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## Chapter 1

### **Functions**

- 1.1 arygcd binary-like gcd algorithms
- 1.1.1 bit num the number of bits

```
bit\_num(a: integer) \rightarrow integer
```

Return the number of bits for a

#### 1.1.2 binarygcd – gcd by the binary algorithm

```
binarygcd(a: integer, b: integer) \rightarrow integer
```

Return the greatest common divisor (gcd) of two integers a, b by the binary gcd algorithm.

#### 1.1.3 arygcd i – gcd over gauss-integer

```
arygcd_i(a1: integer, a2: integer, b1: integer, b2: integer) \rightarrow (integer, integer)
```

Return the greatest common divisor (gcd) of two gauss-integers a1+a2i, b1+b2i, where "i" denotes the imaginary unit.

If the output of  $arygcd_i(a1, a2, b1, b2)$  is (c1, c2), then the gcd of a1+a2i and b1+b2i equals c1+c2i.

†This function uses (1+i)-ary gcd algorithm, which is an generalization of the binary algorithm, proposed by A.Weilert[2].

### 1.1.4 arygcd w – gcd over Eisenstein-integer

```
\begin{array}{c} \mathtt{arygcd\_w(a1:} \ integer, \ \mathtt{a2:} \ integer, \ \mathtt{b1:} \ integer, \ \mathtt{b2:} \ integer) \\ \longrightarrow (integer, \ integer) \end{array}
```

Return the greatest common divisor (gcd) of two Eisenstein-integers  $\mathtt{a1}+\mathtt{a2}\omega$ ,  $\mathtt{b1}+\mathtt{b2}\omega$ , where " $\omega$ " denotes a primitive cubic root of unity.

If the output of arygcd\_w(a1, a2, b1, b2) is (c1, c2), then the gcd of a1+a2 $\omega$  and b1+b2 $\omega$  equals c1+c2 $\omega$ .

†This functions uses  $(1-\omega)$ -ary gcd algorithm, which is an generalization of the binary algorithm, proposed by I.B. Damgård and G.S. Frandsen [1].

#### Examples

```
>>> arygcd.binarygcd(32, 48)
16
>>> arygcd_i(1, 13, 13, 9)
(-3, 1)
>>> arygcd_w(2, 13, 33, 15)
(4, 5)
```

## Bibliography

- [1] Ivan Bjerre Damgard and Gudmund Skovbjerg Frandsen. Efficient algorithms for the gcd and cubic residuosity in the ring of eisenstein integers. Journal of Symbolic Computation, Vol. 39, No. 6, pp. 643–652, 2005.
- [2] André Weiler. (1+i)-ary gcd computation in  $\mathbb{Z}[i]$  as an analogue to the binary gcd algorithm. *Journal of Symbolic Computation*, Vol. 30, No. 5, pp. 605–617, 2000.