EEE302 CONTROL SYSTEMS
LECTURE ASSIGNMENT

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5 QUESTION HOMEWORK-3: LOOT-ROCUS METHOD

Assignment - 3

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Question - 1)

» Plot the root loci for the closed-loop control system with

$$G(s) = \frac{K(s+1)}{s^2}, \quad H(s) = 1$$

Solution =

→ The open loop transfer function is $\boxed{G(s)H(s) = \frac{K(s+1)}{s^2}}$

* We first locate the open-loop poles and zero on the complex plane. A root locus exists on the negative real axis between -1 and -∞. Since the open loop transfer function involves two poles and one zero, there is possibility that a circular root loci exists.

→ $\angle \frac{K(s+1)}{s^2} = \mp 180^\circ(2k+1)$

→ $\angle s+1 - 2\angle s = \mp 180^\circ(2k+1)$, by substituting $s = \sigma + jw$, we obtain

→ $\angle \sigma + jw + 1 - 2\angle \sigma + jw = \mp 180^\circ(2k+1)$

→ $\tan^{-1}(w/\sigma+1) - 2\tan^{-1}\frac{w}{\sigma} = \mp 180^\circ(2k+1)$

→ $\tan^{-1}(w/\sigma+1) - \tan^{-1}\frac{w}{\sigma} = \tan^{-1}\frac{w}{\sigma} \mp 180^\circ(2k+1)$

→ Take tangents of both sides,

$$\tan\left[\tan^{-1}\left(\frac{w}{\sigma+1}\right) - \tan^{-1}\frac{w}{\sigma}\right] = \tan\left[\tan^{-1}\frac{w}{\sigma} \mp 180^\circ(2k+1)\right]$$

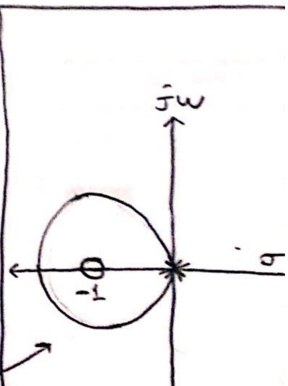
→ $\frac{w}{\sigma+1} - \frac{w}{\sigma} = \frac{w}{\sigma}\left(1 + \frac{w}{\sigma+1}\frac{w}{\sigma}\right)$

→ $w\left[(\sigma+1)^2 + w^2 - 1\right] = 0$

→ $w = 0$ or $(\sigma+1)^2 + w^2 = 1$

* These two equations are the equations for the root loci for the system. The first equation is for the real axis. The real axis from $s=-1$ to $s=-\infty$ corresponds to a root locus for $K \geq 0$.

* In the present system, K is positive. The second equation is an equation of a circle with the center at $\sigma = -1$, $w = 0$ and the radius is equal to 1.



root-locus diagram

Question-2) Show that the root loci for a control system with

$$G(s) = \frac{k(s^2+6s+10)}{s^2+2s+10}, \quad H(s)=1$$

are arcs of the circle centered at the origin with radius equal to $\sqrt{10}$.

Solution: $$1+G(s)H(s) = \frac{(1+k)s^2+(2+6k)s+10+10k}{s^2+2s+10}$$

→ The characteristic equation $= (1+k)s^2+(2+6k)s+10+10k=0$

has two roots at

$$s = -\frac{1+3k}{1+k} \mp j\frac{\sqrt{k^2+14k+9}}{1+k}$$

→ If we write $s = x \mp jy$, that is

$$x = -\frac{1+3k}{1+k}, \quad y = \frac{\sqrt{k^2+14k+9}}{1+k}$$

then,

$$x^2+y^2 = \left(\frac{1+3k}{1+k}\right)^2 + \frac{k^2+14k+9}{(1+k)^2} = \frac{10(k+1)^2}{(1+k)^2} = 10$$

→ This indicates that the root loci are on a circle about the origin of radius $\sqrt{10}$.

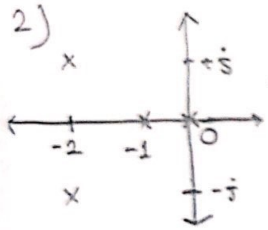
Question-3) Plot the root loci for the closed-loop control system with

$$G(s) = \frac{K}{s(s+1)(s^2+4s+5)}, \quad H(s) = 1$$

Solution= 1) Open-loop poles $\Rightarrow s = 0, -1, -2+j1, -2-j1$

Open-loop zeros $\Rightarrow$ None

2)



$\rightarrow$ The asymptotes of root locus. Calculate the angle of asym.

$$\text{Angle of asymptotes} = \frac{\mp 180(2K+1)}{4-0} \quad K = 0,1,2,3$$

$$= \mp 45^\circ, \mp 135^\circ, \mp 225^\circ, \mp 315^\circ$$

- The Abscisso of intersection of the asymptotes with real axis is,

$$s = \frac{(0+1+2+j+2-j)+0}{4-0} = -1.25$$

3) The breakaway and breaking points are determined by using characteristic equation.

$$1+G(s)H(s)=0 \longrightarrow 1+\frac{K}{s(s+1)(s^2+4s+5)} = s(s+1)(s^2+4s+5)+K = 0$$

$\rightarrow K = -s(s+1)(s^2+4s+5) = -(s^4+5s^3+9s^2+5s)$

$\rightarrow \frac{dK}{ds} = -(4s^3+15s^2+18s+5) \rightarrow$ Breakaway points are $-1.6785+j0.6028$, $-1.6785-j0.6028$, $\boxed{-0.3930}$

$\rightarrow$ Since there are no zeros on the real axis, the breaking point is absent

$\rightarrow$ Routh's criterion is used to determine the value of K.

4) Routh's criterion is used to determine the value of K.

$$s^4+5s^3+9s^2+5s+K=0$$

s^4	1	9	K
s^3	5	5	0
s^2	8	K	0
s^1	$\frac{40-5K}{8}$	0	0
s^0	K		

$\rightarrow$ The value of K,

$$\frac{40-5K}{8} = 0$$

$$5K = 40$$

$$\underline{\underline{K = 8}}$$

The value of s at which the root loci cross the imaginary axis is obtained from s^2 row from the Routh array

$$8s^2+K=0$$

$$s^2 = \frac{-K}{8}, \text{ since } K = 8$$

$$\boxed{s = \mp j}$$

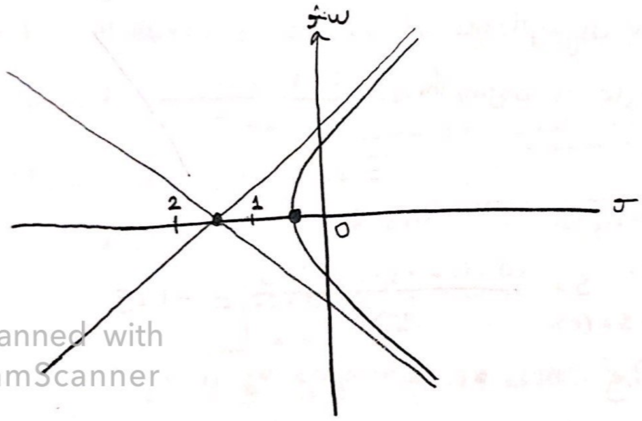
* The angle of departure of root loci from the complex poles are determined using the equations,

$\theta = 180 -$ Sum of angles made by all the remaining poles at the pole at which the angle is being calculated $+$ Sum of angles made by all zeros at the pole at which the angle is being calculated.

$$\theta = 180 - 153.43 - 135 - 90 + 0$$

$$\theta = -198.43^\circ$$

Angle of departure from the open-loop pole at $s = -2 + j$



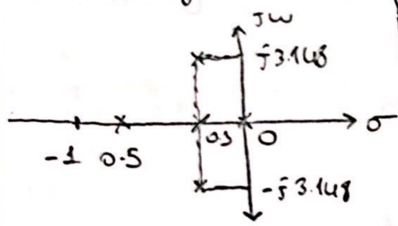
Question 4) Plot the root loci for the system with

$$G(s) = \frac{K}{s(s+0.5)(s^2+0.6s+10)}, \quad H(s)=1$$

Solution: Open loop poles are, $s = 0, -0.5, -0.3 \mp j3.148$

Open loop zeros are absent.

* The number of root loci branches is four, starts at open loop poles and ends at infinity.



Root loci exist on the negative real axis between 0 and -0.5

Determine angle of asymptotes of the root loci

$$\theta = \frac{180(2k-1)}{4-0} \quad k=0,1,2,3 \rightarrow \theta = 45, 135, 225, 315$$

$$s = -\frac{0+0.5+0.3+0.3-0}{4-0} = \underline{\underline{-0.275}}$$

Determine breakaway point.

$\rightarrow 1+G(s)H(s)=0$

$\rightarrow s(s+0.5)(s^2+0.6s+10)+K=0 \rightarrow (s^2+0.5s)(s^2+0.6s+10)+K=0$

$\rightarrow s^4+1.1s^3+10.3s^2+5s+K=0 \rightarrow K=-(s^4+1.1s^3+10.3s^2+5s)$

$\rightarrow \dfrac{dK}{ds} = -(4s^3+3.3s^2+20.6s+5)=0 \rightarrow$ TO find breakaway point it should be equal to zero.

$\rightarrow \boxed{s=-0.2496}, \; s=-0.2876 \cancel{\mp} j2.2189$ it is not valid because it is complex

(Because point $s=-0.2496$ lies between two poles, it is breakaway point

$s^4+1.1s^3+10.3s^2+5s+K=0$

$\downarrow$

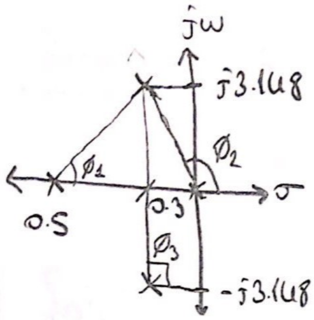
s^4	1	10.3	K
s^3	1.1	5	0
s^2	5.754	K	0
s^1	$\frac{28.77-1.1K}{5.754}$	0	0
s^0	K	0	0

$\rightarrow \dfrac{28.77-1.1K}{5.754} > 0$ and $K>0$

$\rightarrow 0 < K < 26.15 \rightarrow$ The crossover point with imaginary axis can be find by s^2

$\rightarrow 5.754s^2+K=0$

$\rightarrow \boxed{s = \mp j2.1318}$



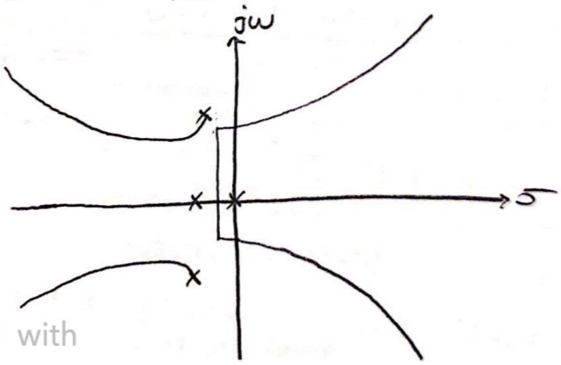
* Join all other poles to this pole and measure the angles.

$\phi_1 = \tan^{-1}\left[\frac{3.148}{0.5-0.3}\right] = 86.364^\circ$

$\phi_2 = 180 - \tan^{-1}\left[\frac{3.148}{0.3}\right] = 95.44^\circ$

$\phi_3 = 90^\circ$

* $\phi_d = 180^\circ - 86.364^\circ - 95.44^\circ - 90^\circ + 0^\circ = -91.804^\circ$
* Root locus branch leaving pole $s = -0.3 + j3.148$ will depart tangentially to the line whose angle is $\phi_d = -91.804^\circ$
* for $s = -0.3 - j3.148 \longrightarrow \phi_d = +91.804^\circ$

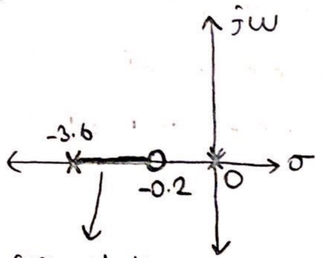


Question-5) Plot the root loci for a closed-loop control system with

$$G(s) = \frac{K(s+0.2)}{s^2(s+3.6)}, \quad H(s) = 1$$

Solution:

* Poles are 0, 0, −3.6
* Zeros are −0.2



represent the existance of root path

Angle of asymptotes $= \dfrac{\mp 180(2K+1)}{3-1}$ $\quad K = 0-1$ (3−1=2 numbers)

$\boxed{\theta = \mp 90^\circ, \mp 270^\circ}$

$$\sigma = \frac{-3.6-0-0-(-0.2)}{3-1} = \underline{\underline{-1.7}}$$

$$1 + \frac{K(s+2)}{s^2(s+3.6)} = \boxed{s^3+3.6s^2+Ks+0.2K=0}$$

characteristic equation

s^3	1	K
s^2	3.6	$0.2K$
s^1	$\frac{3.4K}{3.6}$	0
s^0	$0.2K$	0

$\rightarrow \dfrac{3.4K}{3.6} = 0$ and $K = 0$

* The auxilary equation the left Routh array given by

$3.6s^2 + 0.2K = 0$

put $K=0$ and $s = jw \rightarrow w = 0$

* At point $w=0$, the root locus is tangent to the jw axis because of the presence of a double pole at the origin. Therefore, there are no point where the root locus branches cross the imaginary axis.

* $s^3 + 3.6s^2 + Ks + 0.2K = 0$

* $K = -\dfrac{(s^3+3.6s^2)}{s+0.2}$

* $\dfrac{dK}{ds} = -\dfrac{(s+0.2)(3s^2+7.2s)-(s^3+3.6s^2)}{(s+0.2)^2}$

* $\dfrac{dK}{ds} = 0 \longrightarrow s(s+1.6685)(s+0.4315) = 0$

$s = 0$
$s = -1.6685$
$s = -0.4315$