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Chapter 1

Classes

1.1 quad – Quadratic Form

- Classes
 - **ReducedQuadraticForm**
 - **ClassGroup**
- Functions
 - **class_formula**
 - **class_number**
 - **class_group**
 - **class_number_bsgs**
 - **class_group_bsgs**

1.1.1 ReducedQuadraticForm – Reduced Quadratic Form Class

Initialize (Constructor)

ReducedQuadraticForm(f: list, unit: list) → *ReducedQuadraticForm*

Create ReducedQuadraticForm object.

f, unit must be list of 3 integers [a, b, c], representing a quadratic form $ax^2 + bxy + cy^2$. unit represents the unit form.

Operations

operator	explanation
$M * N$	Return the composition form of M and N .
$M ** a$	Return the a -th powering of M .
M / N	Division of form.
$M == N$	Return whether M and N are equal or not.
$M != N$	Return whether M and N are unequal or not.

Methods

1.1.1.1 inverse

`inverse(self)` → *ReducedQuadraticForm*

Return the inverse of `self`.

1.1.1.2 disc

`disc(self)` → *ReducedQuadraticForm*

Return the discriminant of `self`.

1.1.2 ClassGroup – Class Group Class

Initialize (Constructor)

`ClassGroup(disc: integer, cl: integer, element: integer=None)`
→ *ClassGroup*

Create ClassGroup object.

Methods

1.1.3 class_formula

class_formula(*d*: *integer*, uprbd: *integer*) → *integer*

Return the approximation of class number h with discriminant d using class formula.

$$\text{class formula } h = \frac{\sqrt{|d|}}{\pi} \prod_p \left(1 - \left(\frac{d}{p} \right) \frac{1}{p} \right)^{-1}.$$

Input number d must be int, long or **Integer**.

1.1.4 class_number

class_number(*d*: *integer*, limit_of_d: *integer*=1000000000) → *integer*

Return the class number with the discriminant d by counting reduced forms.

d is not only fundamental discriminant.

Input number d must be int, long or **Integer**.

1.1.5 class_group

class_group(*d*: *integer*, limit_of_d: *integer*=1000000000) → *integer*

Return the class number and the class group with the discriminant d by counting reduced forms.

d is not only fundamental discriminant.

Input number d must be int, long or **Integer**.

1.1.6 class_number_bsgs

class_number_bsgs(*d*: *integer*) → *integer*

Return the class number with the discriminant d using Baby-step Giant-step algorithm.

d is not only fundamental discriminant.

Input number d must be int, long or **Integer**.

1.1.7 class_group_bsgs

```
class_group_bsgs(d: integer, cl: integer, qin: list)
    → integer
```

Return the construction of the class group of order p^{exp} with the discriminant $disc$, where $qin = [p, exp]$.

Input number d , cl must be int, long or **Integer**.

Examples

```
>>> quad.class_formula(-1200, 100000)
12
>>> quad.class_number(-1200)
12
>>> quad.class_group(-1200)
(12, [ReducedQuadraticForm(1, 0, 300), ReducedQuadraticForm(3, 0, 100),
ReducedQuadraticForm(4, 0, 75), ReducedQuadraticForm(12, 0, 25),
ReducedQuadraticForm(7, 2, 43), ReducedQuadraticForm(7, -2, 43),
ReducedQuadraticForm(16, 4, 19), ReducedQuadraticForm(16, -4, 19),
ReducedQuadraticForm(13, 10, 25), ReducedQuadraticForm(13, -10, 25),
ReducedQuadraticForm(16, 12, 21), ReducedQuadraticForm(16, -12, 21)])
>>> quad.class_number_bsgs(-1200)
12L
>>> quad.class_group_bsgs(-1200, 12, [3, 1])
([ReducedQuadraticForm(16, -12, 21)], [[3L]])
>>> quad.class_group_bsgs(-1200, 12, [2, 2])
([ReducedQuadraticForm(12, 0, 25), ReducedQuadraticForm(4, 0, 75)],
[[2L], [2L, 0]])
>>> quad.reducePDF([1, 2, 3])
(1, 0, 2)
```

Bibliography