Inferring personal intake recommendations of phosphorous and potassium for end-stage renal failure patients by simulating with hierarchical Bayesian multivariate model

Supplementary Materials

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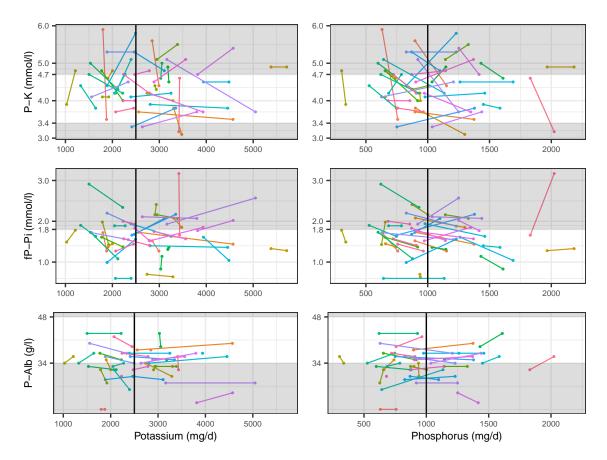
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This notebook reproduces in detail the analysis of personal diet recommendations described in the main article.

The notebook execution generates all the figures and tables that are included in the article and produces also the referenced supplementary figures. The article is accompanied by a PDF rendition of the notebook that shows all the supplementary figures and important parts of the program code so that the analysis can be followed in detail. The executable RMarkdown notebook with data can be found in a public GitHub repository of the corresponding author.

Dialysis patient data

Let us explore then how the intake of dietary potassium and phosphorus correspond in data to plasma concentrations of potassium, phosphate and albumin.



Supplementary Figure S 1: Figure shows the progress between two observations of plasma potassium (P-K), fasting plasma phosphate (fP-Pi) and plasma albumin (P-Alb) concentrations of the studied patients. White regions in the panels show the recommended concentration levels, P-K 3.4 - 4.7 mmol/l, fP-Pi < 1.8 mmol/l and P-Alb 34 - 48 g/l. Vertical black lines denote the commonly recommended maximum intakes of these nutrients. The goal is to find personal intake levels that keep the concentrations within recommended normal ranges, if possible. The figure is plotted with ggplot2 package for R language (v 3.3.5, https://ggplot2.tidyverse.org).

Aim of this analysis is to find such personal levels of potassium and phosphorous intakes that keep all these concentrations in their recommended levels marked with white regions, if possible.

Development of nutrition reaction model

In this work we construct personal generative models for concentrations that allow conditioning personal diet recommendations. Essential part of these models is the personal reactions to nutrients and other predictors. Simultaneous reactions on all considered concentrations are modeled as multivariate model that has all the concentrations are response variables.

For comparison, we estimate multivariate systems with and without cross-model covariance. With cross-model covariance estimated, the model corresponds to seemingly unrelated model system, and without the model is Bayesian network with separated local distributions. This more simple model is estimated first with only potassium and phosphorous concentrations as responses.

```
initial_graph <- mebn.fully_connected_bipartite_graph(datadesc_fat_epros)

pk_fppi_targets <- datadesc_fat_epros[datadesc_fat_epros$Name %in% c('pk','fppi'),]</pre>
```

In Bayesian network both responses were estimated separately. Next, we estimate a multivariate model where both distribution are estimated during single sampling. It does not factorize into separate distributions, but allows using more data.

Next we like to add plasma albumin concentration as a third constraint in the model, but unfortunately 8 of 37 patients have missing albumin measurements. From such a small dataset we don't want to remove any patients and so we predict the missing albumin levels and impute them to dataset.

For prediction, we estimate the model without missing concentrations in P-Alb. The rows with missing values are held out from the density estimation, but patients are kept in the model for estimating their parameters.

```
assumed_targets = pk_fppi_palb_targets,
group_column = "potilas",
local_estimation = mebn.multivariate_sampling,
local_model_cache =
    "models/BLMM_gamma_qr_multivariate3/fat_epros_missing_palb",
stan_model_file = "mebn/v2/BLMM_gamma_qr_mv_cv.stan",
normalize_values = TRUE)
```

Personal data imputation models

Now we can create personal models for patients that have missing plasma albumin measurements. These models are then used for making personal predictions for replacing missing values.

Then we predict missing values with these data imputing models

```
evidence = evidence,
                                  stan_model_file = "diet/posterior_prediction.stan")
  # personal_predictions contains predictions multivariate predictions
  # P-K and fP-Pi are known, but P-Alb is missing
  posterior <- rstan::extract(posterior_prediction, par= "posterior[3]")</pre>
  # use predicted posterior mean for missing P-Alb value
  # - this is i:th NA value in dialysis
  predicted_palb <- mean(posterior$`posterior[3]`)</pre>
  # rows in 'rows_with_missing_palb' and 'original_palb' are in same order
  # so we can imput in the NA values at 'original_palb'
  for (m in 1:imput_length)
      if (is.na(original_palb[m]))
          original_palb[m] <- predicted_palb</pre>
          break:
 }
}
dialysis2 <- dialysis
dialysis2$palb <- original palb
saveRDS(dialysis2, "data/DIALYSIS_imputed_palb.rds")
With these data imputations we can estimate a cross-covariance model with three responses
pk_fppi_palb_targets <- datadesc_fat_epros[datadesc_fat_epros$Name %in% c('pk','fppi','palb'),]
initial_graph <- mebn.fully_connected_bipartite_graph(datadesc_fat_epros)</pre>
dialysis_imputed <- readRDS("data/DIALYSIS_imputed_palb.rds")</pre>
no_holdout <- rep(0, nrow(dialysis_imputed))</pre>
dialdiet gamma mv3 <- mebn.bipartite multivariate(reaction graph = initial graph,
                                    inputdata = dialysis_imputed,
                                    targetdata = no_holdout,
                                    predictor_columns = assumedpredictors_fat_epros,
                                    assumed_targets = pk_fppi_palb_targets,
                                    group_column = "potilas",
                                    local_estimation = mebn.multivariate_sampling,
                                    local_model_cache =
                                      "models/BLMM_gamma_qr_multivariate3/imputed_palb",
                                    stan_model_file = "mebn/v2/BLMM_gamma_qr_mv_cv.stan",
                                    normalize_values = TRUE)
write.graph(dialdiet_gamma_mv3,
            "graphs/dialysis_gamma_multivariate3_imputed.graphml", "graphml")
```

Finally, we consider also the effect of dialysis treatment type as new level of grouping in data. This allows us to estimate average effects of nutrition for each treatment type and then also personal effects within those

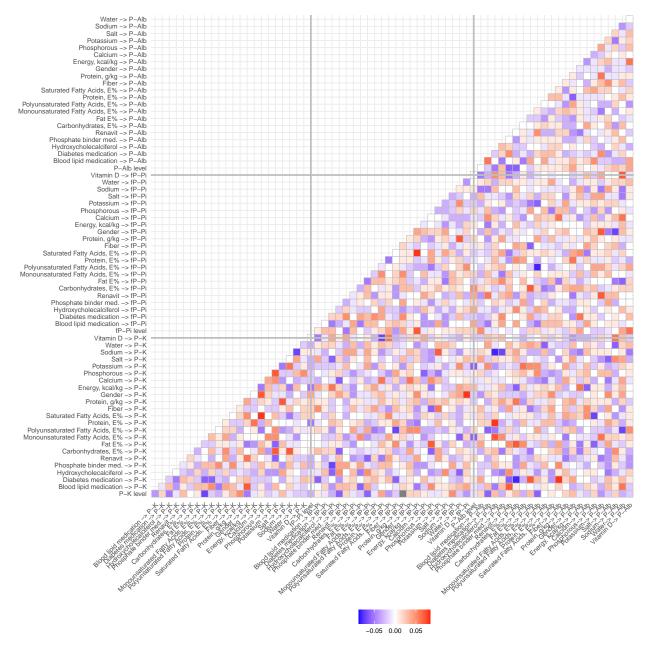
treatments.

```
pk_fppi_palb_targets <- datadesc_fat_epros[datadesc_fat_epros$Name %in% c('pk','fppi','palb'),]</pre>
no_holdout <- rep(0, nrow(dialysis_imputed))</pre>
initial_graph <- mebn.fully_connected_bipartite_graph(datadesc_fat_epros)</pre>
dialysis_imputed <- readRDS("data/DIALYSIS_imputed_palb.rds")</pre>
# add the dialysis treatment type as a grouping factor
dialysis_imputed$hoitoryhma <- as.factor(dialysis_imputed$hoitomuoto)</pre>
# and sort the data by treatment/patient/observation
dialysis imputed <- dialysis imputed[order(dialysis imputed$hoitoryhma,
                                            dialysis_imputed$potilas, dialysis_imputed$havainto),]
dialdiet_gamma_mv3_two_level <- mebn.bipartite_two_level_multivariate(</pre>
                                    reaction_graph = initial_graph,
                                    inputdata = dialysis_imputed,
                                    targetdata = no_holdout,
                                    predictor_columns = assumedpredictors_fat_epros,
                                    assumed_targets = pk_fppi_palb_targets,
                                    group_column = "hoitoryhma",
                                    subject_column = "potilas",
                                    local_estimation = mebn.two_level_multivariate_sampling,
                                    local model cache =
                                      "models/BLMM_gamma_mv_cross/two_levels",
                                    stan model file =
                                      "mebn/v2/BLMM_gamma_two_level_grouping.stan",
                                    normalize values = TRUE)
write.graph(dialdiet_gamma_mv3_two_level,
            "graphs/dialysis_gamma_two_level_grouping.graphml", "graphml")
```

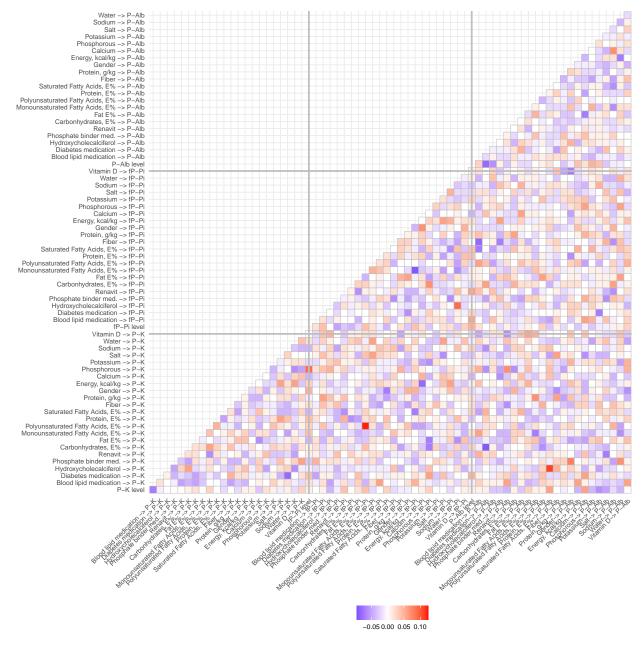
Model analysis

Here we look into the estimated correlation matrices of effects. The model estimates how much the effects of nutrients vary between different types of dialysis treatment and further how the effects vary personally within a treatment. These correlation plots show how the effects are correlated.

```
## Scale for fill is already present.
## Adding another scale for fill, which will replace the existing scale.
```



Supplementary Figure S 2: Correlation plot of matrix C_g that includes the estimated within-model and cross-model correlations in level of dialysis treatment



Supplementary Figure S 3: Correlation plot of matrix C_b that includes the estimated within-model and cross-model correlations of personal effects

Supplementary Table S 1: Table shows 40 highest positive or negative correlations between treatment effects of potassium and phosphorous with other treatment effects. This structure of correlations is used in estimating the personal effects based on personal intake and matching concentrations.

Treatment-level effects of nu		
Effect 1	Effect 2	Correlation
Renavit -> fP-Pi	Potassium-> P-Alb	0.074
Gender -> fP-Pi	Potassium-> P-Alb	0.071
Sodium -> P-K	Potassium -> fP-Pi	0.065
Protein, g/kg -> P-Alb	Potassium-> P-Alb	0.063
Sodium -> P-K	Potassium -> P-K	0.058
Calcium -> P-K	Potassium -> P-K	0.053
Monounsaturated Fatty Acids, E% -> P-K	Potassium -> P-K	0.052
Polyunsaturated Fatty Acids, E% -> P-Alb	Potassium -> P-K	0.050
Diabetes medication -> P-Alb	Potassium -> P-K	0.047
Vitamin D -> fP-Pi	Potassium-> P-Alb	0.045
Blood lipid medication -> fP-Pi	Potassium -> fP-Pi	0.044
Phosphorous -> P-Alb	Potassium-> P-Alb	0.043
Polyunsaturated Fatty Acids, E% -> fP-Pi	Potassium -> P-K	-0.042
Salt -> P-K	Potassium-> P-Alb	-0.043
Hydroxycholecalciferol -> P-K	Potassium -> P-K	-0.045
Sodium -> fP-Pi Sodium -> P-Alb Monounsaturated Fatty Acids, E% -> P-Alb P-Alb level Polyunsaturated Fatty Acids, E% -> P-Alb	Potassium-> P-Alb Potassium-> P-Alb Potassium-> P-K Potassium -> P-F	-0.048 -0.049 -0.051 -0.056 -0.059
Potassium -> P-K	Phosphorous -> P-K	0.066
Blood lipid medication -> P-K	Phosphorous -> fP-Pi	0.054
Calcium -> P-K	Phosphorous -> fP-Pi	0.051
Monounsaturated Fatty Acids, E% -> P-K	Phosphorous -> P-K	0.050
Protein, E% -> fP-Pi	Phosphorous -> fP-Pi	0.044
Salt -> fP-Pi	Phosphorous-> P-Alb	0.043
Polyunsaturated Fatty Acids, E% -> fP-Pi	Phosphorous-> P-K	0.042
Calcium -> P-K	Phosphorous -> P-K	0.038
Saturated Fatty Acids, E% -> fP-Pi	Phosphorous -> fP-Pi	0.036
Hydroxycholecalciferol -> P-K	Phosphorous -> fP-Pi	-0.037
Saturated Fatty Acids, E% -> fP-Pi	Phosphorous -> P-K	-0.037
Carbonhydrates, E% -> P-K	Phosphorous -> P-K	-0.039
Phosphorous -> fP-Pi	Phosphorous-> P-Alb	-0.039
Salt -> P-Alb	Phosphorous -> fP-Pi	-0.039
Carbonhydrates, E% -> fP-Pi	Phosphorous -> P-K	-0.040
Phosphate binder med> P-Alb	Phosphorous -> fP-Pi	-0.042
Polyunsaturated Fatty Acids, E% -> fP-Pi	Phosphorous -> fP-Pi	-0.048
Saturated Fatty Acids, E% -> P-Alb	Phosphorous -> fP-Pi	-0.053
Carbonhydrates, E% -> P-K	Phosphorous -> fP-Pi	-0.060
Calcium -> fP-Pi	Phosphorous -> P-K	-0.063

Supplementary Table S 2: Table shows 40 highest positive or negative correlations between personal effects of potassium and phosphorous with other personal effects. This structure of correlations is used in estimating the personal effects based on personal intake and matching concentrations.

Personal effects of nutries		
Effect 1	Effect 2	Correlation
Sodium -> fP-Pi Gender -> P-Alb Fiber -> P-K Polyunsaturated Fatty Acids, E% -> P-K P-Alb level	Potassium-> P-Alb Potassium -> P-K Potassium -> fP-Pi Potassium -> P-K Potassium-> P-Alb	0.049 0.046 0.044 0.043 0.041
Sodium -> P-Alb Blood lipid medication -> P-K Salt -> P-Alb P-Alb level Saturated Fatty Acids, E% -> P-K	Potassium -> fP-Pi Potassium -> P-K Potassium -> P-K Potassium -> fP-Pi Potassium-> P-Alb	0.037 0.035 0.034 0.032 0.031
Water -> P-K Water -> fP-Pi Hydroxycholecalciferol -> fP-Pi Monounsaturated Fatty Acids, E% -> P-K Phosphorous -> P-K	Potassium -> fP-Pi Potassium -> fP-Pi Potassium -> fP-Pi Potassium -> P-K Potassium -> P-K	-0.030 -0.030 -0.031 -0.033 -0.035
Salt -> P-K Diabetes medication -> P-Alb Water -> P-Alb Calcium -> fP-Pi Phosphorous -> fP-Pi	Potassium-> P-Alb Potassium-> P-K Potassium -> P-K Potassium -> P-K Potassium -> P-K	-0.040 -0.040 -0.046 -0.051 -0.058
fP-Pi level Renavit -> P-Alb Diabetes medication -> P-K Hydroxycholecalciferol -> fP-Pi Saturated Fatty Acids, E% -> fP-Pi	Phosphorous -> P-K Phosphorous -> P-K Phosphorous -> fP-Pi Phosphorous -> P-K Phosphorous-> P-Alb	0.088 0.070 0.056 0.053 0.051
Vitamin D -> P-Alb Protein, g/kg -> fP-Pi Hydroxycholecalciferol -> P-K Diabetes medication -> fP-Pi Monounsaturated Fatty Acids, E% -> P-Alb	Phosphorous -> P-K Phosphorous -> P-K Phosphorous -> fP-Pi Phosphorous -> P-K Phosphorous -> P-K	0.051 0.049 0.041 0.039 0.036
P-K level Blood lipid medication -> P-Alb Monounsaturated Fatty Acids, E% -> P-K Calcium -> P-K Hydroxycholecalciferol -> P-K	Phosphorous-> P-Alb Phosphorous-> P-Alb Phosphorous-> P-Alb Phosphorous -> fP-Pi Phosphorous -> P-K	-0.037 -0.037 -0.039 -0.044 -0.050
Fiber -> P-Alb Fat E% -> fP-Pi Water -> P-K Diabetes medication -> P-Alb Carbonhydrates, E% -> P-K	Phosphorous-> P-Alb Phosphorous -> fP-Pi Phosphorous -> P-K Phosphorous -> P-K Phosphorous -> fP-Pi	-0.051 -0.053 -0.055 -0.058 -0.069

Overview of nutritional effects

Following tables include all the estimated nutrition effects in general, dialysis treatment and personal levels.

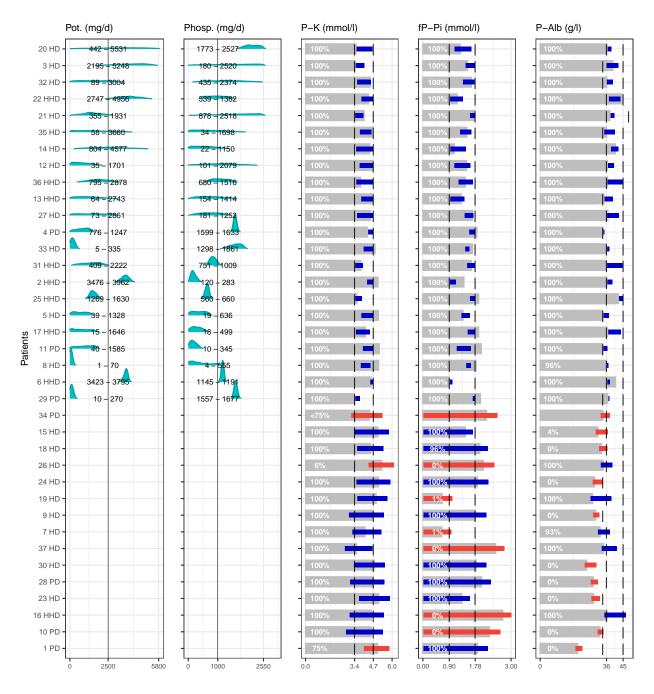
Supplementary Table S4: Nutrition effect magnitudes from nutrients and other modeled features $(j=1,\ldots,22)$ to blood concentrations $(i=1,\ldots,3)$ for analyzed patients $(p=1,\ldots,37)$ in all three additive levels of the model. General effects $(\hat{\beta}_{ij})$ are shown to vary between patients in home hemodialysis, hospital hemodialysis, and peritoneal dialysis. First column of each dialysis type (avg) shows typical effect of the treatment $(\hat{\beta}_{ij}+\hat{g}_{ijk},k=1,\ldots,3)$ that can further vary personally. Minimum and maximum of these personal effects are shown within each treatment $(\hat{\beta}_{ij}+\hat{g}_{ijk}+\hat{b}_{ijp})$. The table is sorted in decreasing order of between-treatment variation $(\hat{\sigma}_g)$ and all the estimates include their 90%-credible intervals.

Nutrient	Conc.	c. General effect	Home hemodialysis			Hospital hemodialysis			Peritoneal dialysis		
			avg	min	max	avg	min	max	avg	min	max
Water	P-Alb	-2.04 [-2.18; 2.97]	-2.07 [-5.76; 1.76]	-2.60 [-8.07; 2.01]	-2.06 [-6.13; 2.47]	-0.23 [-3.21; 2.52]	-0.60 [-4.28; 2.84]	0.11 $[-3.60; 3.77]$	-1.19 [-5.93; 4.22]	-1.53 [-6.39; 4.00]	-1.12 [-5.86; 4.42]
Blood lipid medication	P-Alb	0.36 [0.16; 7.76]	1.25 [-6.00; 10.22]	0.24 [-8.77; 11.01]	1.97 [-6.60; 12.07]	0.14 [-5.39; 5.61]	-1.17 [-7.39; 5.64]	$\begin{bmatrix} 1.19 \\ [-4.35; 6.96] \end{bmatrix}$	1.14 [-5.92; 10.13]	$\begin{bmatrix} 0.60 \\ [-8.54; 9.56] \end{bmatrix}$	1.81 [-5.82; 11.07]
Hydroxycholecalci		0.50 [0.73; 3.68] -0.08	-0.42 [-5.08; 2.69] 1.85	-0.52 $[-5.31; 2.40]$ 1.22	-0.19 [-5.07; 2.73] 2.25	-0.51 [-2.17; 1.19] 0.88	-0.82 [-2.69; 1.19] -0.32	-0.25 $[-2.28; 2.02]$ 1.59	-0.94 [-3.42; 1.72] 1.64	-1.08 [-3.85; 1.76] 2.03	-0.83 [-3.48; 1.84] 2.60
Calcium Sodium	P-Alb P-Alb	[0.73; 6.03] 1.22	[-2.93; 7.46] 2.23	[-3.36; 6.27] 2.00	[-3.61; 9.68] 2.80	[-3.47; 4.95] -0.87	[-5.36; 4.17] -1.39	[-3.38; 6.13] -0.28	[-2.90; 6.84] -3.02	[-3.76; 8.83] -3.28	[-3.21; 11.67] -2.60
		[0.87; 15.01] 1.60	[-5.69; 12.83] 3.76	[-6.13; 11.84] 3.28	[-5.80; 15.93] 4.52	[-6.51; 5.45] 1.82	[-7.65; 5.06] 1.27	[-6.33; 7.10] 2.27	[-13.00; 7.48] -0.39	[-13.36; 7.35] -0.81	[-12.82; 8.02] -0.31
Salt Gender	P-Alb P-Alb	[2.14; 10.81] 1.67	[-7.68; 12.53] 0.36	[-8.23; 11.96] -0.53	[-6.23; 12.73] 1.69	[-3.68; 6.93] 2.73	[-4.49; 6.56] 1.43	[-4.09; 9.07] 4.13	[-11.05; 8.80] 1.64	[-12.25; 8.04] 0.77	[-11.56; 8.59] 2.73
Phosphate	P-Alb	[1.42; 10.14] -2.78	[-6.76; 6.92] -2.68	[-11.15; 7.42] -4.26	[-8.20; 10.28] -1.59	[-3.79; 9.68] -1.95	[-7.72; 9.22] -4.25	[-3.80; 11.67] -0.96	[-7.19; 9.96] -0.37	[-11.13; 10.90] -1.32	[-7.01; 13.42] 0.84
binder med.	P-Alb	[-2.94; 4.87] 1.60	[-10.50; 7.83] -0.40	[-18.77; 8.61] -1.39	[-9.86; 8.98] 0.69	[-9.93; 6.69] -1.40	[-16.99; 7.20] -2.93	[-11.15; 10.81] -0.18	[-8.93; 8.19] 1.06	[-10.67; 7.77] -0.18	[-10.26; 12.01] 2.15
Energy, kcal/kg Carbohydrates,	P-Alb	[1.83; 15.39] 1.29	[-12.86; 13.00] 2.67	[-13.20; 11.42] 2.28	[-12.25; 15.89] 3.71	[-12.93; 9.31] 1.80	[-14.33; 8.17] 0.91	[-12.79; 11.23] 2.33	[-11.93; 16.56] 0.82	[-13.30; 15.81] 0.16	[-12.02; 18.26] 1.33
E%		[1.56; 15.35] 1.81	[-13.38; 18.00] 2.82	[-13.04; 17.08] 1.80	[-11.99; 18.43] 3.92	[-13.38; 15.62] 1.39	[-15.21; 15.17] 0.71	[-11.97; 15.69] 2.13	[-15.89; 15.37] 0.38	[-16.99; 15.88] -0.65	[-15.87; 15.52] 0.87
Monounsaturated Fatty Acids, E%	P-Alb	[1.07; 13.45]	[-3.72; 12.69]	[-5.70; 12.61]	[-3.14; 13.42]	[-4.01; 6.82]	[-5.18; 6.70]	[-3.10; 7.39]	[-7.41; 8.35]	[-9.54; 8.54]	[-6.41; 8.64]
Saturated Fatty Acids, E%	P-Alb	-2.58 [-2.40; 4.58]	-3.39 [-10.30; 3.85]	-4.07 [-11.26; 3.42]	-2.80 [-9.88; 5.06]	-2.23 [-9.00; 4.21]	-2.80 [-9.46; 4.27]	-1.17 [-7.44; 4.97]	-2.19 [-9.54; 5.17]	-2.48 [-9.24; 5.09]	-1.63 [-9.61; 5.69]
Hydroxycholecalci	f ₽:∕A lb	1.71 [1.49; 10.13] -0.93	2.12 [-5.96; 12.13] -1.21	1.61 [-6.54; 10.42] -1.43	3.09 [-5.98; 13.35] -0.89	0.90 [-5.30; 7.19] -2.05	0.54 [-6.56; 8.47] -2.42	1.68 [-6.08; 9.36] -1.64	3.16 [-6.04; 13.07] -1.17	2.24 [-7.66; 12.55] -1.27	3.47 [-6.70; 13.94] -0.92
Polyunsaturated Fatty Acids, E%	P-Alb	[-1.37; 5.86]	[-5.89; 3.58]	[-6.40; 4.03]	[-5.79; 4.35]	[-5.86; 1.40]	[-6.01; 1.27]	[-5.96; 2.41]	[-5.52; 3.44]	[-6.31; 3.49]	[-5.68; 3.78]
Potassium	P-Alb	-3.14 $[-1.47; 4.12]$	-3.89 [-9.99; 1.30]	-5.04 $[-12.20; 0.79]$	-3.63 [-9.67; 2.06]	-1.06 [-4.53; 1.96]	-1.97 $[-8.76; 2.01]$	0.34 $[-3.64; 3.96]$	-0.24 [-5.53; 5.54]	-1.02 $[-7.03; 5.17]$	0.27 [-5.92; 8.05]
Renavit	P-Alb	10.04 [11.10; 24.48] 3.19	11.73 [-8.72; 25.68] 2.32	10.55 [-8.97; 24.74] 1.52	12.77 [-7.93; 28.05] 3.00	9.33 [-9.77; 24.03] 3.40	7.75 [-10.19; 22.86] 2.52	10.62 [-10.79; 25.61] 4.14	8.25 [-12.70; 22.49] 3.50	7.28 [-12.78; 21.69] 2.88	9.12 [-12.99; 23.78] 3.95
Fat E%	P-Alb	[3.56; 19.59] -0.19	[-16.46; 20.22] -2.10	[-17.42; 20.12] -3.09	[-15.81; 20.48] -1.87	[-12.94; 18.61] 0.27	[-15.43; 17.92] -0.41	[-12.29; 19.78] 1.73	[-14.06; 19.88] 0.80	[-14.63; 19.85] 0.10	[-14.63; 21.22] 1.22
Phosphorous	P-Alb P-Alb	[-0.07; 6.56] 0.33	[-10.73; 5.28] -1.35	[-12.68; 4.94] -2.72	[-12.44; 7.81] -0.19	[-5.57; 5.95] -0.28	[-6.77; 6.08] -0.95	[-4.98; 8.63] 0.98	[-7.31; 8.48] 1.09	[-8.68; 8.11] 0.74	[-6.42; 8.81] 1.62
Protein, g/kg Vitamin D	P-Alb	[-0.24; 13.11] 1.60	[-16.07; 11.86] 1.55	$\begin{bmatrix} -17.16; 10.61 \\ 0.79 \end{bmatrix}$	[-15.66; 14.04] 2.39	[-11.68; 11.80] 0.79	[-12.82; 11.39] 0.11	[-11.00; 15.10] 1.45	[-13.08; 14.12] 1.50	[-12.95; 13.72] 1.24	[-12.93; 15.03] 2.22
Fiber	P-Alb	[1.61; 4.01] 1.19	[-1.01; 4.27] 1.43	[-3.52; 4.84] 1.00	[-0.82; 6.41] 2.02	[-1.25; 2.87] 1.34	[-3.45; 3.32] 0.70	[-1.76; 4.78] 2.01	[-1.49; 4.58] 1.60	[-2.88; 5.29] 1.28	[-1.24; 6.60] 1.92
Diabetes	P-Alb	[1.07; 7.55] 0.67	[-2.36; 5.60] 0.45	[-3.40; 6.23] -0.22	[-2.56; 7.51] 1.58	[-1.91; 4.70] -0.12	[-3.35; 4.61] -0.98	[-2.06; 6.49] 1.55	[-3.85; 6.61] 0.49	[-4.24; 6.85] 0.03	[-4.02; 7.59] 1.40
medication		[0.79; 8.48] 0.28	[-6.07; 8.60] 0.25	[-8.31; 8.33] -0.21	[-5.59; 9.25] 0.61	[-6.22; 5.63] 0.40	[-7.54; 5.88] 0.04	[-6.13; 11.84] 0.68	[-7.22; 8.87] 0.26	[-9.63; 9.68] -0.14	[-8.00; 11.06] 0.70
Phosphate binder med.	P-K	[0.16; 2.75] 0.50	[-1.83; 2.88] 0.61	[-2.39; 2.53] 0.33	[-2.15; 3.88] 0.82	[-1.23; 2.46] 0.41	[-2.01; 2.50] 0.15	[-1.49; 3.63] 0.73	[-1.45; 2.40] 0.12	[-2.83; 2.70] -0.05	[-1.44; 3.12] 0.18
Phosphate binder med.	fP-Pi	[0.17; 2.94]	[-0.97; 2.71]	$\left[-2.11; 2.69\right]$	[-1.44; 3.28]	[-0.90; 2.00]	[-1.86; 1.99]	[-1.34; 3.02]	[-1.71; 1.81]	$\left[-2.23; 1.87\right]$	[-1.91; 2.04]
Protein, E%	P-Alb	0.64 $[0.79; 7.41]$	2.08 [-4.84; 8.78]	1.70 $[-5.21; 8.26]$	$\begin{bmatrix} 2.67 \\ [-4.80; 10.24] \end{bmatrix}$	1.72 $[-5.00; 8.28]$	$\begin{bmatrix} 1.17 \\ [-5.17; 7.86] \end{bmatrix}$	$\begin{bmatrix} 2.40 \\ [-5.78; 9.59] \end{bmatrix}$	$\begin{bmatrix} 1.27 \\ [-5.70; 8.44] \end{bmatrix}$	0.88 [-7.16; 8.59]	1.78 [-5.68; 8.87]
Protein, g/kg	P-K	-0.22 [-0.41; 2.74]	0.09 [-2.42; 2.57]	-0.06 [-2.63; 2.54]	0.21 [-3.09; 2.96]	0.03 [-2.50; 2.41]	-0.22 [-3.16; 2.65]	0.18 $[-2.37; 2.65]$	-0.33 [-3.63; 2.99]	-0.36 [-3.95; 3.03]	-0.25 [-3.63; 3.12]
Carbohydrates, E%	P-K	1.05 [1.01; 4.69]	$\begin{bmatrix} 1.26 \\ [-2.36; 4.61] \end{bmatrix}$	$\begin{bmatrix} 1.12 \\ [-2.64; 4.81] \end{bmatrix}$	$\begin{bmatrix} 1.32 \\ [-2.46; 4.82] \end{bmatrix}$	0.85 $[-2.16; 3.43]$	$\begin{bmatrix} 0.67 \\ [-2.37; 3.40] \end{bmatrix}$	$\begin{bmatrix} 0.93 \\ [-2.40; 3.47] \end{bmatrix}$	$\begin{bmatrix} 0.74 \\ [-2.67; 4.07] \end{bmatrix}$	0.68 [-2.99; 4.04]	0.83 [-2.73; 4.47]
Calcium	P-K	-0.09 [-0.18; 1.54]	-0.46 [-2.59; 0.97]	-0.63 [-3.83; 1.03]	-0.36 [-2.93; 1.41]	0.01 [-0.94; 1.17]	-0.15 [-1.92; 1.29]	0.38 $[-0.76; 2.19]$	-0.13 [-1.62; 1.29]	-0.37 $[-2.41; 1.41]$	0.05 [-1.49; 1.56]
Energy, kcal/kg	P-K	0.30 [0.03; 4.98] 0.11	-0.19 [-2.92; 2.96] 0.93	-0.30 [-3.37; 2.87] 0.74	-0.11 [-2.54; 2.94] 1.09	-0.14 [-2.52; 2.41] 0.18	-0.24 [-2.71; 2.37] -0.09	-0.09 [-2.64; 2.53] 0.38	0.29 [-3.70; 4.70] -0.24	0.17 [-3.81; 4.57] -0.50	0.43 [-3.70; 4.95] -0.20
Salt	P-K	[0.12; 2.50]	[-1.37; 2.94]	[-1.96; 2.72]	[-1.35; 3.42]	[-0.79; 1.10]	[-2.57; 1.23]	[-0.64; 1.41]	[-2.89; 1.63]	[-3.94; 1.64]	[-3.15; 1.68]
Gender	P-K	-0.04 [0.06; 2.63] -0.65	0.41 [-1.36; 2.47] 0.11	0.35 $[-1.54; 2.50]$ -0.21	0.57 $[-1.36; 2.75]$ 0.36	0.26 [-1.17; 1.78] -0.32	0.07 [-1.57; 1.95] -0.61	0.39 $[-1.40; 2.49]$ 0.10	0.92 [-1.97; 4.50] -0.55	0.76 $[-2.39; 4.48]$ -0.96	1.07 [-1.90; 4.28] -0.23
Blood lipid medication	P-K	[-0.60; 1.82]	[-2.42; 4.45]	[-3.88; 4.27]	[-2.46; 5.41]	[-1.68; 0.98]	[-2.45; 0.86]	[-1.30; 1.86]	[-3.45; 1.73]	[-5.15; 1.91]	-0.23 [-3.25; 3.33]

Supplementary Table S4 (continued)

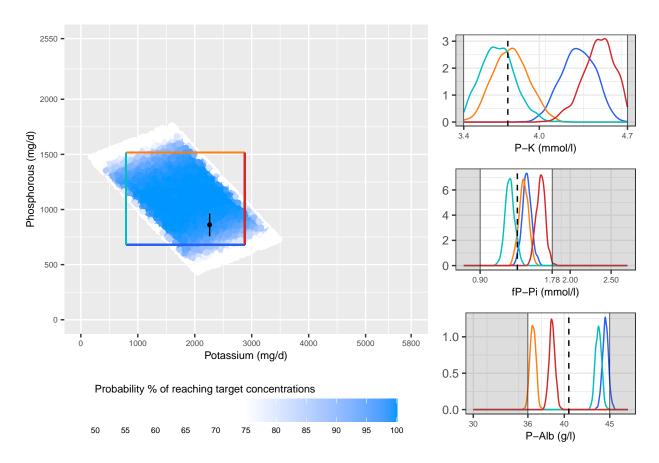
Nutrient	Conc.	c. General effect	Home hemodialysis			Hospital hemodialysis			Peritoneal dialysis		
			avg	min	max	avg	min	max	avg	min	max
Carbohydrates, E%	fP-Pi	0.29 [0.27; 2.87]	0.00 [-1.88; 1.73]	-0.07 [-1.88; 1.68]	0.03 [-1.82; 1.79]	0.16 [-1.40; 1.58]	0.08 [-1.66; 1.53]	0.27 [-1.08; 1.59]	0.06 [-1.90; 1.87]	0.01 [-1.89; 1.85]	0.08 [-1.99; 1.97]
Monounsaturated Fatty Acids, E%	Р-К	$0.43 \\ [0.27; 3.02]$	$\begin{array}{c} \textbf{0.76} \\ [-0.85; 3.37] \end{array}$	$0.72 \\ [-0.93; 3.46]$	$0.93 \\ [-0.87; 3.74]$	$\begin{array}{c} \textbf{0.52} \\ [-0.75; 2.02] \end{array}$	$0.28 \\ [-1.18; 1.77]$	$0.66 \\ [-0.96; 3.02]$	$\begin{array}{c} {\bf 0.58} \\ [-1.48; 2.81] \end{array}$	$0.45 \\ [-1.80; 2.80]$	$0.72 \\ [-1.20; 2.68]$
Sodium	fP-Pi	0.19 [0.16; 1.93]	0.63 [-0.91; 3.46]	0.57 $[-0.85; 3.26]$	0.70 [-0.86; 3.47]	0.12 [-0.65; 0.72]	0.05 $[-0.76; 0.68]$	0.20 [-0.57; 0.85]	0.41 [-1.65; 3.99]	0.36 [-1.60; 3.80]	0.47 [-1.68; 4.15]
Sodium	P-K	-0.15 [-0.29; 2.35]	-0.06 [-1.88; 2.31]	-0.15 [-1.98; 2.23]	-0.02 [-1.88; 2.32]	-0.25 [-1.74; 1.63]	-0.40 [-1.86; 1.45]	-0.05 [-1.76; 2.26]	-0.54 [-2.64; 2.16]	-0.60 [-2.74; 2.22]	-0.39 [-2.55; 2.47]
Potassium	P-K	-0.46 $[-0.13; 2.02]$	-0.06 [-1.48; 1.55]	-0.32 [-2.14; 1.32]	0.14 [-1.57; 1.76]	0.14 $[-0.58; 1.02]$	-0.11 [-1.40; 1.27]	0.50 $[-0.52; 1.64]$	-0.21 [-1.66; 0.98]	-0.51 [-2.22; 1.17]	0.16 $[-1.50; 2.45]$
Diabetes medication	fP-Pi	0.20 [0.03; 1.86]	0.19 [-0.71; 1.33]	0.04 $[-1.14; 1.37]$	0.37 $[-0.74; 1.72]$	0.05 $[-0.68; 0.77]$	-0.09 $[-1.12; 0.90]$	0.10 [-0.76; 1.09]	0.17 [-0.95; 1.93]	0.04 $[-1.28; 2.04]$	0.33 [-1.12; 2.12]
Polyunsaturated Fatty Acids, E%	fP-Pi	$0.22 \\ [0.03; 2.26]$	$\begin{array}{c} \textbf{-0.52} \\ [-1.63; 0.31] \end{array}$	-0.53 [$-1.81; 0.37$]	-0.50 [-1.62; 0.40]	-0.34 [-0.89; 0.09]	-0.40 [$-1.04; 0.10$]	-0.29 [$-0.80; 0.16$]	-0.25 [-0.97; 0.62]	-0.28 [-1.06; 0.64]	-0.21 [-0.93; 0.64]
Blood lipid medication	fP-Pi	0.02 [-0.29; 3.27]	-0.81 [-3.36; 1.66]	-0.95 [-3.36; 2.11]	-0.67 [-3.58; 1.79]	-0.51 [-1.24; 0.19]	-0.82 [-2.45; 0.29]	-0.33 [-1.19; 0.49]	-0.14 [-1.96; 2.11]	-0.33 [-2.22; 1.70]	-0.07 [-2.23; 2.54]
Diabetes medication	P-K	5.41 [0.26; 23.43]	-0.17 [-1.92; 2.00]	-0.35 $[-2.73; 2.46]$	0.07 $[-1.90; 2.35]$	-0.13 [-1.29; 1.14]	-0.31 [-2.35; 1.47]	0.08 [-1.18; 1.81]	-0.65 [-4.90; 2.38]	-0.76 [-4.82; 2.23]	-0.54 [-5.11; 2.94]
Energy, kcal/kg	fP-Pi	0.50 [0.64; 2.17]	0.58 [-0.90; 2.05]	0.46 [-0.95; 1.93]	0.68 [-0.90; 2.25]	0.71 [-0.72; 1.96]	0.63 [-0.76; 1.91]	0.80 [-0.72; 2.08]	0.51 [-1.79; 2.22]	0.45 [-1.83; 2.21]	0.56 [-1.88; 2.41]
Fat E%	fP-Pi	0.21 [0.24; 2.17]	0.51 [-1.48; 2.46]	0.46 [-1.50; 2.43]	0.57 [-1.46; 2.74]	0.57 [-1.22; 2.58]	0.50 [-1.26; 2.55]	0.63 [-1.25; 2.79]	0.18 [-1.91; 2.15]	0.13 [-1.82; 2.12]	0.24 [-2.00; 2.17]
Renavit	P-K	0.94 [-0.01; 7.50]	0.29 [-3.83; 6.53]	0.11	0.58	0.76 [-2.53; 5.74]	0.49 [-3.01; 5.34]	0.93 [-2.38; 5.89]	0.88 [-3.09; 6.47]	0.92 [-3.27; 6.59]	1.08
Water	P-K	0.16 [0.10; 1.79]	0.18 [-0.59; 1.08]	-0.04 [-1.81; 1.35]	0.36 $[-0.54; 1.31]$	-0.24 [-0.92; 0.44]	-0.48 [-1.67; 0.65]	0.01 [-0.69; 0.56]	-0.10 [-1.53; 1.13]	-0.33 [-1.93; 1.27]	0.01 $[-1.53; 1.54]$
Water	fP-Pi	$\begin{bmatrix} 0.11 \\ [0.12; 0.73] \end{bmatrix}$	$\begin{bmatrix} 0.14 \\ [-0.29; 0.60] \end{bmatrix}$	$\begin{bmatrix} 0.03 \\ -0.88; 0.69 \end{bmatrix}$	0.24 [-0.37; 0.89]	$\begin{bmatrix} 0.17 \\ -0.13; 0.55 \end{bmatrix}$	$\begin{bmatrix} 0.00 \\ -0.30; 0.34 \end{bmatrix}$	0.23 [-0.20; 0.87]	$\begin{bmatrix} 0.05 \\ -0.81; 0.99 \end{bmatrix}$	-0.08 [-1.18; 0.97]	0.21 [-0.69; 1.08]
Fiber	P-K	-0.92 [-0.10; 1.42]	0.23 $[-0.72; 1.48]$	-0.07 [-1.45; 1.17]	0.46 [-0.85; 1.85]	$\begin{bmatrix} 0.03 \\ [-0.93; 0.94] \end{bmatrix}$	-0.29 [-1.45; 0.96]	0.28 [-1.04; 1.33]	0.47 $[-1.04; 2.48]$	0.31 [-1.32; 2.59]	0.62 [-1.21; 2.79]
Renavit	fP-Pi	0.14 [-0.02; 3.06]	0.36 $[-2.03; 2.93]$	0.28 [-2.04; 2.94]	0.39 [-2.03; 2.96]	$\begin{bmatrix} 0.20 \\ [-1.60; 1.78] \end{bmatrix}$	$\begin{bmatrix} 0.14 \\ [-1.73; 1.80] \end{bmatrix}$	0.26 $[-1.63; 1.89]$	$\begin{bmatrix} 0.77 \\ [-2.49; 4.08] \end{bmatrix}$	$\begin{bmatrix} 0.70 \\ [-2.59; 3.99] \end{bmatrix}$	0.83 $[-2.47; 4.02]$
Vitamin D	P-K	-0.17 $[-0.23; 1.11]$ -0.80	-0.15 [-0.94; 0.60] -0.84	-0.25 [-1.23; 0.78] -0.91	-0.11 [-0.91; 0.79] -0.78	-0.18 [-0.65; 0.45] -0.81	-0.33 [-1.04; 0.46] -0.86	$\begin{bmatrix} 0.02 \\ [-0.44; 0.60] \\ -0.74 \end{bmatrix}$	-0.31 [-1.24; 0.72] -0.64	-0.49 [-1.32; 0.42] -0.65	-0.22 [-1.21; 0.95] -0.60
Protein, g/kg	fP-Pi	[-0.87; 1.10] -0.38	[-2.37; 0.91] -0.14	[-2.49; 0.86] -0.26	[-2.52; 0.99] -0.06	[-2.03; 0.65] - 0.25	[-2.16; 0.57] -0.40	[-2.03; 0.74] -0.01	[-2.32; 1.06] - 0.51	[-2.29; 1.10] -0.65	[-2.38; 1.12] -0.41
Fat E% Protein, E%	P-K fP-Pi	[-0.24; 3.29] 0.20	[-4.80; 3.86] 0.12	[-5.16; 3.77] 0.08	$[-4.85; 4.02] \\ 0.16$	$[-4.41; 3.08] \\ 0.24$	$\begin{bmatrix} -4.67; 2.98 \\ 0.20 \end{bmatrix}$	$\begin{bmatrix} -4.12; 3.17 \\ 0.30 \end{bmatrix}$	[-5.07; 3.44] 0.14	[-5.50; 3.48] 0.09	[-4.82; 3.39] 0.17
Monounsaturated		[0.19; 1.87]	[-0.65; 0.79] 0.06	0.03	[-0.68; 0.81]	[-0.49; 0.84] -0.13	[-0.56; 0.87] -0.16	[-0.39; 0.92] -0.05	[-1.03; 1.09] -0.02	[-1.04; 1.05] -0.06	0.02
Fatty Acids, E%		[-0.05; 1.77]	[-0.73; 1.05]	[-0.80; 1.06]	[-0.77; 1.21]	[-0.89; 0.62]	[-1.04; 0.63]	[-0.88; 0.72]	[-0.95; 0.94]	[-1.01; 0.97]	[-0.91; 1.00]
Polyunsaturated Fatty Acids, E%	P-K	$0.29 \\ [0.23; 1.84]$	0.28 [-1.11; 1.73]	$ 0.21 \\ [-1.28; 1.85] $	$0.38 \\ [-1.21; 2.24]$	$ \begin{array}{c} 0.15 \\ [-0.71; 1.10] \end{array} $	[-1.02; 1.07]	$0.25 \\ [-0.76; 1.48]$	0.32 [-0.91; 1.86]	0.19 [-1.05; 1.44]	$[-0.45 \\ [-0.81; 2.22]$
Phosphorous	P-K	0.39 [0.52; 2.31]	0.25 $[-1.41; 2.02]$	0.15 $[-1.57; 1.79]$	0.36 $[-1.54; 2.84]$	0.21 [-1.20; 1.69]	0.10 [-1.37; 1.54]	0.37 $[-1.30; 2.61]$	0.51 [-1.42; 2.55]	0.45 $[-1.44; 2.39]$	0.59 $[-1.49; 2.61]$
Protein, E%	P-K	0.49 [0.32; 3.27]	0.23 [-0.93; 2.24]	$\begin{bmatrix} 0.13 \\ [-1.23; 2.20] \end{bmatrix}$	0.25 $[-0.99; 2.24]$	0.28 [-1.23; 2.16]	0.15 $[-1.46; 2.25]$	$\begin{bmatrix} 0.42 \\ [-1.03; 2.26] \end{bmatrix}$	0.54 [-1.25; 3.15]	0.46 $[-1.49; 3.14]$	0.52 $[-1.32; 3.09]$
Saturated Fatty Acids, E%	P-K	$\begin{bmatrix} 1.01 \\ [1.03; 3.03] \end{bmatrix}$	0.86 $[-1.48; 3.10]$	$ 0.77 \\ [-1.34; 2.93] $	0.95 $[-1.50; 3.18]$	0.82 [-0.95; 2.82]	0.64 [-1.47; 2.69]	0.95 $[-0.86; 3.08]$	1.06 [-1.01; 3.19]	$ \begin{bmatrix} 1.01 \\ [-1.17; 3.28] \end{bmatrix} $	$\begin{bmatrix} 1.18 \\ -0.85; 3.25 \end{bmatrix}$
Saturated Fatty Acids, E%	fP-Pi	-0.36 [-0.29; 0.79]	-0.53 [-2.02; 0.56]	-0.58 [$-2.04; 0.57$]	$-0.49 \ [-2.07; 0.67]$	$\begin{array}{c} \textbf{-0.32} \\ [-1.22; 0.50] \end{array}$	-0.39 [-1.34; 0.50]	-0.27 $[-1.24; 0.65]$	-0.05 $[-1.14; 0.96]$	-0.06 $[-1.19; 0.98]$	-0.01 [-1.15; 1.03]
Fiber	fP-Pi	-0.21 [-0.15; 1.06]	-0.11 [-0.60; 0.37]	-0.18 [-0.79; 0.36]	-0.08 [-0.56; 0.41]	-0.11 [-0.66; 0.30]	-0.19 [-0.79; 0.21]	-0.05 $[-0.69; 0.56]$	-0.13 [-1.11; 0.96]	-0.16 $[-1.25; 0.97]$	-0.09 [-1.15; 1.09]
Calcium	fP-Pi	-0.25 $[-0.14; 0.42]$	-0.04 [-0.54; 0.56]	-0.11 [-0.93; 0.69]	0.17 $[-0.63; 1.12]$	-0.15 [-0.59; 0.22]	-0.30 [-1.19; 0.29]	0.01 [-0.56; 0.66]	-0.06 [-0.96; 0.74]	-0.24 [-1.12; 0.84]	0.00 [-1.04; 1.04]
Hydroxycholecalc	if €P oPi	1.51 [0.48; 7.33]	0.52 $[-2.27; 3.05]$	0.40 [-2.44; 3.08]	0.70 [-2.10; 3.25]	0.49 [-0.30; 1.38]	0.40 [-0.64; 1.38]	0.59 [-0.42; 1.77]	0.17 [-2.04; 1.54]	-0.05 [-2.09; 1.56]	0.26 [-2.08; 1.65]
Gender	fP-Pi	-0.13 [-0.09; 0.88]	-0.02 [-1.32; 1.06]	-0.14 $[-1.51; 1.14]$	0.06 $[-1.46; 1.22]$	-0.14 [-1.12; 0.76]	-0.28 [-1.42; 0.95]	0.05 $[-1.04; 1.10]$	-0.04 [-1.34; 1.14]	-0.07 [-1.41; 1.22]	0.02 $[-1.53; 1.54]$
Vitamin D	fP-Pi	0.10 [0.15; 0.54]	0.06 $[-0.41; 0.58]$	-0.07 [-0.69; 0.46]	$0.13 \\ [-0.54; 1.01]$	0.15 $[-0.14; 0.41]$	0.05 $[-0.44; 0.52]$	0.29 $[-0.14; 0.71]$	0.14 $[-0.40; 0.64]$	0.16 $[-0.52; 0.77]$	0.22 $[-0.35; 0.82]$
Potassium	fP-Pi	0.07 [0.04; 0.88]	0.00 [-0.72; 0.76]	-0.05 [-0.82; 0.77]	0.04 [-0.72; 0.78]	0.06 [-0.34; 0.52]	0.03 [-0.51; 0.53]	0.09 [-0.35; 0.56]	0.17 [-0.56; 0.97]	0.15 $[-0.68; 1.03]$	0.21 [-0.48; 1.05]
Phosphorous	fP-Pi	0.08 [0.06; 1.41]	0.15 [-1.00; 1.13]	0.10 [-1.19; 1.23]	0.21 [-1.14; 1.21]	-0.02 [-0.79; 0.79]	-0.09 [-0.95; 0.81]	0.08 [-0.63; 0.76]	0.16 [-0.91; 1.43]	0.05 $[-1.22; 1.43]$	0.20 [-0.90; 1.45]
Salt	fP-Pi	0.61 [0.12; 3.65]	-0.66 [-3.44; 0.90]	-0.71 [-3.64; 0.86]	-0.62 [-3.37; 0.96]	-0.11 [-0.52; 0.39]	-0.16 [-0.91; 0.42]	-0.04 [-0.54; 0.56]	-0.66 [-3.86; 1.21]	-0.73 [-4.08; 1.25]	-0.64 [-3.74; 1.22]

Overview of personal recommendations



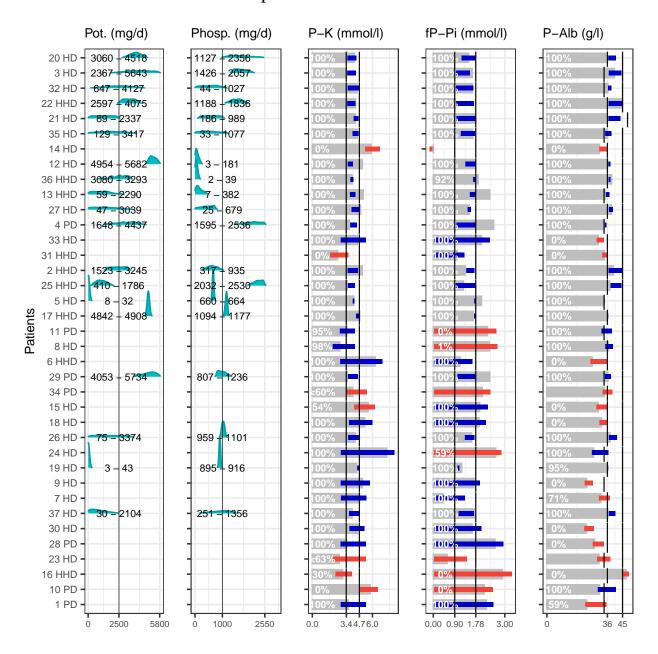
Supplementary Figure S 4: The figure shows in two left-most panels personal recommendations of potassium and phosphorous intake $(\hat{Q}^{min} - \hat{Q}^{max})$ with predictive distributions of \hat{Q} illustrated in the background. Each row shows a numeric label of the patient and the type of patient's dialysis treatment (HD = hospital hemodialysis, HHD = home hemodialysis, PD = peritoneal dialysis). Personal recommendations are given for those patients whose resulting plasma concentration levels could be predicted in either over 90% probability. Three right-most panels show the estimated concentrations that match the recommended intake. In these panels, grey bars indicate the estimated concentration without the effect of potassium and phosphorous intake, μ_{q0} . Blue bars indicate the range of concentration that is resulted from modifying phosphorous and potassium intake. It is required that these simulated concentrations stay within the personal target ranges denoted with vertical black lines. The dark blue bar indicates satisfying this requirement in $P^{max} > 90\%$ confidence and $P^{max} > 80\%$ confidence would be indicated with light blue, but there aren't any in the results. The best exact probabilities P^{max}_m are shown with percentage figures. Red bars indicate that even the best concentration estimation has $P^{max} < 80\%$ confidence and personal recommendations are not given for these patients. For these patients, the red and blue bars show the whole reachable ranges. The figure is plotted with ggplot2 package for R language (v 3.3.5, https://ggplot2.tidyverse.org).

Personal recommendation for a patient



Supplementary Figure S 5: Figure shows detailed intake recommendation for patient 10 from the previous recommendation table. The intake plot on the left shows the posterior samples of diet configurations that result concentrations of plasma potassium (P-K), fasting plasma phosphate (fP-Pi) and plasma albumin (P-Alb) to stay within their recommended limits. These levels are marked with vertical solid lines in the concentration panels. The black point in the middle of intake plot represents the patient's current potassium and phosphorous intake. Current concentrations matching this intake are shown with dashed vertical lines. Reported recommendation is shown with a rectangle that contains 95% of diet proposals that result recommended concentrations over 90% accuracy. Colouring of the rectangle sides match the concentration estimates in the right hand panels. The figure is plotted with ggplot2 package for R language (v 3.3.5, https://ggplot2.tidyverse.org).

Personal recommendations for predicted reactions



Supplementary Figure S 6: The figure shows personal recommendations of potassium and phosphorous intake $(\hat{Q}^{min} - \hat{Q}^{max})$ based on cross-validation predictions of nutrition effects, \hat{b}_k . Other parameters of the figure are similar to Supplementary Fig. S4. The figure is plotted with ggplot2 package for R language (v 3.3.5, https://ggplot2.tidyverse.org).