CURVES

Part 1

① What is Curve? And it's Representations...

Explicit , Implicit & Parametric Representation.

* ② Parametric Curves.

③ Parametric & Geometric Continuity.

$\underbrace{\text{Parametric \& Geometric Continuity}}$ → How much smooth a curve is, What is its order. How much continuous it is.

④ Spline Curve → How to create & design curves.

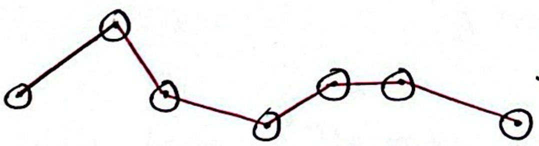
* * * ⑤ Hermite Spline Curve

* * * * * ⑥ Bezier Spline Curve

(i) Curves & Representations.

Curves are: A set of points draw with a pen.
A set of finite / infinite points, when joined together will result in a curve.

Eg.1



→ Curve

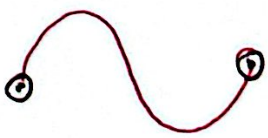
The more number of points within the start and end point. The more smoother.

Eg.2

→ Is this a curve?

→ minimum 2 points.

But when we imagine a curve we think of this:



∅ Anything drawn by joinining multiple lines are they curves? → Yes! Circles?

What about the area bounded by a circle → "Region"

Representations ~

(i) Explicit Form
(ii) Implicit Form
(iii) Parametric Form.

∅ Explicit form :

$y = f(x)$ — x is independent variable

dependent variable — y

$y = \underbrace{mx + c}_{f(x)}$

∅ Single valued function (for y)

∅ For vertical lines $x = d$?
No way to represent using single valued function

∅ Implicit form :

$f(x, y) = 0$

ex: eqn of a circle →

$\underbrace{x^2 + y^2 - r^2}_{f(x,y)} = 0$

∅ y can be multiple valued function of x
1 value of x → 2 values of y

∅ Continuity hard to detect.

∅ Parametric form : {The most convinient form}

$x = f_x(t)$
$y = f_y(t)$
$z = f_z(t)$

t → Parameter (t)

f_z → function for z with parameter t.

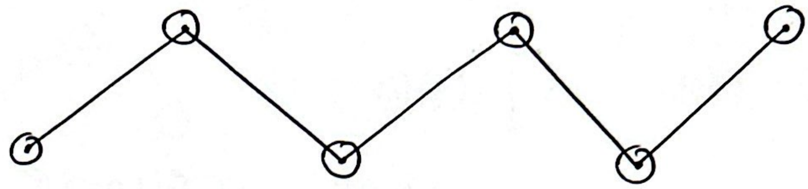
ex: $x = x_0 + t(x_1 - x_0)$
$y = y_0 + t(y_1 - y_0)$

∅ Easy to specify, modify & control.

ii) Parametric Curve

For any geometrical figure to be drawn, we need points.

Piecewise Linear Curve:



Ø The mathematical eqn will be in linear form. $y = mx + c$

Does this look like a curve?

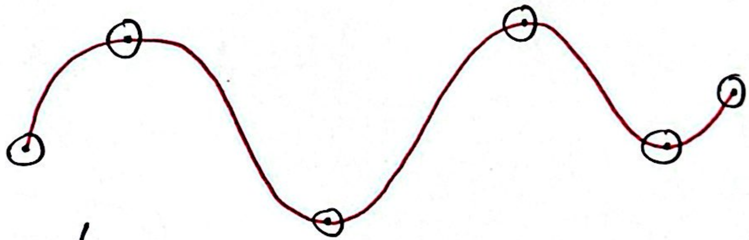
→ By definitions yes, but by looks no.

(*) → we need bends within our curve.

Piecewise Polynomial Curve:

→ The eqn won't be linear. Rather: $x^3 + x^2 + x + 5$

$x^3 \to x^4 \to x^5 \to x^6$ can be anything.



Say an equation of a curve → $x^3 + x^2 + 6$

degree (3) = 4 points used

" (4) = 5 " "

" (2) = 3 " "

$y = mx + c$ → [" (1) = 2 " "] → line.

So, in this example: 6 points

5th degree polynomial, the more number of points the higher the degree will be. And as such, the computational complexity increases.

Ø If we decrease the number of points → will look more like a line

Ø If we increase the number of points → complexity increases.

So, the **best** option is to stick to curves drawn using 4 points $\rightarrow$ x^{3}

{ Cubic polynomial Curve ? }

$\hookrightarrow$ eqn : $Q(t)$

$$Q(t) = [x(t) \quad y(t) \quad z(t)]$$

where,

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$

general form

similarly,

$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$

$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

(Can be converted to matrix form.)

t matrix

$$T = [t^3 \quad t^2 \quad t \quad 1]$$

coefficient matrix

$$C = \begin{bmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \\ d_x & d_y & d_z \end{bmatrix}$$

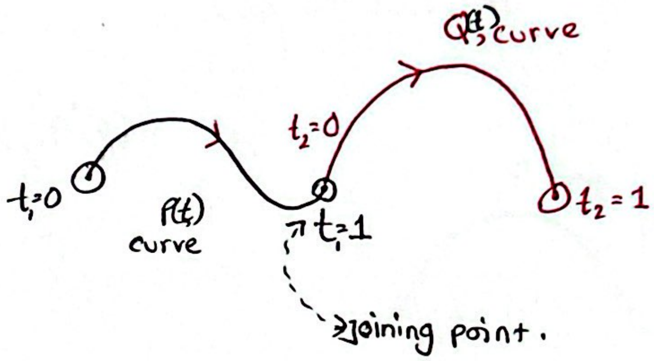
$$Q(t) = T \cdot C$$

(*) Since higher degree curves are computationally complex we can divide them into smaller manageable curves of degree 3. These smaller curves are called Splines. And join them.

(iii) Parametric & Geometric Continuity

Continuity : How smooth a curve is.

Parametric Continuity :- Deals with parametric eqn. We basically compare the parametric eqn's associated with two curves joined at a point to check how much continuous they are in a parametric way.



C = parametric continuity

G = Geometric continuity.

1) Zero order parametric continuity : (C^0)

if the point where curve P ends is the same as the point where curve Q starts. Then they follow zero order parametric continuity.

So, if.

$P(1) = Q(0)$ → respective values of t.

Hence, it is C^0 → zero order.

2) first order parametric continuity : (C^1)

So,

$P'(1) = Q'(0)$

&

$P(1) = Q(0)$

Take first curve P's derivative from its end point $t_1=1$. Then take the second curves Q's derivative from its starting point $t_2=0$. If they match, then they follow first order continuity.

3) Second order parametric Continuity : (C^2)

$$P'(1) = Q'(0) \;\&\; P''(1) = Q''(0) \;\&\; P(1) = Q(0)$$

C^n = if the derivatives match through to n^{th} derivative.

> **Note:** The higher the order of Parametric continuity, the smoother the curve will look.

Ø Geometric Continuity : How continuse two curves are in a geometric (shape) way.

1) Zero order geometric Continuity : G^0

$P(1) = Q(0)$ → Condition same as parametric continuity.

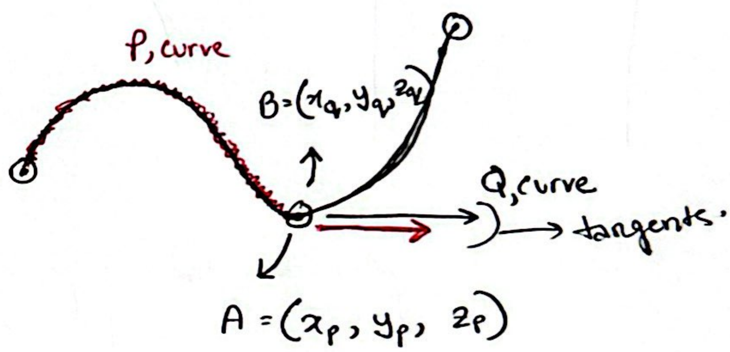
2) First order geometric Continuity : G^1

if the first curve P's tangent at $t_1 = 1$ is proportional (can be equal) to curve Q's tangent at $t_2 = 0$.

So,

$$A'(1) = K(B'(0))$$

K → Constant, since it is proportional.



So, if two curves are :

$$C^1 \Rightarrow G^1$$

if two curves are C^1 continuos they must be G^1 continuos

But,

$$G^1 \not\Rightarrow C^1$$

if two curves are G^1 continuos they might not necessarily be C^1 continuos.

we can write like this as well:

$$A'(x_p, y_p, z_p) = K\, B'(x_q, y_q, z_q)$$

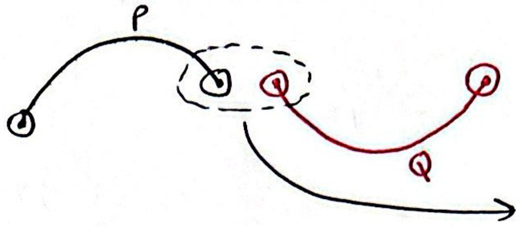
K → may or may not be equal, but must be proportional.

3) Second order geometric Continuity : G^2

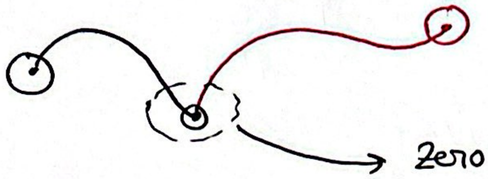
if $A''(1) = K(B''(0))$

→ if they are proportional, then we can say they are 2nd order Geometric continuos.

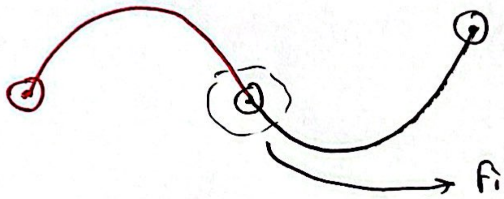
Examples :



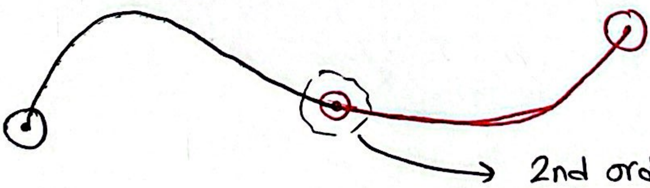
No continuity.



Zero order continuity. (same point but not smooth)



first order continuity. (little smoother)



2nd order continuity (a bit more smoother)