Practical Measurement of Voltage-Controlled Current Source Output Impedance for Applications in Transcranial Electrical Simulation

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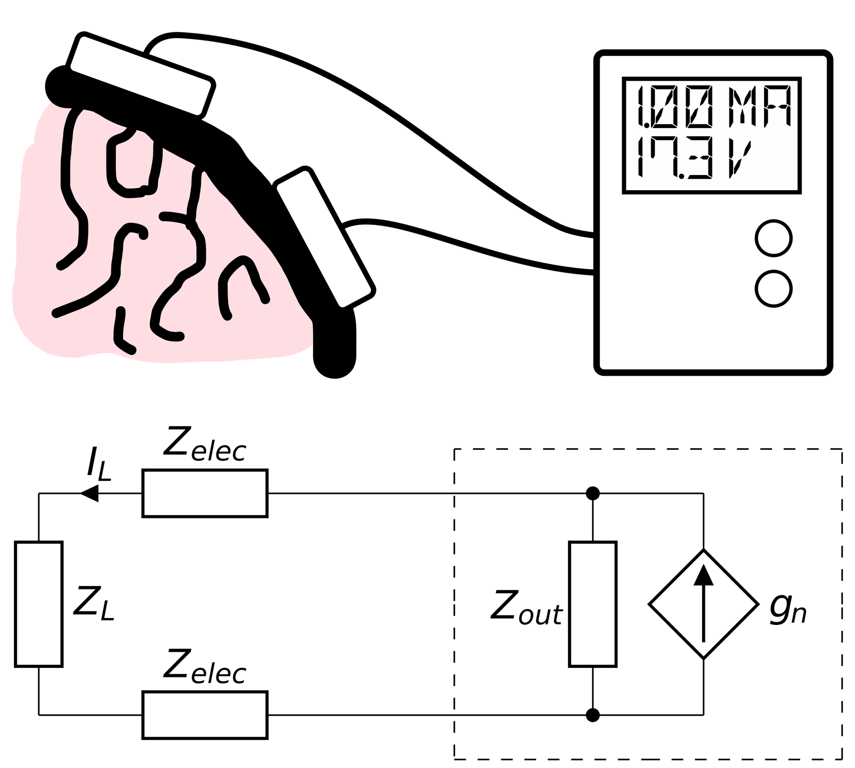
Abstract

A voltage-controlled current source (VCCS) is a key component in the electrical stimulation of neuronal tissue. We present a method for the evaluation of output impedance across frequency, of VCCS circuits with applications in transcranial electrical stimulation (tES).

Two VCCS circuit architectures, one with a single-ended, and one with a differential output, are theoretically analysed and simulated, and the method is validated by matching these results with empirical measurements. Monte Carlo simulation of the measurement method itself is used to predict its range of applicability under real-world conditions.

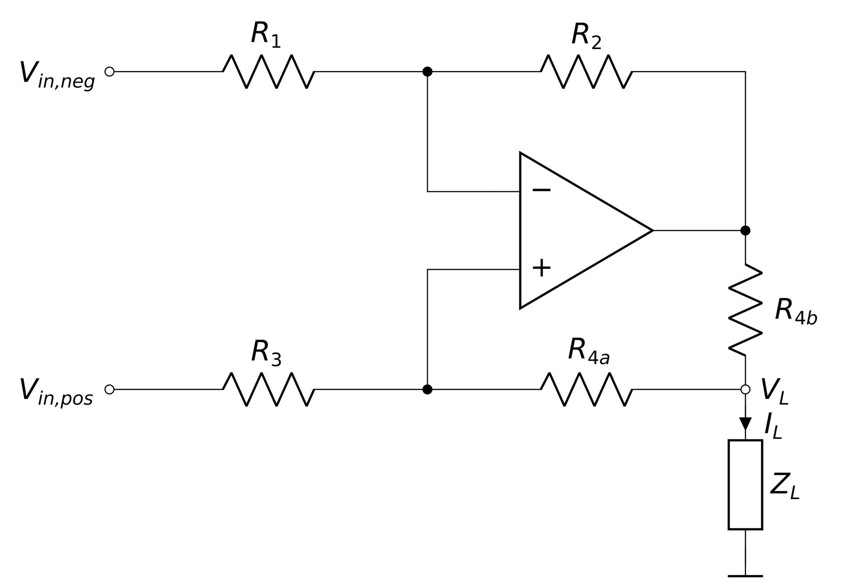
Both circuits, as well as the method used to determine output impedance, are easy to reproduce, using generic hardware components at low cost. The method described has immediate practical application value in neuroscience research, where it can be used in the deployment and further development of VCCS circuit architectures.



Situation sketch (top panel) and equivalent circuit (bottom panel). In a typical tES session, the device is adjusted to supply a given current I_L into a load Z_L via two electrodes, each with interfacial impedance Z_{elec}. We assume the device is battery powered or in any case galvanically isolated; the ground node is thus elided in the bottom panel.

Analysis

Single-ended "improved" Howland source:



Under the conditions:

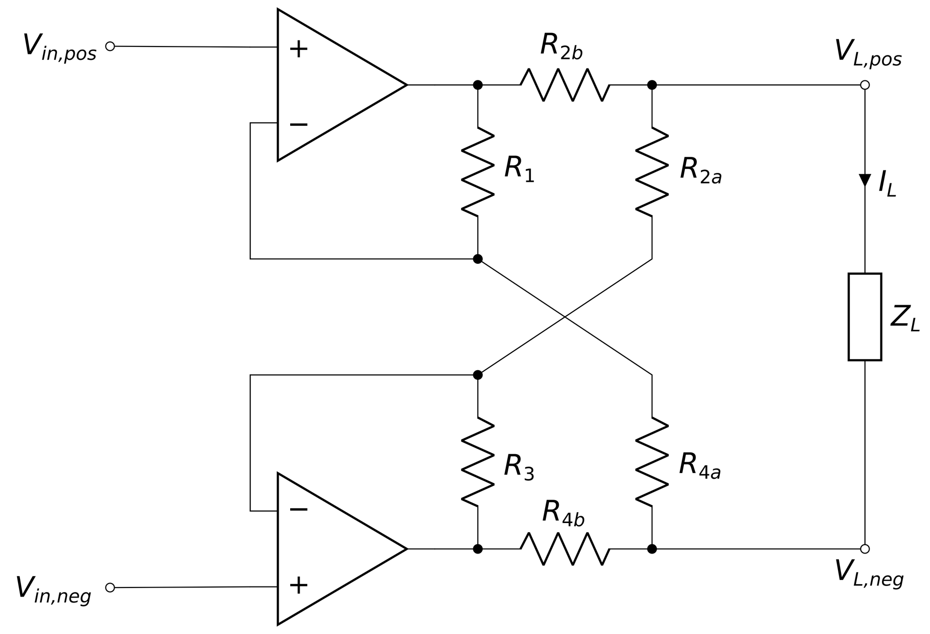
$$R_1 = R_3$$
$$R_2 = R_{4a} + R_{4b}$$

The transfer function is given by:

$$I_L = g_m(V_{in,pos} - V_{in,neg})$$

with $g_m = R_2 \,/\, (R_3 R_{4b})$.

Fully differential "improved" Howland source:



Under the conditions:

$$R_{2a} = R_{4a}$$
$$R_{2b} = R_{4b}$$
$$R_1 = R_3 = R_{2a} + R_{2b}$$

The transfer function is given by:

$$I_L = g_m(V_{in,pos} - V_{in,neg})$$

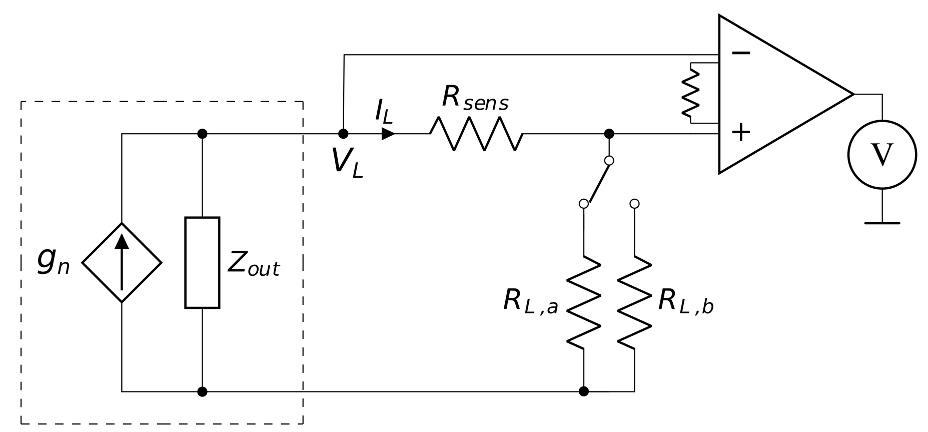
with $g_m = 1 \,/\, R_{2b}$.

Measurement method

Definition of output impedance:

$$Z_{out} = \frac{\partial V_L}{\partial I_L}$$

Method for determining output impedance:



The dashed box indicates the non-ideal VCCS, which has a finite output impedance Z_{out}. The ground node is assumed internal to g_n. When load resistors $R_{L,a}$ and $R_{L,b}$ are alternately connected, and the voltage (resp. current) in each case is measured as $V_{L,a}$ and $V_{L,b}$ (resp. $I_{L,a}$ and $I_{L,b}$), the output impedance is given by:

$$Z_{out} = \frac{R_{L,a}\, R_{L,b}\,(V_{L,b} - V_{L,a})}{V_{L,a}\, R_{L,b} - V_{L,b} R_{L,a}} = \frac{I_{L,b}\, R_{L,b} - I_{L,a}\, R_{L,a}}{I_{L,a} - I_{L,b}}$$

Although both equations require the measurement of a complex load voltage (or current), in practice only the (scalar) magnitude is available with sufficient accuracy and precision. If we substitute magnitudes, we obtain an expression for a new, real-valued quantity $\hat{Z}_{out}$, equal to the true magnitude $|Z_{out}|$ at DC.

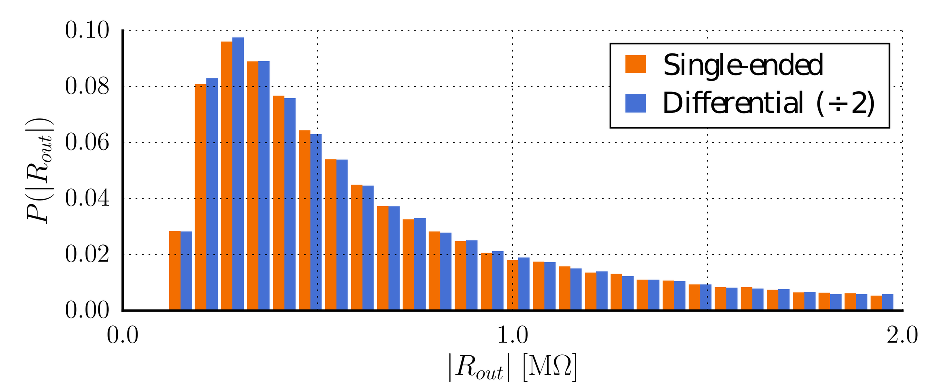
$$\hat{Z}_{out} = \frac{R_{L,a}\, R_{L,b}\,(|V_{L,b}| - |V_{L,a}|)}{|V_{L,a}| R_{L,b} - |V_{L,b}| R_{L,a}} = \frac{|I_{L,b}| R_{L,b} - |I_{L,a}| R_{L,a}}{|I_{L,a}| - |I_{L,b}|}$$

The quantity $\hat{Z}_{out}$ can be used to determine the full, complex output impedance Z_{out}, under the assumption that Z_{out} can be modeled as a resistor R_{out} in parallel with a capacitor C_{out}. In this case, $|V_L|$ can be written as:

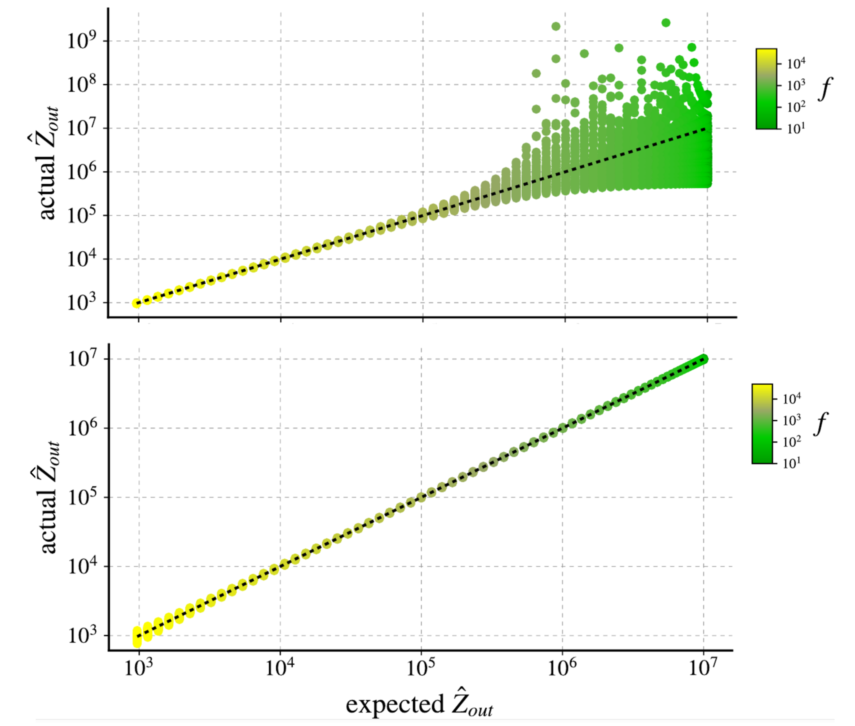
$$|V_L| = g_m(V_{in,pos} - V_{in,neg})\,\frac{R_L \| R_{out}}{\sqrt{1 + (\omega (R_L \| R_{out}) C_{out})^2}}$$

Simulation results

Monte Carlo simulation of absolute DC output resistance for a resistor tolerance of 0.1% and ideal op-amps shows a factor 2 improvement of the differential with respect to the single-ended circuit. Note that the output resistance of the differential VCCS is scaled by a factor ½ in this plot to allow easier visual comparison.



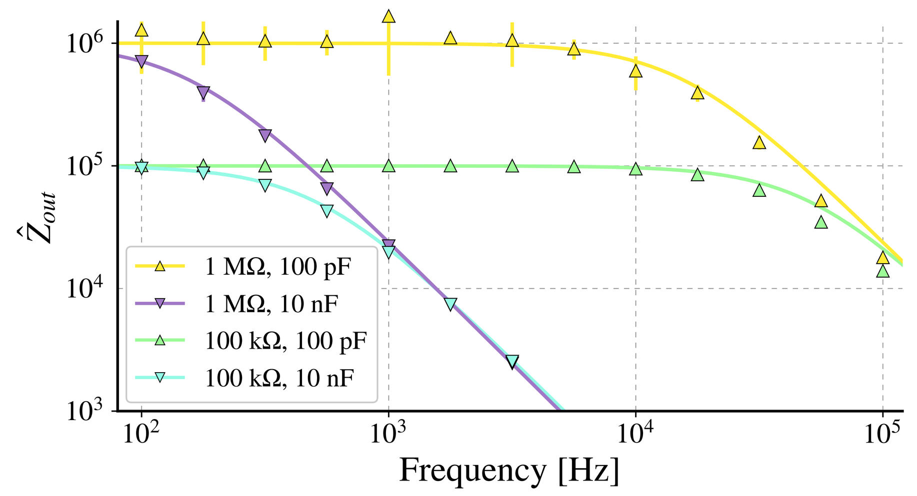
Measurement artefact is introduced by tolerance in the probe resistors. Sensitivity of the measured Z_{out} to tolerance of the probe resistors $R_{L,a}$ and $R_{L,b}$ is plotted for the voltage-based method (top) and current-based method (bottom). For each point, colour indicates the frequency at which it was measured; impedance drops at higher frequencies due to the presence of C_{out}.



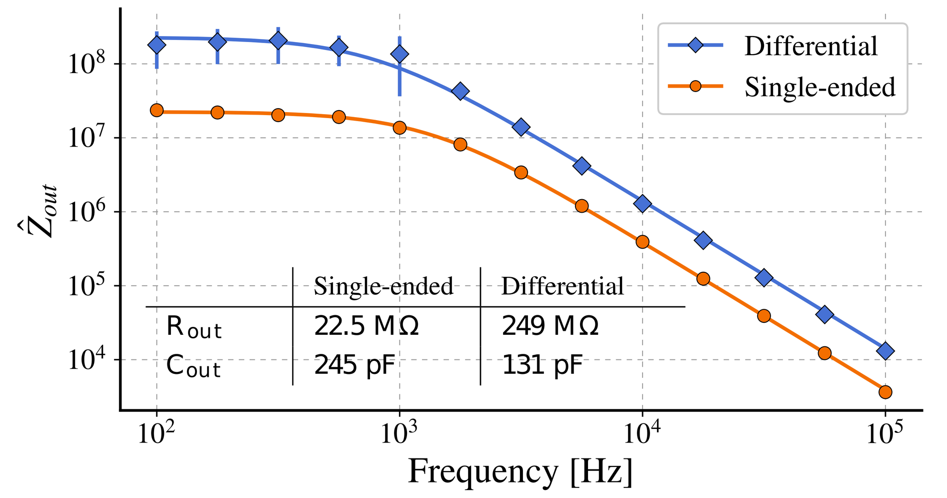
The voltage-based method (top panel) is most sensitive to variations in probe resistor values in the region of highest output impedance. This makes it an unsuitable method for measuring the output impedance of a current source. The current-based method (bottom panel) exhibits sensitivity only at values of $\hat{Z}_{out}$ that are so low as to be outside the range of interest for a VCCS.

Empirical results

The method was first validated using a reference circuit with known values, where the Thévenin equivalent of a current source was constructed using a voltage source in series with a resistor (representing R_{out}) and a capacitor (representing C_{out}). Results show good agreement between empirical and actual values.



The method was then used on the VCCS circuits, and R_{out} and C_{out} were fitted to the data points for each circuit. Data points were well fit with the RC model of VCCS output impedance. Theory predicts a factor 2 improvement in output resistance and capacitance for the differential with respect to the single-ended circuit.



	Single-ended	Differential
R_{out}	22.5 MΩ	249 MΩ
C_{out}	245 pF	131 pF

Conclusion

Only the current-based method for measuring VCCS output impedance has practical application value.

We can infer the full, complex-valued Z_{out} on the basis of the proxy measurement of $\hat{Z}_{out}$.

The method is suitable for circuits with a fully differential output, and was able to provide reliable data up to about 100 MΩ at frequencies up to 100 kHz.

IEEE BioCAS 2021 Berlin
Biomedical Circuits and Systems Conference
Virtual Conference | October 6-9, 2021

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