EEE302 CONTROL SYSTEMS
PRE-LABORATORY REPORT

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ASSIGNMENT NUMBER	:	**4**

OBJECTIVES OF THE LABORATORY ASSIGNMENT:

Objectives of this lab are learning to draw root locus on MATLAB and observing Kp range change in Simulink.

QUESTION-1 (ON PAPER)

Control LAB-4

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Prelab (On Paper)

* Unity feedback system which has a $G(s) = \frac{K(s+2)}{s^2-4s+13}$ transfer function.

i) Draw root locus.

ii) Determine the points where the root loci cross the imaginary axis and find the gain on these points.

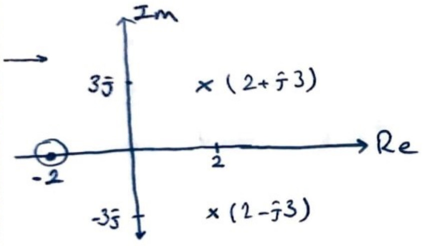
iii) Determine the angle of departure from the complex-conjugate open-loop poles.

iv) Determine the points where the root loci cross the real axis.

Solution =

→ Given, $G(s) = \frac{K(s+2)}{s^2-4s+13}$ → $\boxed{\text{zero} \Rightarrow s=-2}$ → $\boxed{\text{poles} \Rightarrow s = 2 \mp 3j}$

→



* Centroid, $\sigma = \frac{(\Sigma \text{ Real part of pole} - \Sigma \text{ real part of zero})}{\text{pole number} - \text{zero number}}$

* $\sigma = \frac{2+2-(-2)}{2-1} = \underline{\underline{6}}$

* Angle of asymptotes, $\theta_\ell = \frac{(2\ell+1)}{p-z} \times 180°$ $\ell = 0$ (p: 2, z: 1)

→ $\boxed{\theta = 180°}$

* Point of intersection of Root locus with respect to imaginary axis.

→ Routh's Stability

* Closed-loop character equation ⇒ $1 + G(s) = 1 + \frac{K(s+2)}{s^2-4s+13}$

* $s^2 + s(K-4) + (13+2K) = 0$

s^2	1	13+2K
s	K-4	0
1	13+2K	

→ $K-4=0$

$\underline{\underline{K=4}}$

s^2 row = auxiliary eq.

$s^2 + (13+2K) = 0$

$s^2 = -21$ $\boxed{w = \mp j4.58}$

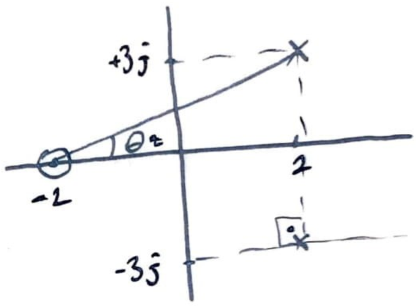
ii) Root locus will cross the imaginary axis at $\underline{\underline{\omega = \mp j4.58}}$

Gain at $j4.58$

$$K = \left| \frac{-(s^2-4s+13)}{s+2} \right| \quad , \quad \begin{array}{l} \text{when } s = +4.58 , \ \underline{\underline{K = 2.38}} \\ \text{when } s = -4.58 , \ \underline{\underline{K = 20.27}} \end{array}$$

iii) Angle of departure from complex conjugate pole.

$$\theta_D = 180^\circ - \angle \text{Pole} + \angle \text{Zero}$$



$$\theta_z = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$\theta_p = 90^\circ$$

$$\rightarrow \theta_D = 180 - 90^\circ + 36.87^\circ = \underline{\underline{126.87^\circ}}$$

iv) Point where root loci cross the real axis,

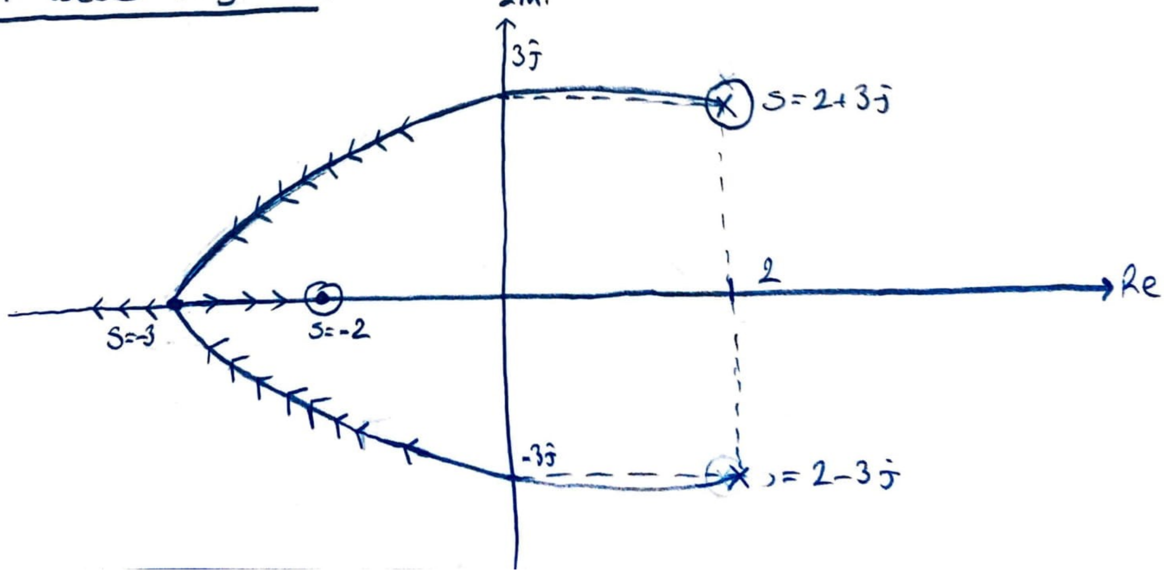
→ Break in and break away point

$$K = \frac{-s^2+4s-13}{s+2} \quad , \quad \frac{dK}{ds} = 0$$

$$\rightarrow s^2 - 4s - 21 = 0 \quad \begin{array}{l} \rightarrow s_1 = 7 \rightarrow \\ \rightarrow \boxed{s_2 = -3} \end{array}$$

Since $s=7$ doesn't lie in the Root locus it is not the point

Root locus Diagram i)



QUESTION-2 (ON PAPER)

1) Movement equation of the system in the following figure can be writed as $m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$. Assume that $M=1$ kg, $b=10$ Ns/m, and $k=20$ N/m.

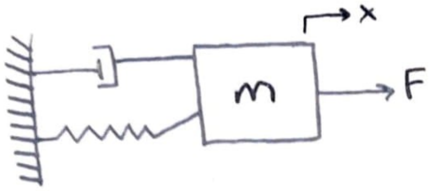


Fig. System

i) If system output $x(t)$ is controlled by proportional controller, Find Kp ranges which make to underdamped, critically damped, over damped, undamped system's response. Is there a Kp range that makes the system stable? If there is state this range.

Solution =

→ Given system equation: $m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$

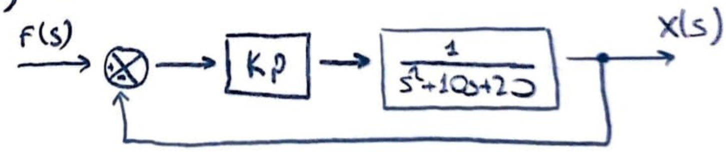
$m = 1$ kg
$b = 10$ Ns/m
$k = 20$ N/m

→ $\ddot{x}(t) + 10\dot{x}(t) + 20x(t) = f(t)$

* Taking Laplace Transform = $s^2X(s) + 10sX(s) + 20X(s) = F(s)$

→ $F(s) = [s^2 + 10s + 20]\,X(s)$

→ $\boxed{\dfrac{X(s)}{F(s)} = \dfrac{1}{s^2+10s+20}}$ →



→ $\dfrac{X(s)}{F(s)} = \dfrac{Kp/s^2+10s+20}{1 + Kp/s^2+10s+20} = \dfrac{Kp}{s^2+10s+20+Kp}$ → $s^2 + 2\xi w_n + w_n^2 = 0$

$2\xi w_n = 10$

$\boxed{\xi w_n = 5}$

$w_n^2 = 20 + Kp$

$\boxed{w_n = \sqrt{20+Kp}}$

$\boxed{\xi = \dfrac{5}{\sqrt{20+Kp}}}$

- To get K_p range for different system behaviour,

a) Underdamped : ($0 < \xi < 1$)

$$0 < \frac{5}{\sqrt{20+K_p}} < 1 \Rightarrow \boxed{K_p > 5}$$

b) Critically Damped ($\xi = 1$)

$$\frac{5}{\sqrt{20+K_p}} = 1 \Rightarrow \boxed{K_p = 5}$$

c) Overdamped ($\xi > 1$)

$$\frac{5}{\sqrt{20+K_p}} > 1 \Rightarrow \boxed{K_p < 5}$$

d) Undamped ($\xi = 0$)

$$\frac{5}{\sqrt{20+K_p}} = 0 \Rightarrow \boxed{K_p = \infty}$$

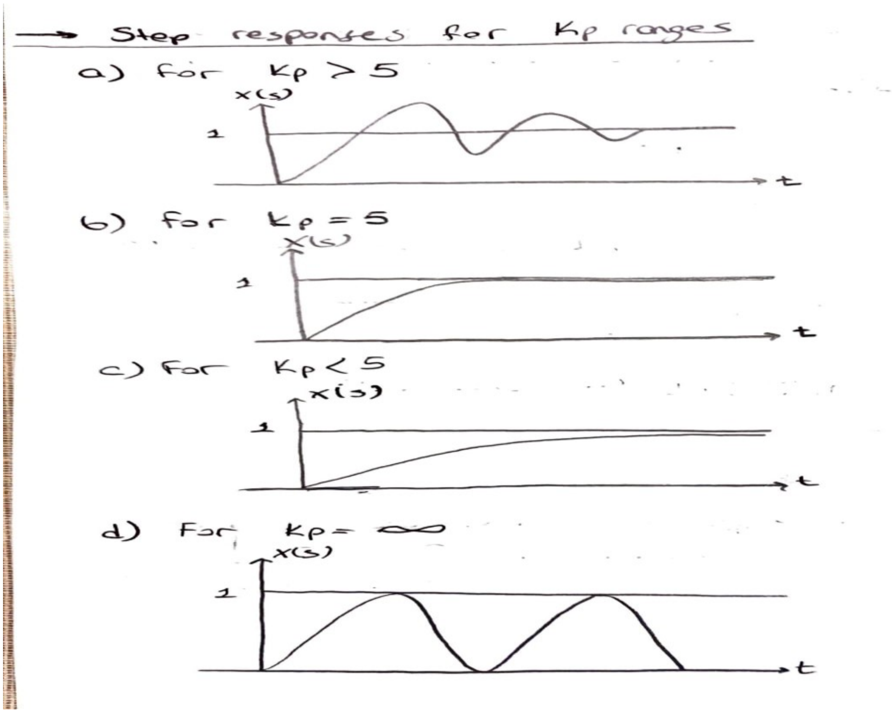
- for system to be stable, we can use Routh Criterion to get results.

→ Character equation = $\boxed{s^2 + 10s + (20+K_p) = 0}$

s^2	1	$20+K_p$
s^1	10	0
s^0	$20+K_p$	0

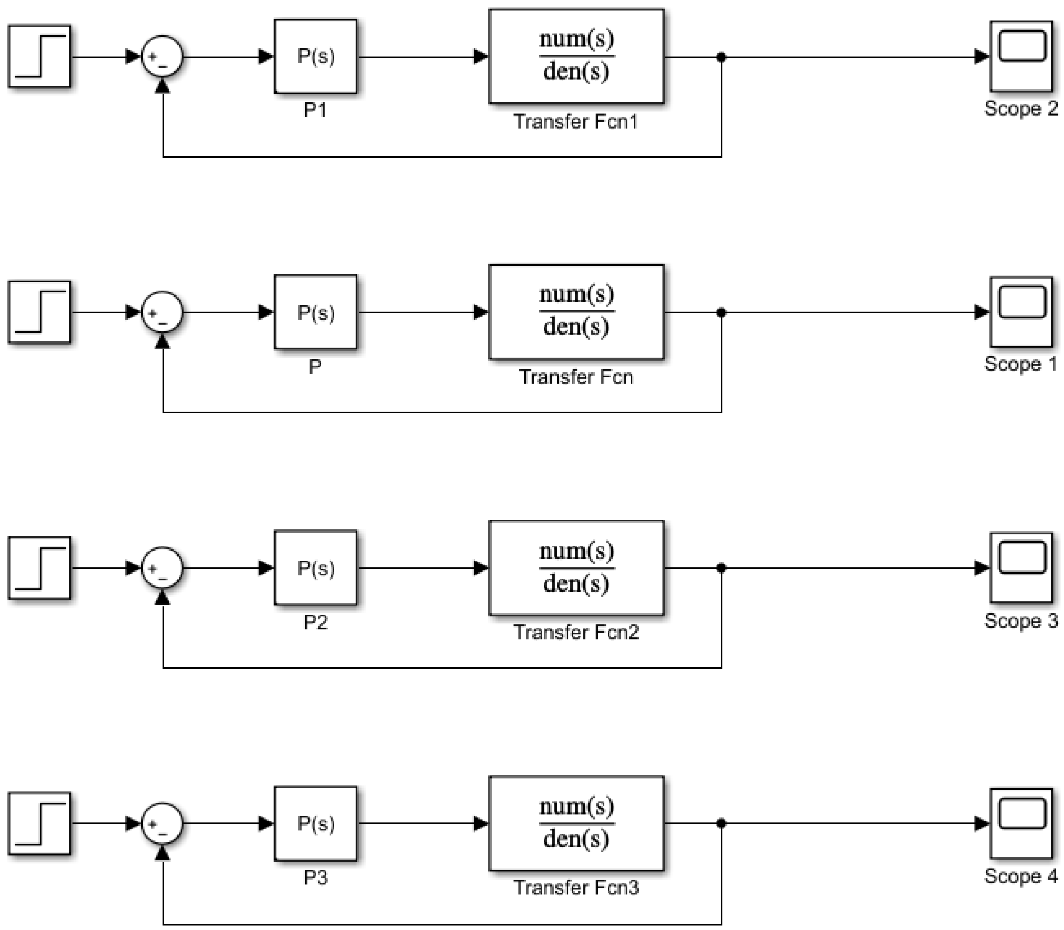
$20 + K_p > 0$

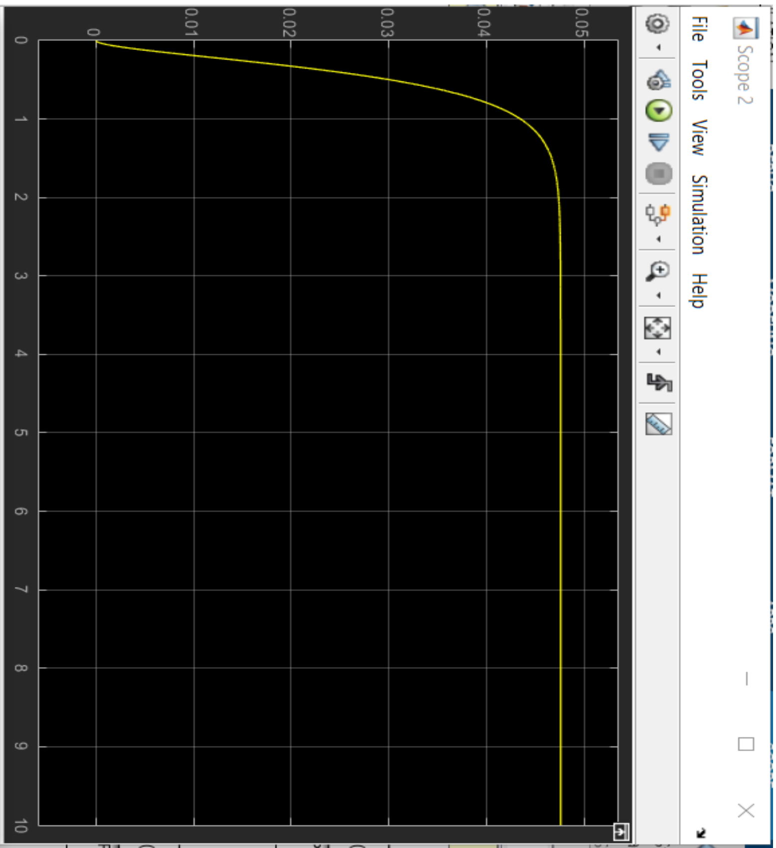
$\boxed{K_p > -20}$ → for system to be stable



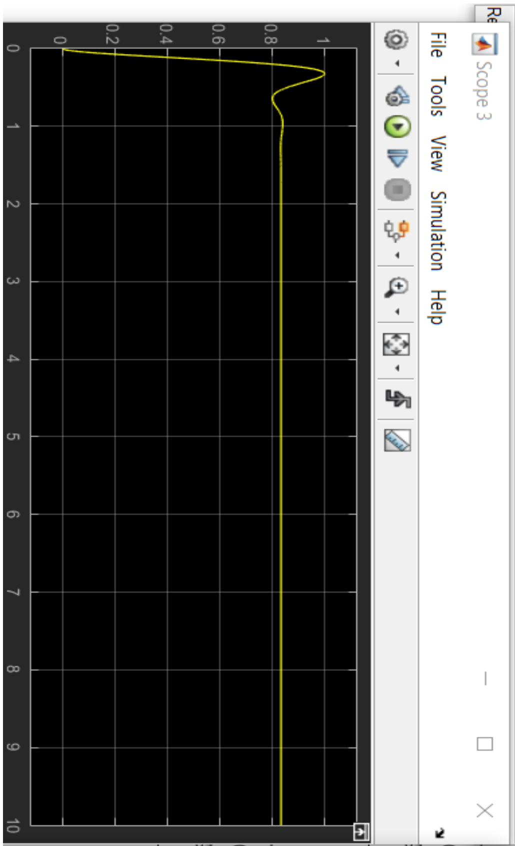
QUESTION-2 (SIMULINK)

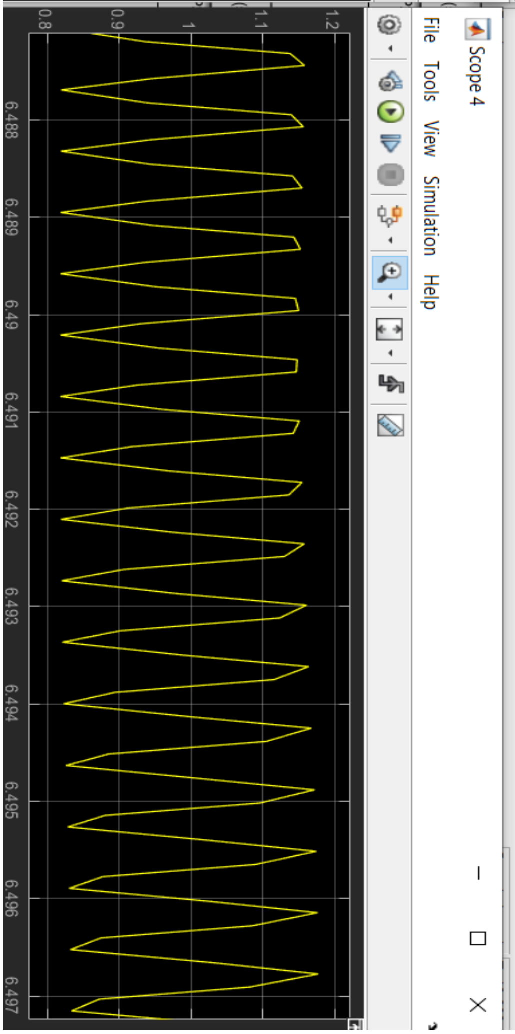
Kp = 1, 5, 100, and very high respectively.




Scope 2
File Tools View Simulation Help
0
0.01
0.02
0.03
0.04
0.05
0 1 2 3 4 5 6 7 8 9 10

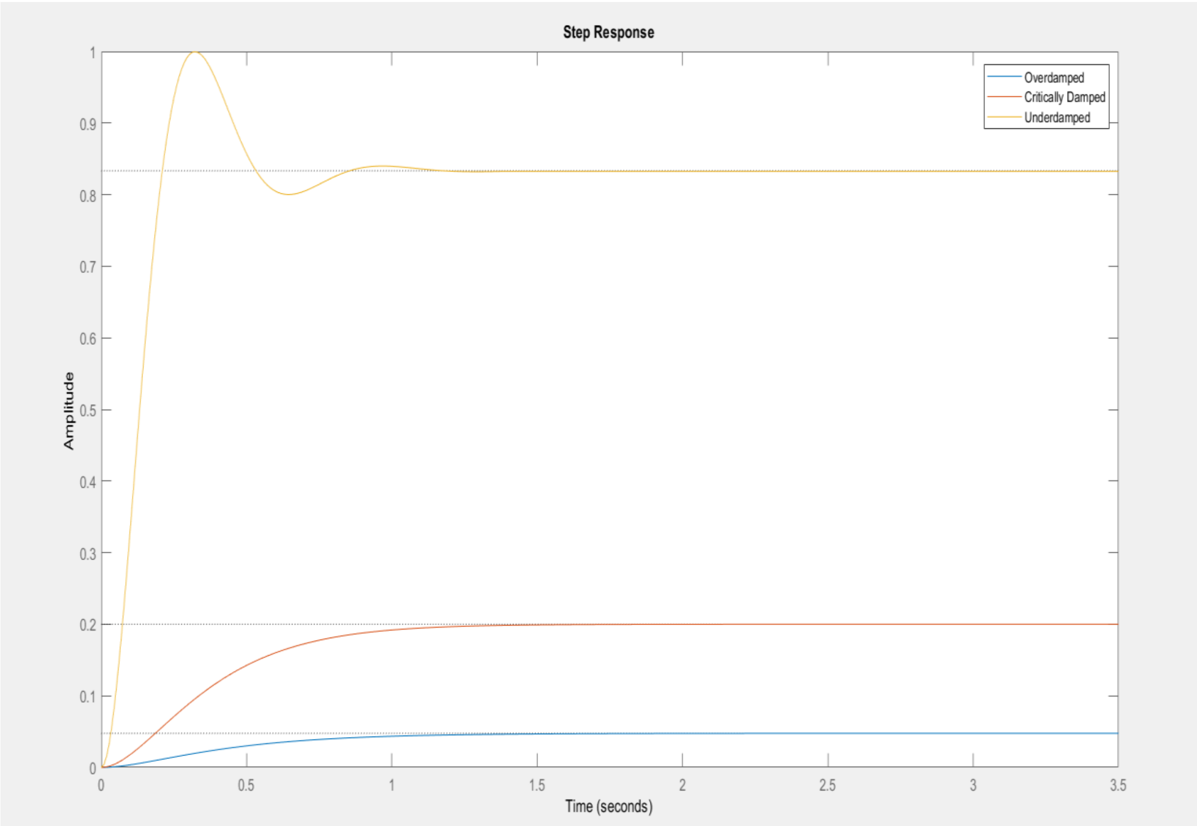

Scope 1
File Tools View Simulation Help
0
0.05
0.1
0.15
0.2
0 1 2 3 4 5 6 7 8 9 10

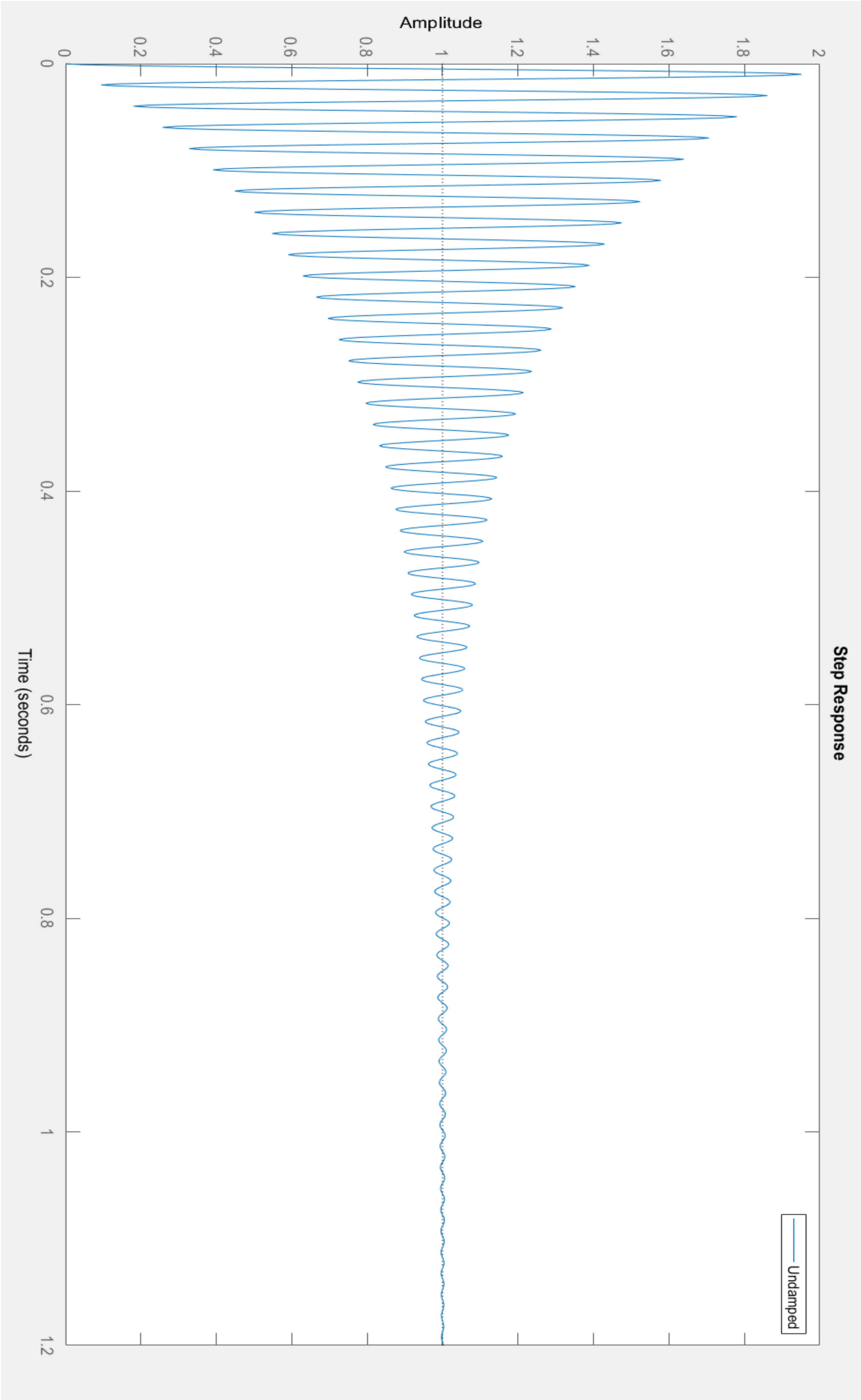

Scope 3
File Tools View Simulation Help
0
0.2
0.4
0.6
0.8
1
0 1 2 3 4 5 6 7 8 9 10


Scope 4
File Tools View Simulation Help
0.8
0.9
1
1.1
1.2
6.488 6.489 6.49 6.491 6.492 6.493 6.494 6.495 6.496 6.497

QUESTION-2 (M-FILE)

```
clc
close all
s=tf('s');
G= 1/(s^2+10*s+20);

kp = 1; % For Overdamped System
sys1 = feedback(kp*G,1);

kp = 5; % For Critically Damped System
sys2 = feedback(kp*G,1);

kp = 100; % For Underdamped System
sys3 = feedback(kp*G,1);

%kp = 99999; % For Undamped System
%sys4 = feedback(kp*G,1);

step(sys1,sys2,sys3)
legend('Overdamped','Critically Damped','Underdamped')

disp('for k=1')
pole(sys1)

disp('for k=5')
pole(sys2)

disp('for k=100')
pole(sys3)
```



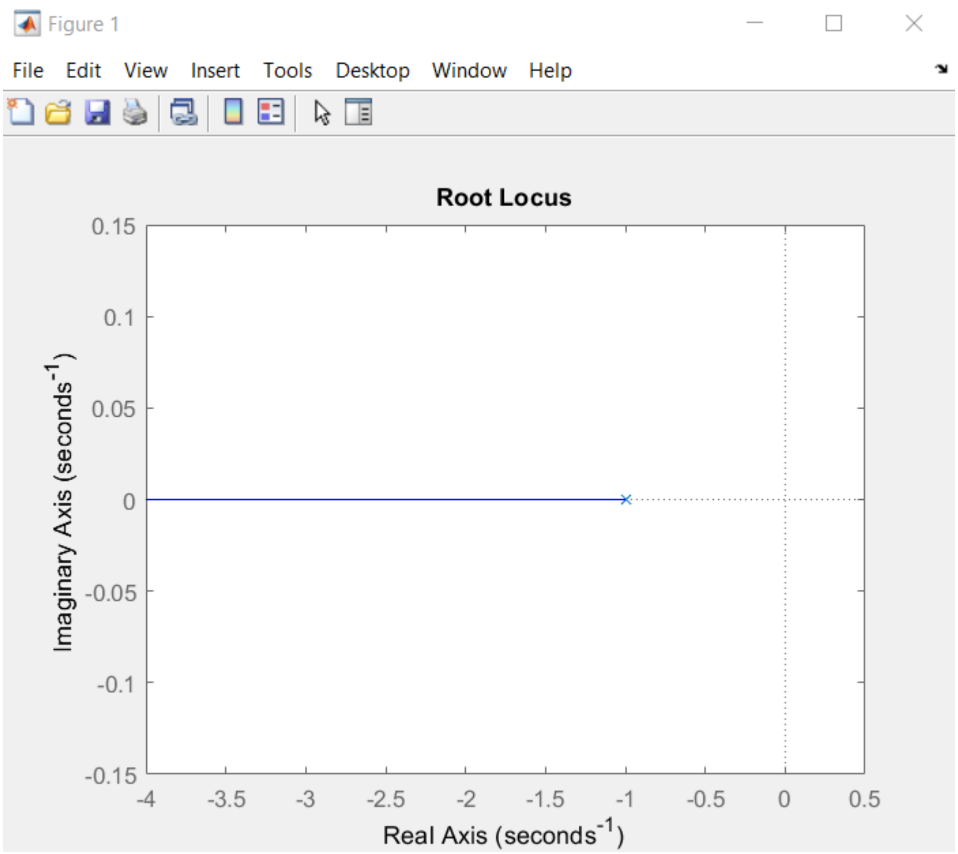

Step Response
Amplitude
Time (seconds)
Undamped
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

```
for k=1

ans =

    -7
    -3

for k=5

ans =

    -5
    -5

for k=100

ans =

  -5.0000 + 9.7468i
  -5.0000 - 9.7468i

for k=99999

ans =

   1.0e+02 *

  -0.0500 + 3.1622i
  -0.0500 - 3.1622i
```

QUESTION-3 (M-FILE)

A-

```
clc;

% Question A
num=[1]; % Numerator
denum=[1 1]; % Denominator
sys=tf(num,denum) % Transfer Function
rlocus(sys)        % Root Locus

% i
num=[1];
denum2=[1 3 2];
sys2=tf(num,denum2)
rlocus(sys2)

% ii
num=[1];
denum3=[1 5 4];
sys3=tf(num,denum3)
rlocus(sys3)
```



I)

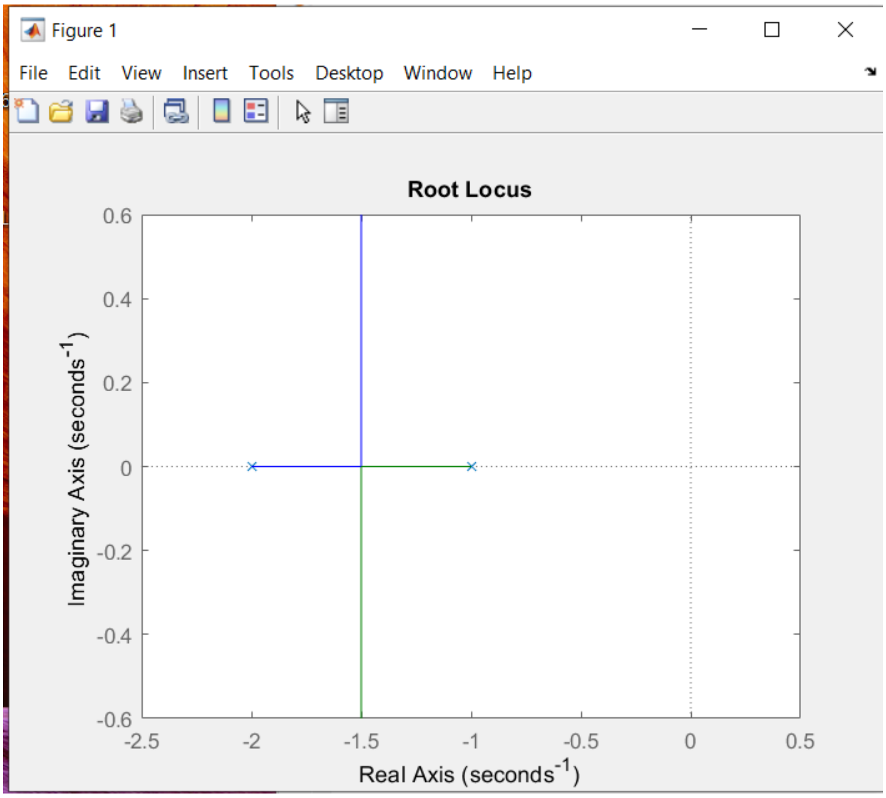


Figure 1
File Edit View Insert Tools Desktop Window Help
Root Locus
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Imaginary Axis (seconds^{-1})
-2.5
-2
-1.5
-1
-0.5
0
0.5
Real Axis (seconds^{-1})

II)

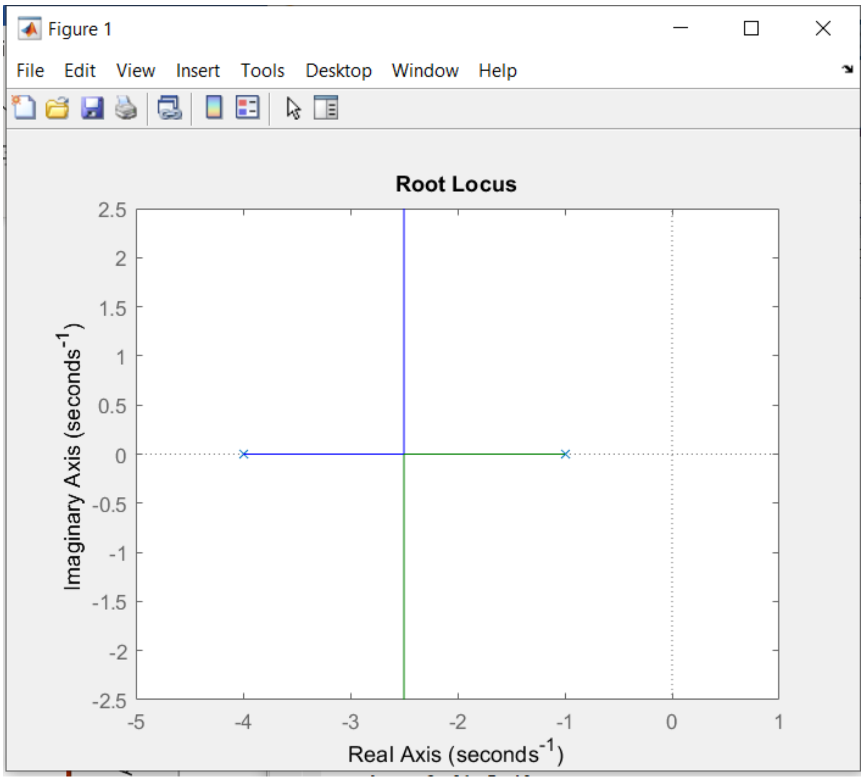
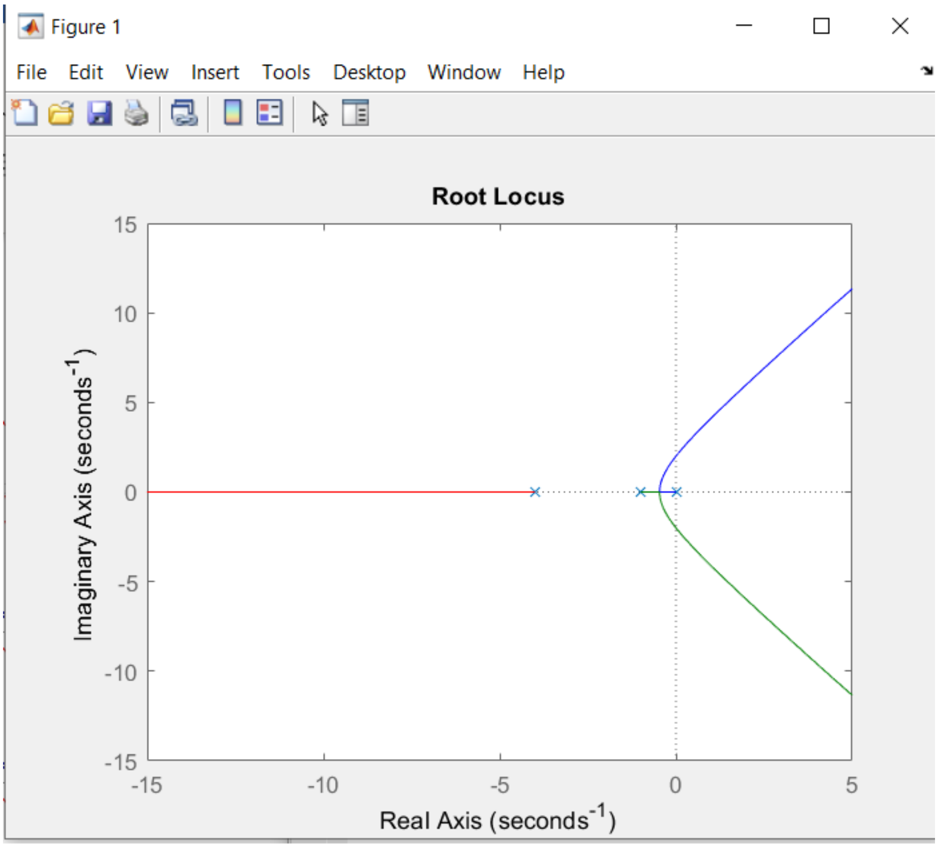


Figure 1
File Edit View Insert Tools Desktop Window Help
Root Locus
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
Imaginary Axis (seconds^{-1})
-5
-4
-3
-2
-1
0
1
Real Axis (seconds^{-1})

B-

```
clc;

% Question B
num=[1]; % Numerator
denum=[1 5 4 0]; % Denominator
sys=tf(num,denum) % Transfer Function
rlocus(sys)       % Root Locus

% i
num=[1 5];
denum2=[1 5 4 0];
sys2=tf(num,denum2)
rlocus(sys2)

% ii
num=[1 2];
denum3=[1 5 4 0];
sys3=tf(num,denum3)
rlocus(sys3)

% iii
num=[1 0.5];
denum4=[1 5 4 0];
sys4=tf(num,denum4)
rlocus(sys4)
```



I)

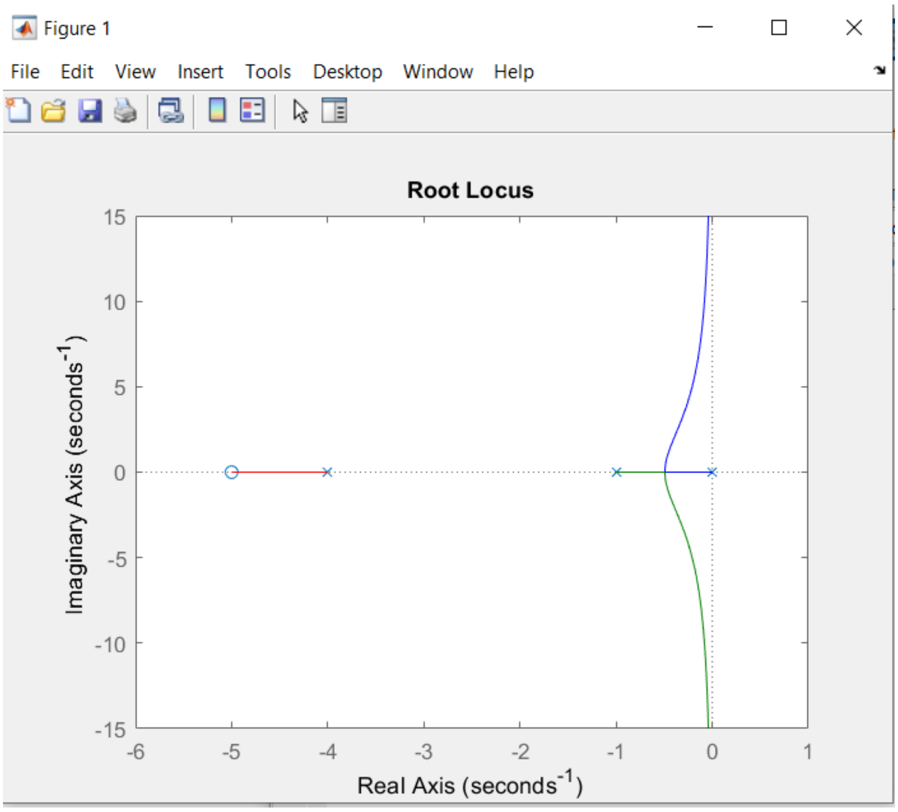


Figure 1
File Edit View Insert Tools Desktop Window Help
Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
15
10
5
0
-5
-10
-15
-6 -5 -4 -3 -2 -1 0 1

II)

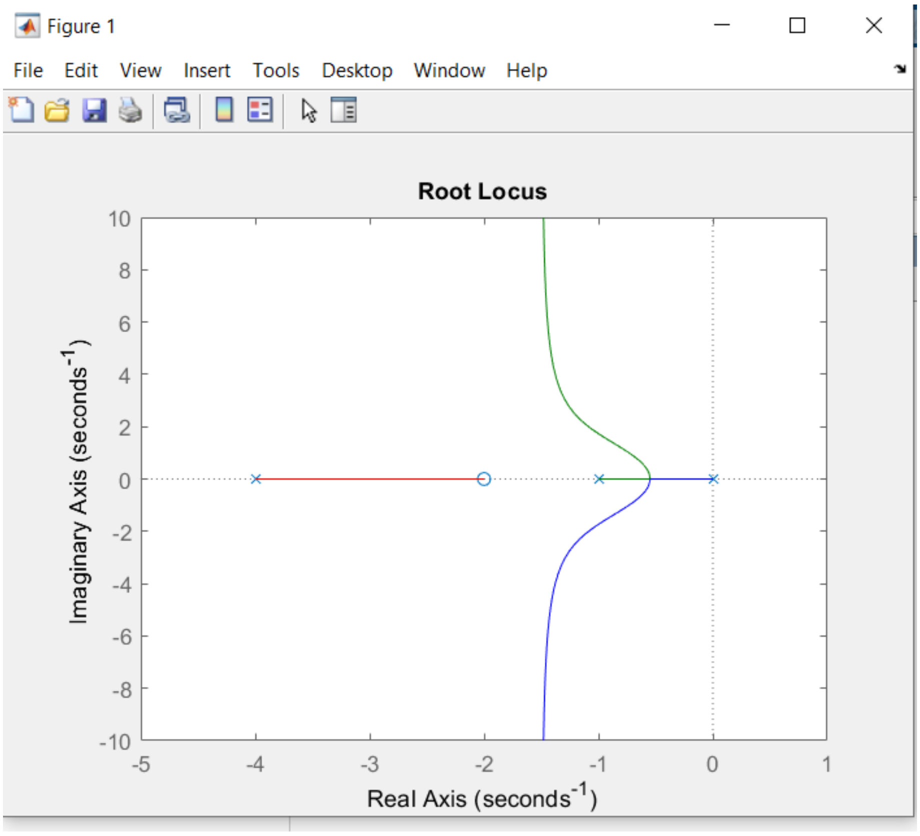
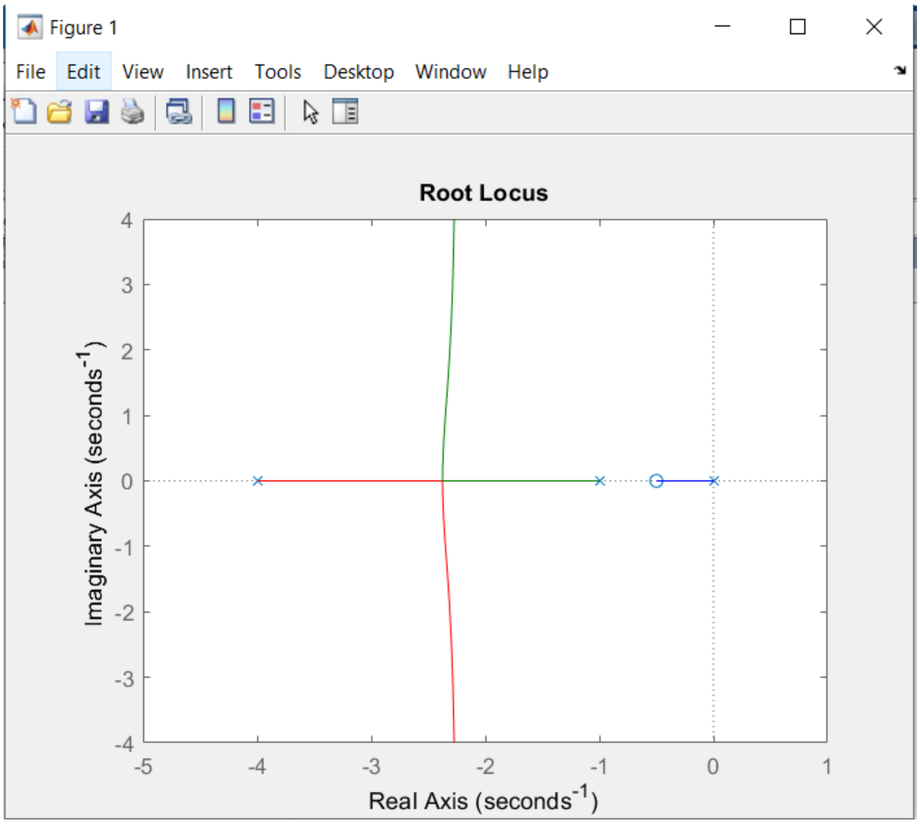


Figure 1
File Edit View Insert Tools Desktop Window Help
Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
10
8
6
4
2
0
-2
-4
-6
-8
-10
-5 -4 -3 -2 -1 0 1

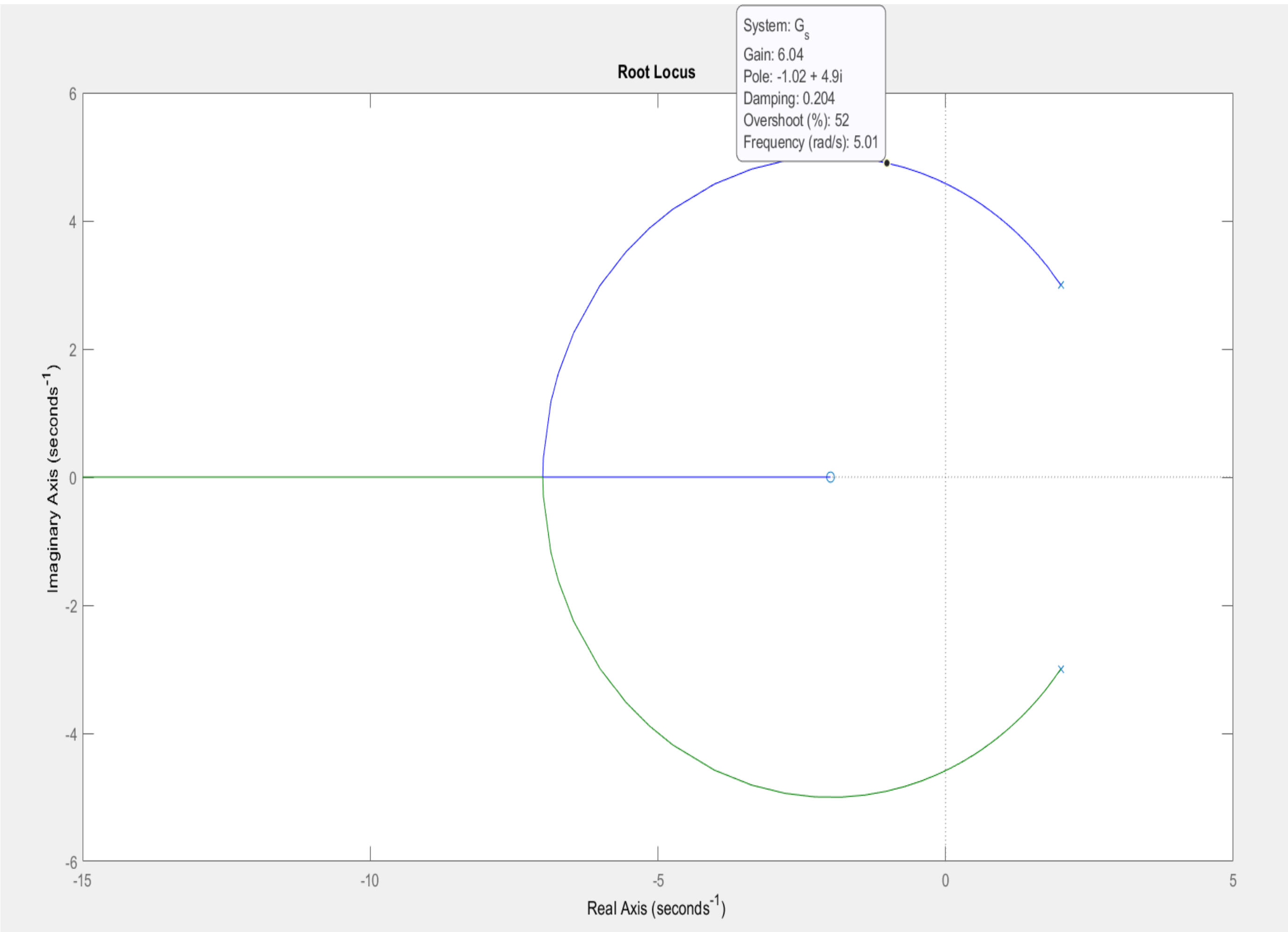
III)



C-

```
clc;
num=[1 2]; % numerator
den=[1 -4 13]; % Denominator
G_s=tf(num,den) % Transfer function
rlocus(G_s) % Root locus

K=6.04; % DC gain
G_s=tf(K.*num,den) % Transfer function

H_s=feedback(G_s,1) % Closed Loop Transfer function with Unit Negative feedback
step(H_s) % Unit Step Response

% Command Window
G_s =

  6.04 s + 12.08
  --------------
  s^2 - 4 s + 13

H_s =

     6.04 s + 12.08
  --------------------
  s^2 + 2.04 s + 25.08
```

Root-Locus



Here at 52 % Overshoot ,gain is 6.04