

Emergent Hybrid Computation in Gradient-Free Evolutionary Networks

Payton Miller

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Abstract

This paper analyzes the representational structures that emerge in neural networks trained via evolutionary selection rather than gradient-based optimization. While standard deep learning avoids saturation due to the vanishing gradient problem, gradient-free evolution demonstrates that saturation serves as a functional mechanism for discovering hybrid digital-analog representations. We formalize this as a partitioned state space where k saturated neurons establish discrete operational modes, while $n - k$ continuous neurons facilitate fine-grained modulation. Through systematic experimentation across 13 configurations, we empirically validate that saturation emerges when networks must selectively attend to a subset of available inputs—whether due to explicitly irrelevant dimensions or natural redundancy in complex input spaces. Our results demonstrate that evolution dynamically allocates k based on task demands, achieving $k = 0$ for clean continuous tasks where all inputs are relevant, and $k \rightarrow n$ when selective filtering becomes necessary.

1 Introduction

In conventional gradient-based neural network training, neuron saturation is typically avoided to prevent gradients from vanishing, which halts the learning process. Consequently, techniques like batch normalization and ReLU activations are utilized to maintain neurons in non-saturated regimes.

However, in evolutionary training regimes

where selection pressure replaces gradient descent, saturation carries no penalty. Empirical observation reveals that networks spontaneously develop partial saturation, evolving toward a hybrid configuration where some neurons operate as discrete switches while others maintain analog modulation. This represents an emergent discovery of an efficient computational structure combining digital and analog benefits.

We propose that saturation functions as a **selective attention mechanism**. When a network receives more input dimensions than are necessary for the task, or when inputs contain redundant or irrelevant information, saturated neurons emerge to gate attention toward task-relevant signals. This occurs naturally in complex tasks with high-dimensional input spaces, and can be induced experimentally by adding explicitly irrelevant dimensions.

The key insight is not that “noise causes saturation,” but rather that **saturation emerges whenever the network must learn to selectively attend to a subset of its inputs**. In our controlled experiments, we inject random dimensions to create this pressure artificially. In naturalistic tasks with complex input spaces, this pressure exists inherently—not all inputs are equally relevant to all decisions.

2 Formalization of the State Space

The representational capacity of a hidden layer with n neurons is determined by the ratio of saturated (k) to continuous ($n - k$) units.

2.1 Pure Binary vs. Pure Continuous

Consider a hidden layer with n neurons using tanh activation, outputting values in $[-1, +1]$.

Table 1: Pure Configuration Comparison

Config	States	Search	Express
Binary (n=8)	256	✓	Limited
Continuous (n=8)	∞	Hard	✓
Hybrid (k, n-k)	$2^k \times \infty^{n-k}$	✓	✓

2.2 The Hybrid Manifold

A hybrid configuration creates a state space defined as:

$$S = 2^k \times \infty^{n-k} \quad (1)$$

where k is the number of saturated neurons and $n - k$ is the number of continuous neurons. Geometrically, each saturated neuron functions as a hyperplane bisecting the input space. With k saturated neurons, the input space is partitioned into 2^k distinct regions. Within each region, the remaining continuous neurons provide smooth, infinite-resolution control.

2.3 Geometric Interpretation

Each saturated neuron creates a decision boundary that partitions the input space. With $k = 1$, we get 2 regions. With $k = 2$, we get 4 regions. With $k = 3$, we get 8 regions. In general, k saturated neurons create 2^k discrete regions.

Within each region, continuous neurons provide smooth interpolation—*infinite states* for fine-grained control.

3 Experimental Validation

To validate the theory, we conducted a systematic sweep across 13 configurations testing the relationship between input relevance, compression ratio, hidden layer capacity, and emergent saturation levels.

3.1 Experimental Setup

Task: Sine wave approximation ($y = \sin(x)$, $x \in [-2\pi, 2\pi]$)

Architecture: Input expansion (Fourier basis) → hidden layer → single output

Training: GENREG evolutionary algorithm with selection pressure based on MSE fitness

Saturation threshold: $|h| > 0.95$ classified as saturated

3.2 Core Finding: Selective Attention Drives Saturation

The critical finding is that **compression ratio alone does not cause saturation**. Saturation emerges only when the network must learn to selectively attend to a subset of inputs—that is, when some input dimensions are more relevant to the task than others.

In our controlled experiments, we create this condition by introducing explicitly irrelevant input dimensions (random values). However, this is a proxy for a more general phenomenon: in complex real-world tasks, input spaces naturally contain varying degrees of relevance. Some sensors may be redundant, some features may be irrelevant to specific decisions, and some input combinations may matter more than others.

Table 2: Control Tests: Compression Without Irrelevant Inputs

Test	Signal Dims	Compression	Final k
C1	16	2:1	0/8 (0%)
C2	64	8:1	0/8 (0%)
C3	256	32:1	0/8 (0%)

Despite 32:1 compression in C3, the network maintains $k = 0$ —all neurons remain continuous. When every input dimension carries task-relevant information (even under extreme compression), the network preserves continuous processing to extract maximum information from all channels.

This demonstrates that saturation is not a response to compression pressure, but rather a mechanism for **learned selective attention**:

saturated neurons function as binary gates that partition the input space into “relevant” and “irrelevant” regions.

3.3 Irrelevant Input Scaling Experiment

To isolate the effect of selective attention pressure, we systematically varied the number of task-irrelevant input dimensions while holding task-relevant signal constant at 16 dimensions. The irrelevant dimensions were populated with random values, ensuring they carried no information useful for the task.

Table 3: Saturation Scaling with Task-Irrelevant Inputs

Test	Irrel.	Total	k	Ratio	MSE
N0	0	16	0/8	0%	0.001
N1	16	32	0/8	0%	0.003
N2	48	64	0/8	0%	0.015
N3	112	128	3/8	38%	0.015
N4	240	256	7/8	88%	0.016
N5	496	512	8/8	100%	0.122

Key observation: There exists a threshold (approximately 100 irrelevant dimensions in this configuration) below which networks can tolerate irrelevant inputs through continuous processing. Above this threshold, saturation becomes necessary for effective filtering.

This threshold behavior suggests that continuous neurons have a limited capacity to “average out” irrelevant information. Once the ratio of irrelevant to relevant inputs exceeds this capacity, the network must develop discrete gating mechanisms (saturated neurons) to explicitly filter the input space.

3.4 Hidden Layer Capacity Scaling

With irrelevant inputs fixed at 240 dimensions, we varied hidden layer size:

Critical finding: With limited capacity (4-8 neurons), all neurons saturate. With increased capacity (16-32 neurons), the network maintains a hybrid configuration—approximately 75-78% saturated with 22-25% remaining continuous.

Table 4: Saturation by Hidden Layer Size (240 Irrelevant Dims)

Test	Hidden	Final k	k Ratio	MSE
H1	4	4/4	100%	0.0108
H2	8	8/8	100%	0.0156
H3	16	12/16	75%	0.0305
H4	32	25/32	78%	0.0743

This demonstrates that given sufficient capacity, evolution preserves continuous neurons for fine-grained modulation while allocating others to discrete filtering.

4 Mechanics of Evolutionary Discovery

4.1 The Searchability Argument

Pure continuous networks often present unsearchable fitness landscapes for evolutionary algorithms due to the high probability that random mutations in high-dimensional spaces will not improve fitness. Saturated neurons mitigate this by creating discrete “landmarks”. When a neuron saturates, it is “locked” at ± 1 , effectively reducing the dimensionality of local search while preserving global structure.

Evolution then proceeds in two modes:

- **Exploitation:** Mutating continuous neuron weights for fine-tuning within a region.
- **Exploration:** Large mutations that flip a saturated neuron, jumping to a different region.

4.2 The Variable- k Insight

The allocation of neurons between binary and continuous modes is discovered by evolution based on the degree of selective attention required by the task. Our experiments confirm this with striking clarity:

- **All inputs relevant** (sine, no irrelevant dims): $k = 0$
- **Moderate filtering required** (112 irrelevant dims): $k = 3/8$ (37.5%)

- **Heavy filtering required** (240 irrelevant dims): $k = 7-8/8$ (87.5–100%)
- **Excess capacity available** (32 hidden, 240 irrelevant): $k = 25/32$ (78%)

The total representational capacity is:

$$\Phi = \sum_{k=0}^n \binom{n}{k} \times 2^k \times \infty^{n-k} \quad (2)$$

This space is significantly larger than pure binary or pure continuous configurations and remains searchable because evolution navigates it incrementally.

Generalization to complex tasks: While our controlled experiments use explicitly random irrelevant dimensions, the same mechanism applies to naturalistic tasks where input relevance varies contextually. In separate experiments with high-dimensional continuous control tasks, we observed immediate saturation emergence even without injected irrelevant inputs—the natural structure of the input space (where not all sensors are equally relevant to all actions) creates equivalent selective attention pressure. This confirms that saturation is a general-purpose attention mechanism, not merely a filtering artifact of our experimental design.

4.3 Saturation Trajectory

Empirical observation of training dynamics reveals a characteristic pattern: early generations show low k (broad exploration), intermediate generations show increasing k (modes lock in), and late generations show high k stability (refinement within discovered structure).

Table 5: Theory Validation Results

Claim	Tests	Result
Compression \neq saturation	C1-C3	✓
All-relevant $\rightarrow k \approx 0$	N0, C1-C3	✓
Irrelevant \rightarrow increased k	N0-N5	✓
k scales with irrelevance	N3-N5	✓
Excess capacity \rightarrow hybrid	H3-H4	✓

5 Validation Summary

Of 13 experimental configurations, 11 validated predictions exactly. Two tests (N1, N2) showed $k = 0$ where moderate k was predicted, revealing a threshold effect: irrelevant input dimensions must exceed a critical level before saturation becomes necessary. This refinement strengthens rather than contradicts the theory.

6 Implications

6.1 Biological Parallels

The emergence of hybrid signaling parallels biological neural systems, which utilize both discrete (action potentials) and continuous (graded potentials) signaling. This suggests the digital-analog hybrid may represent a fundamental computational optimum discovered independently by evolution in both biological and artificial systems.

6.2 Quantization Readiness

Networks that evolve toward saturation are pre-adapted for quantization. Because saturation occurs organically during evolution, these models can be quantized for compact hardware deployment with minimal loss—the continuous-to-discrete conversion has already occurred through selection pressure.

6.3 Implications for Task Design

Our results suggest that the structure of inputs significantly impacts emergent computation:

- **All inputs task-relevant:** Networks remain fully analog, maximizing information extraction
- **Mixed relevance inputs:** Hybrid representations emerge, with saturated neurons gating attention
- **High-dimensional naturalistic inputs:** Saturation emerges rapidly as the network learns which input subspaces matter

This has practical implications for applying evolutionary methods:

- 1. Input design matters:** Providing only task-relevant inputs may prevent useful saturation structure from emerging. Richer input spaces allow evolution to discover its own attention patterns.
- 2. Bottleneck architecture:** The hidden layer size relative to input dimensionality controls the pressure for selective attention. Narrower bottlenecks force more aggressive filtering.
- 3. Temporal structure:** For tasks like classification where single-frame inputs may contain ambiguous relevance, temporal presentation (sequences showing the same object under transformation) may help evolution identify which input patterns are consistently relevant.

7 Conclusion

Saturation is a functional feature, not a bug, in gradient-free training. It creates the discrete structure necessary for navigating complex fitness landscapes while enabling selective attention to task-relevant input dimensions.

Our empirical validation across 13 configurations confirms:

1. Partial saturation emerges spontaneously under evolutionary pressure
2. Saturation scales with the need for selective attention, not compression ratio alone
3. A threshold exists below which continuous processing can tolerate irrelevant inputs
4. Given sufficient capacity, hybrid configurations (75–78% saturated) emerge
5. The hybrid state space $2^k \times \infty^{n-k}$ combines searchability with expressiveness

The key insight is that saturated neurons function as learned binary gates implementing selective attention. In controlled experiments, we induce this by adding explicitly irrelevant input dimensions. In complex naturalistic tasks, this pressure exists inherently—evolution discovers which inputs matter and develops gating structures accordingly.

This suggests that the standard gradient-based paradigm, by avoiding saturation, may be systematically excluding efficient hybrid solutions that evolution naturally discovers. Furthermore, the spontaneous emergence of selective attention mechanisms without explicit attention architectures points toward a fundamental computational principle that biological and artificial evolution converge upon independently.

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