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Chapter 1

Functions

1.1 poly.groebner – Gröbner Basis

The groebner module is for computing Gröbner bases for multivariate polynomial ideals.

This module uses the following types:

polynomial :

`polynomial` is the polynomial generated by function `poly.multiutil.polynomial`.

order :

`order` is the order on terms of polynomials.

1.1.1 buchberger – naïve algorithm for obtaining Gröbner basis

`buchberger(generating: list, order: order) → [polynomials]`

Return a Gröbner basis of the ideal generated by given generating set of polynomials with respect to the order.

The argument **`generating`** is a list of `poly.multiutil.Polynomial`; the argument **`order`** is an order.

Be careful, this implementation is very naive.

1.1.2 normal_strategy – normal algorithm for obtaining Gröbner basis

`normal_strategy(generating: list, order: order) → [polynomials]`

Return a Gröbner basis of the ideal generated by given generating set of polynomials with respect to the order. This function uses the ‘normal strategy’.

The argument **`generating`** is a list of `poly.multiutil.Polynomial`; the argument

`order` is an order.

1.1.3 `reduce_groebner` – reduce Gröbner basis

`reduce_groebner(gbasis: list, order: order) → [polynomials]`

Return the reduced Gröbner basis constructed from a Gröbner basis. It satisfies that:

- $\text{lb}(f)$ divides $\text{lb}(g) \Rightarrow g$ is not in reduced Gröbner basis, and
- monic.

The argument `gbasis` is a list of polynomials, a Gröbner basis.

1.1.4 `s_polynomial` – S-polynomial

`s_polynomial(f: polynomial, g: polynomial, order: order)`
`→ [polynomials]`

Return S-polynomial of `f` and `g` with respect to the `order`.

$$S(f, g) = (\text{lc}(g) * T / \text{lb}(f)) * f - (\text{lc}(f) * T / \text{lb}(g)) * g,$$

where $T = \text{lcm}(\text{lb}(f), \text{lb}(g))$.