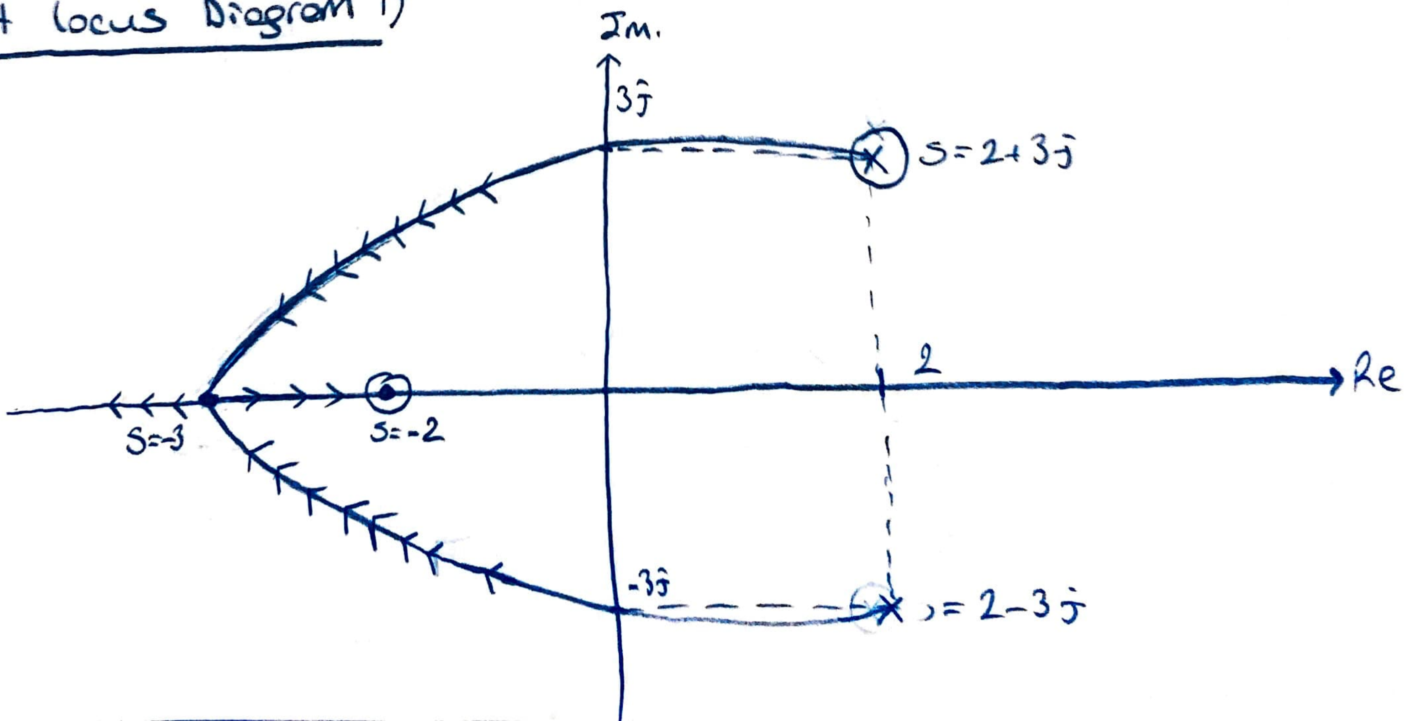
$\rightarrow$ Break in and break away point

$$K = \frac{-s^2 + 4s - 13}{s+2} , \quad \frac{dK}{ds} = 0$$

$\rightarrow s^2 - 4s - 21 = 0 \longrightarrow s_1 = 7 \rightarrow$ Since $s = 7$ doesn't lie in the Root locus it is not the point

$\longrightarrow \boxed{s_2 = -3}$

Root locus Diagram i)



1) Movement equation of the system in the following figure can be writed as $m\ddot{x}(t)+b\dot{x}(t)+kx(t)=f(t)$. Assume that $M=1$ kg, $b=10$ Ns/m, and $k=20$ N/m.

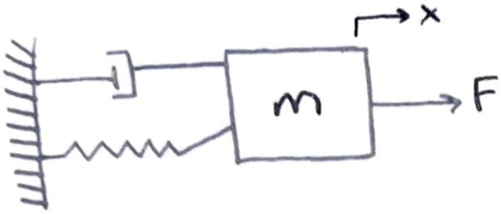


Fig. System

i) If system output $x(t)$ is controlled by proportional controller, Find Kp ranges which make to underdamped, critically damped, over damped, undamped system's response. Is there a Kp range that makes the system stable? If there is state this range.

Solution =

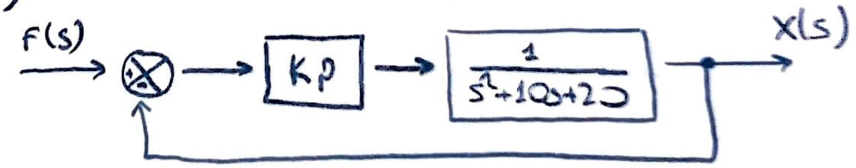
$\rightarrow$ Given system equation: $m\ddot{x}(t)+b\dot{x}(t)+kx(t)=f(t)$ $\quad m=1$ kg, $b=10$ Ns/m, $k=20$ N/m

$\rightarrow$ $\ddot{x}(t)+10\dot{x}(t)+20x(t)=f(t)$

* Taking Laplace Transform = $s^2X(s)+10sX(s)+20X(s)=F(s)$

$\rightarrow$ $F(s)=[s^2+10s+20]X(s)$

$\rightarrow$ $\boxed{\dfrac{X(s)}{F(s)}=\dfrac{1}{s^2+10s+20}}$ $\rightarrow$



$\rightarrow$ $\dfrac{X(s)}{F(s)}=\dfrac{Kp/s^2+10s+20}{1+Kp/s^2+10s+20}=\dfrac{Kp}{s^2+10s+20+Kp}$ $\rightarrow$ $s^2+2\xi w_n+w_n^2=0$

$2\xi w_n=10$

$\boxed{\xi w_n=5}$

$w_n^2=20+Kp$

$\boxed{w_n=\sqrt{20+Kp}}$

$\boxed{\xi=\dfrac{5}{\sqrt{20+Kp}}}$

* To get K_p range for different system behaviour,

a) Underdamped: $(0 < \xi < 1)$

$$0 < \frac{5}{\sqrt{20+K_p}} < 1 \Rightarrow \boxed{K_p > 5}$$

b) Critically Damped $(\xi = 1)$

$$\frac{5}{\sqrt{20+K_p}} = 1 \Rightarrow \boxed{K_p = 5}$$

c) Overdamped $(\xi > 1)$

$$\frac{5}{\sqrt{20+K_p}} > 1 \Rightarrow \boxed{K_p < 5}$$

d) Undamped $(\xi = 0)$

$$\frac{5}{\sqrt{20+K_p}} = 0 \Rightarrow \boxed{K_p = \infty}$$

* for system to be stable, we can use Routh Criterion to get results.

→ Character equation $= \boxed{s^2 + 10s + (20+K_p) = 0}$

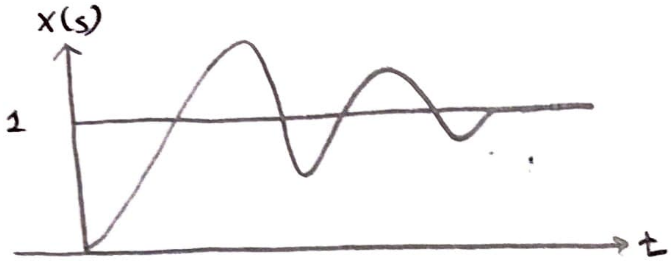
$$\begin{array}{c|cc|}
s^2 & 1 & 20+K_p \\
s^1 & 10 & 0 \\
s^0 & 20+K_p & 0
\end{array}$$

$20 + K_p > 0$

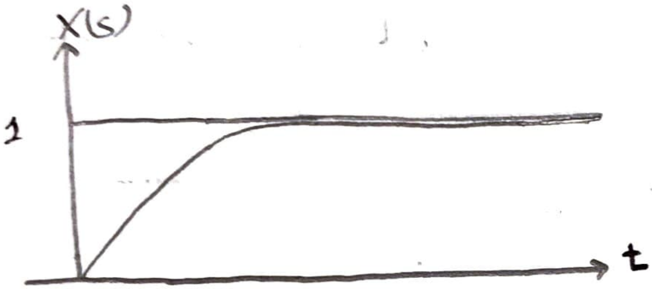
$\boxed{K_p > -20}$ → for system to be stable

→ Step responses for Kp ranges

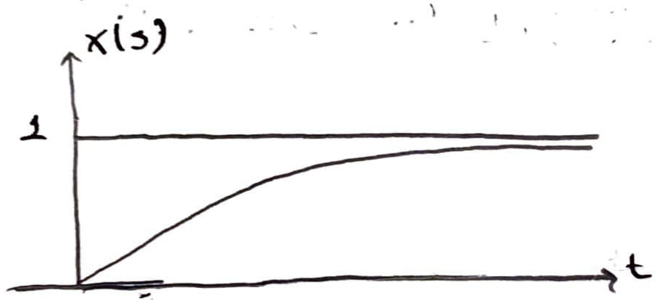
a) for $K_p > 5$



b) for $K_p = 5$



c) For $K_p < 5$



d) For $K_p = \infty$