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# Chapter 1

## Classes

### 1.1 `real` – real numbers and its functions

The module `real` provides arbitrary precision real numbers and their utilities. The functions provided are corresponding to the `math` standard module.

- **Classes**

- `RealField`
- `Real`
- `†Constant`
- `†ExponentialPowerSeries`
- `†AbsoluteError`
- `†RelativeError`

- **Functions**

- `exp`
- `sqrt`
- `log`
- `log1piter`
- `piGaussLegendre`
- `eContinuedFraction`
- `floor`
- `ceil`
- `trunc`
- `sin`
- `cos`

- **tan**
- **sinh**
- **cosh**
- **tanh**
- **asin**
- **acos**
- **atan**
- **atan2**
- **hypot**
- **pow**
- **degrees**
- **radians**
- **fabs**
- **fmod**
- **frexp**
- **ldexp**
- **EulerTransform**

This module also provides following constants:

**e** :  
     **e** is the base of the natural logarithm function, also called Napier’s constant.

**pi** :  
     **pi** is the circular constant, also denoted by  $\pi$ .

**Log2** :  
     **Log2** is the natural logarithm of 2.

**†defaultError** :  
     **defaultError** is the instance of **RelativeError**.

**theRealField** :  
     **theRealField** is the instance of **RealField**.

### 1.1.1 RealField – field of real numbers

The class is for the field of real numbers. The class has the single instance **theRealField**.

This class is a subclass of **Field**.

#### Initialize (Constructor)

**RealField()**  $\rightarrow$  *RealField*

Create an instance of RealField. You may not want to create an instance, since there is already **theRealField**.

#### Attribute

**zero** :  
It expresses The additive unit 0. (read only)

**one** :  
It expresses The multiplicative unit 1. (read only)

#### Operations

operator	explanation
<b>in</b>	membership test; return whether an element is in or not.
<b>repr</b>	return representation string.
<b>str</b>	return string.

## Methods

### 1.1.1.1 `getCharacteristic` – get characterisitic

`getCharacteristic(self)` → *integer*

Return the characteristic, zero.

### 1.1.1.2 `issubring` – subring test

`issubring(self, aRing: Ring)` → *bool*

Report whether another ring contains the real field as subring.

### 1.1.1.3 `issuperring` – superring test

`issuperring(self, aRing: Ring)` → *bool*

Report whether the real field contains another ring as subring.

### 1.1.2 Real – a Real number

Real is a class of real number. This class is only for consistency for other **Ring** object.

This class is a subclass of **CommutativeRingElement**.

All implemented operators in this class are delegated to Float type.

#### Initialize (Constructor)

**Real(value: *number*)**  $\rightarrow$  *Real*

Construct a Real object.

value must be int, long, Float or **Rational**.

## Methods

### 1.1.2.1 `getRing` – get ring object

`getRing(self)`  $\rightarrow$  *RealField*

Return the real field instance.



### 1.1.3 Constant – real number with error correction

Constant provides constant-like behavior for Float calculation context. It caches the constant value and re-computes for more precision by request.

Almost every operators are delegated to the cached value.

#### Initialize (Constructor)

```
Constant(getValue: numbers, err: Error=defaultError)  
→ Constant
```

Construct Constant from Float and error. `getValue` must be Float, and `err` must be **AbsoluteError** or **RelativeError**.

#### Operations

operator	explanation
(err)	Return the value at least as accurate as the given error <code>err</code> . .

#### Examples

```
>>> pi = Constant(piGaussLegendre)  
>>> print pi  
3.14159265358979  
>>> pi + 1  
4.14159265358979  
>>> pi(RelativeError(0,1,2**100)) # for 100 bit precision  
3.1415926535897932384626433832795
```

## Methods

### 1.1.3.1 `inverse` – inverse value

`inverse(self)` → *Constant*

Return the inverse of the number.

### 1.1.3.2 `toRational` – convert to Rational

`toRational(self)` → *Rational*

Return a rational number approximating the number.

### 1.1.4 ExponentialPowerSeries – exponential power series

ExponentialPowerSeries is a class for exponential power serieses, whose n-th term has form  $\frac{a_n x^n}{n!}$ .

#### Initialize (Constructor)

**ExponentialPowerSeries(iterator: *iterator*) → *ExponentialPowerSeries***

Construct an exponential power series with coefficient generated by the given `iterator`, which can be an infinite iterator.

#### Operations

operator	explanation
<code>(x,maxerror)</code>	Return the value of the series with <code>x</code> assigned. The maximum error <code>maxerror</code> must be given as a <b>RelativeError</b> or <b>AbsoluteError</b> instance.

#### Examples

```
>>> expo = ExponentialPowerSeries(itertools.cycle([1]))
>>> expo(.5, defaultError)
Rational(5434422938503507, 3296144130048000)
```

## Methods

### 1.1.4.1 terms – generator of terms of series

`terms(self, x: numbers )` → *ExponentialPowerSeries*

Generator of terms of series with assigned x value. x must be int, long or Float.

### **1.1.5 AbsoluteError – absolute error**

AbsoluteError is the class of absolute error of real numbers.  
this class is deprecated.

### **1.1.6   RelativeError – relative error**

AbsoluteError is the class of relative error of real numbers.  
this class is deprecated.

### 1.1.7 `exp(function)` – exponential value

`exp(x: number, err: Error=defaultError) → number`

Return exponential of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.8 `sqrt(function)` – square root

`sqrt(x: number, err: Error=defaultError) → number`

Return square root of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.9 `log(function)` – logarithm

`log(x: number, base: number=None, err: Error=defaultError)  
→ number`

Return logarithm of a positive number x.

If an additional argument `base` is given, it returns logarithm of x to the base.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.10 `log1piter(function)` – iterator of `log(1+x)`

`log1piter(xx: number) → iterator`

Return iterator for `log(1 + x)`.

### 1.1.11 `piGaussLegendre(function)` – pi by Gauss-Legendre

`piGaussLegendre(err: Error=defaultError) → number`

Return pi by Gauss-Legendre algorithm.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.12 `eContinuedFraction(function)` – Napier's Constant by continued fraction expansion

`eContinuedFraction(err: Error=defaultError) → number`

Return the base of natural logarithm **e** by continued fraction expansion.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.13 floor(function) – floor the number

**floor**(x: *number*) → *integer*

Return the biggest integer not more than x.

### 1.1.14 ceil(function) – ceil the number

**ceil**(x: *number*) → *integer*

Return the smallest integer not less than x.

### 1.1.15 trunc(function) – round-off the number

**trunc**(x: *number*) → *integer*

Return the number of rounded off x.

### 1.1.16 sin(function) – sine function

**sin**(x: *number*, err: *Error*=defaultError) → *number*

Return the sine of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.17 cos(function) – cosine function

**cos**(x: *number*, err: *Error*=defaultError) → *number*

Return the cosine of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.18 tan(function) – tangent function

**tan**(x: *number*, err: *Error*=defaultError) → *number*

Return the tangent of x.

err must be **AbsoluteError** or **RelativeError**.



### 1.1.19 `sinh(function)` – hyperbolic sine function

`sinh(x: number, err: Error=defaultError) → number`

Return the hyperbolic sine of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.20 `cosh(function)` – hyperbolic cosine function

`cosh(x: number, err: Error=defaultError) → number`

Return the hyperbolic cosine of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.21 `tanh(function)` – hyperbolic tangent function

`tanh(x: number, err: Error=defaultError) → number`

Return the hyperbolic tangent of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.22 `asin(function)` – arc sine function

`asin(x: number, err: Error=defaultError) → number`

Return the arc sine of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.23 `acos(function)` – arc cosine function

`acos(x: number, err: Error=defaultError) → number`

Return the arc cosine of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.24 `atan(function)` – arc tangent function

`atan(x: number, err: Error=defaultError) → number`

Return the arc tangent of x.

err must be **AbsoluteError** or **RelativeError**.

### 1.1.25 atan2(function) – arc tangent function

```
atan2(y: number, x: number, err: Error=defaultError)  
    → number
```

Return the arc tangent of y/x.

Unlike **atan**(y/x), the signs of both x and y are considered.

‡It is unrecomended to obtain the value of **pi** with atan2(0,1). **err** must be **AbsoluteError** or **RelativeError**.

### 1.1.26 hypot(function) – euclidian distance function

```
hypot(x: number, y: number, err: Error=defaultError)  
    → number
```

Return  $\sqrt{x^2 + y^2}$ .

**err** must be **AbsoluteError** or **RelativeError**.

### 1.1.27 pow(function) – power function

```
pow(x: number, y: number, err: Error=defaultError)  
    → number
```

Return yth power of x.

**err** must be **AbsoluteError** or **RelativeError**.

### 1.1.28 degrees(function) – convert angle to degree

```
degrees(rad: number, err: Error=defaultError) → number
```

Converts angle rad from radians to degrees.

**err** must be **AbsoluteError** or **RelativeError**.

### 1.1.29 radians(function) – convert angle to radian

```
radians(deg: number, err: Error=defaultError) → number
```

Converts angle deg from degrees to radians.

**err** must be **AbsoluteError** or **RelativeError**.

### 1.1.30 fabs(function) – absolute value

```
fabs(x: number) → number
```

Return absolute value of x

### 1.1.31 `fmod(function)` – modulo function over real

`fmod(x: number, y: number) → number`

Return  $x - ny$ , where  $n$  is the quotient of  $x / y$ , rounded towards zero to an integer.

### 1.1.32 `frexp(function)` – expression with base and binary exponent

`frexp(x: number) → (m,e)`

Return a tuple  $(m,e)$ , where  $x = m \times 2^e$ ,  $1/2 \leq \text{abs}(m) < 1$  and  $e$  is an integer.

†This function is provided as the counter-part of `math.frexp`, but it might not be useful.

### 1.1.33 `ldexp(function)` – construct number from base and binary exponent

`ldexp(x: number, i: number) → number`

Return  $x \times 2^i$ .

### 1.1.34 `EulerTransform(function)` – iterator yields terms of Euler transform

`EulerTransform(iterator: iterator) → iterator`

Return an iterator which yields terms of Euler transform of the given `iterator`.

# Bibliography