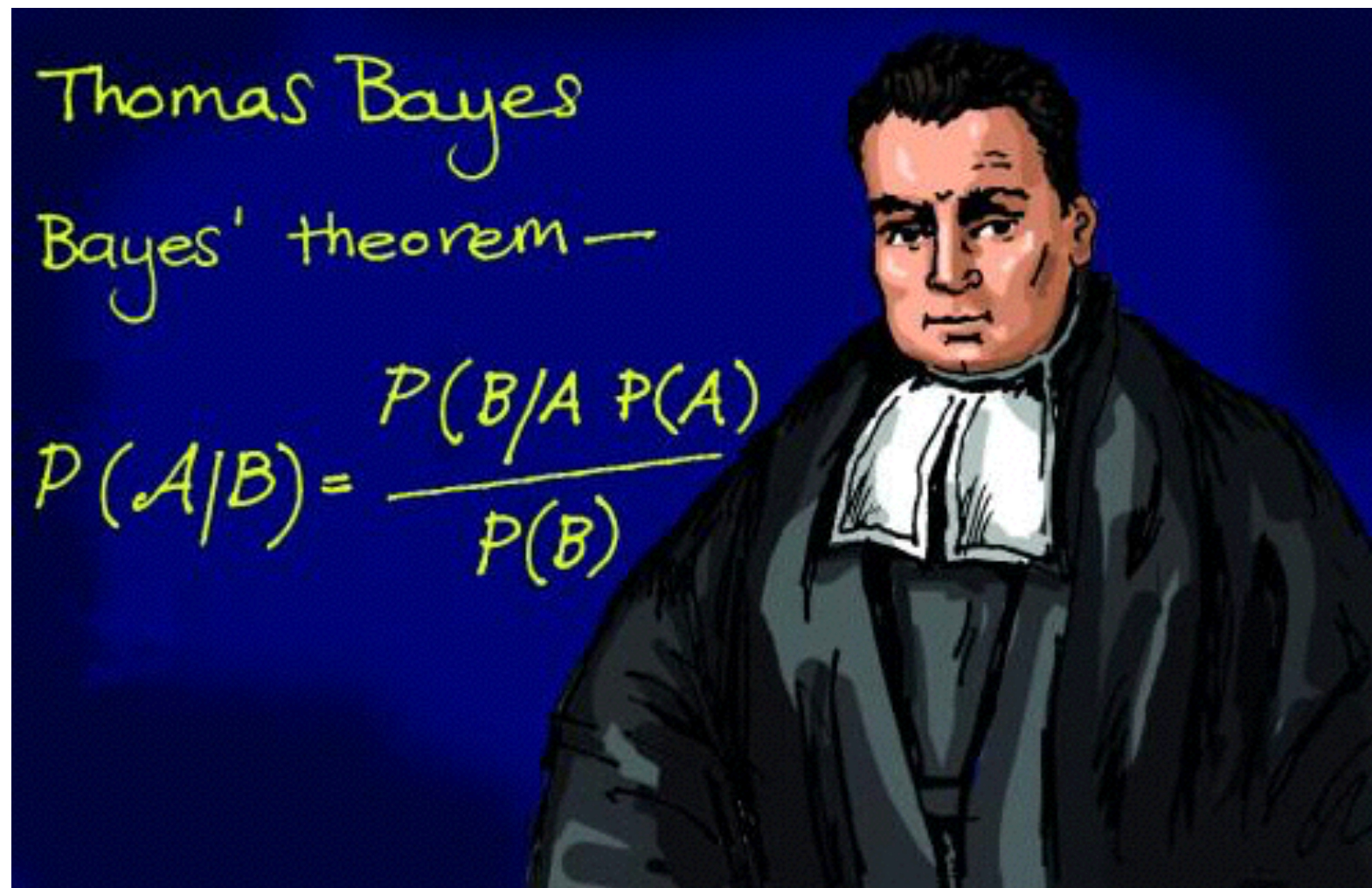


A Brief Overview of Bayesian Inference



Some Facts About Probability

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

$$P(A|B)P(B) = P(B|A)P(A)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$



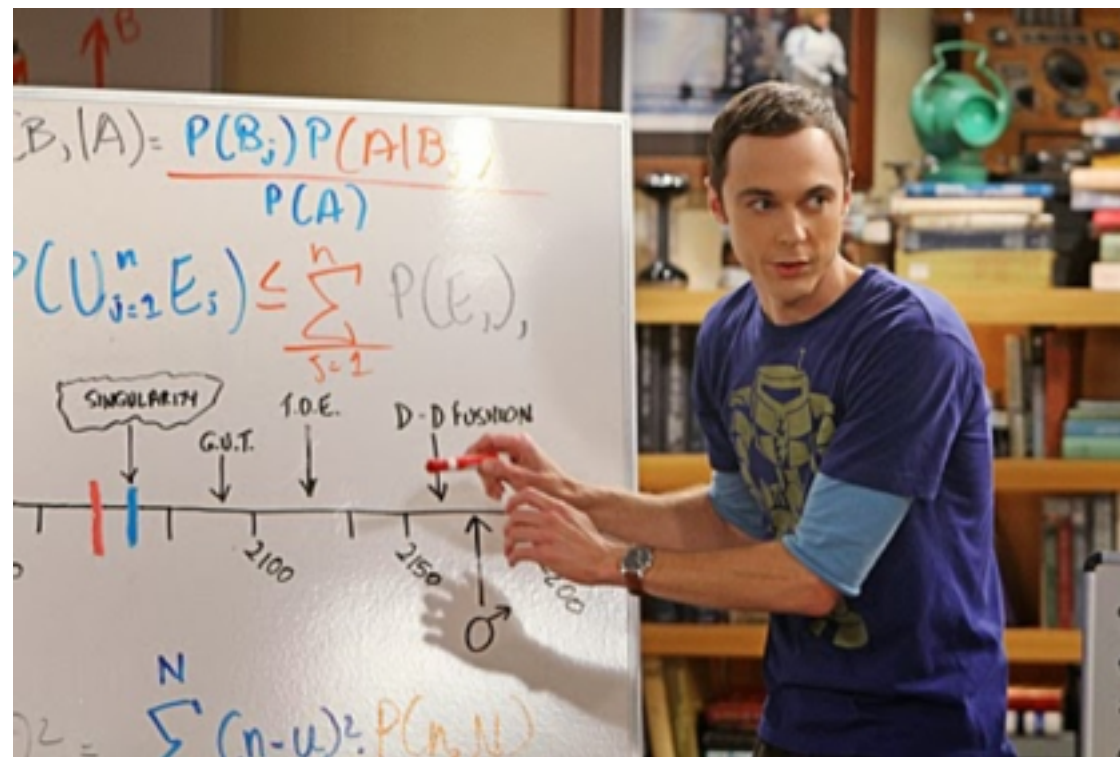
T. Bayes.

Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$



T. Bayes.

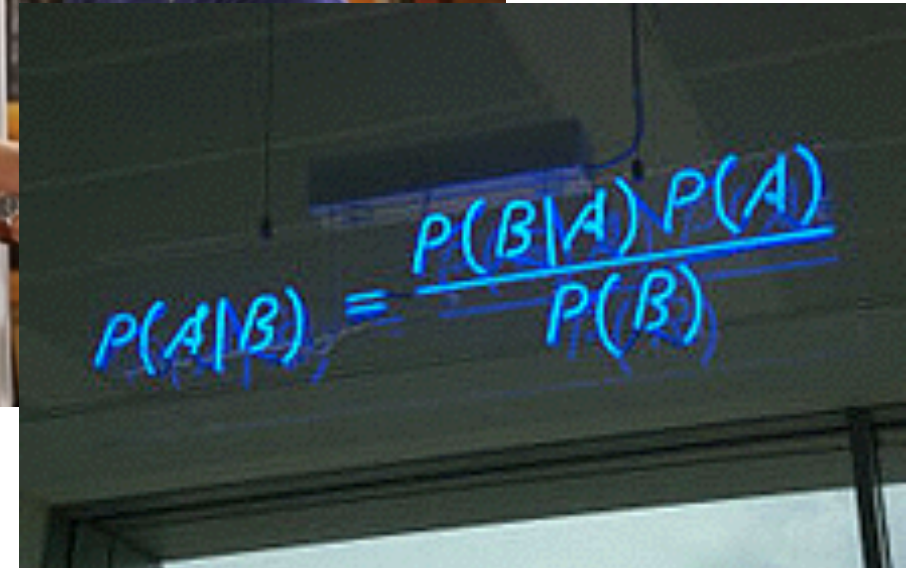
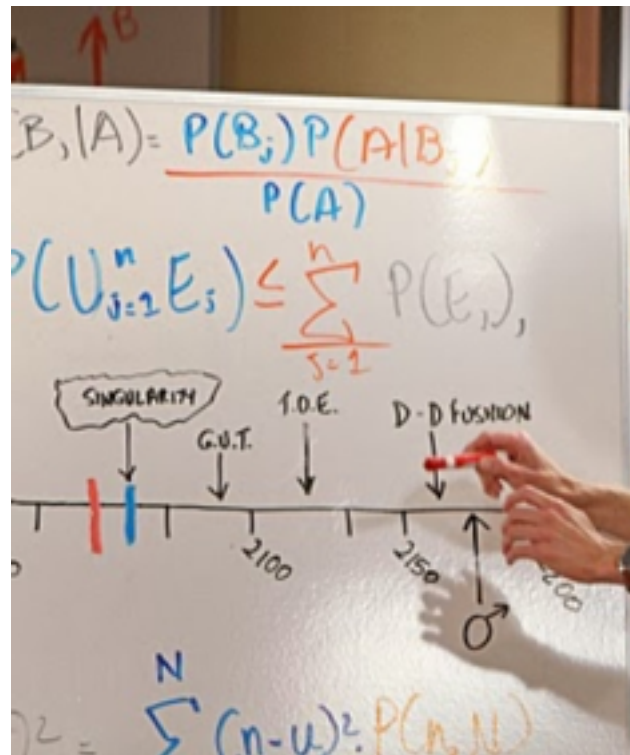


Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$



T. Bayes.

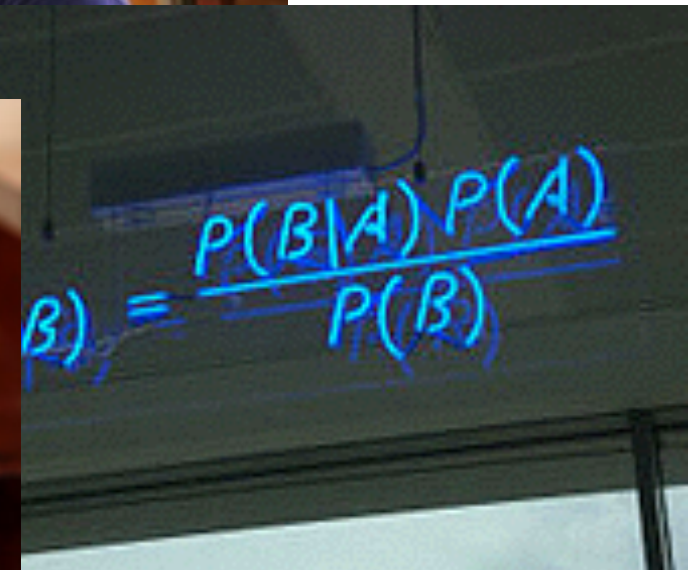
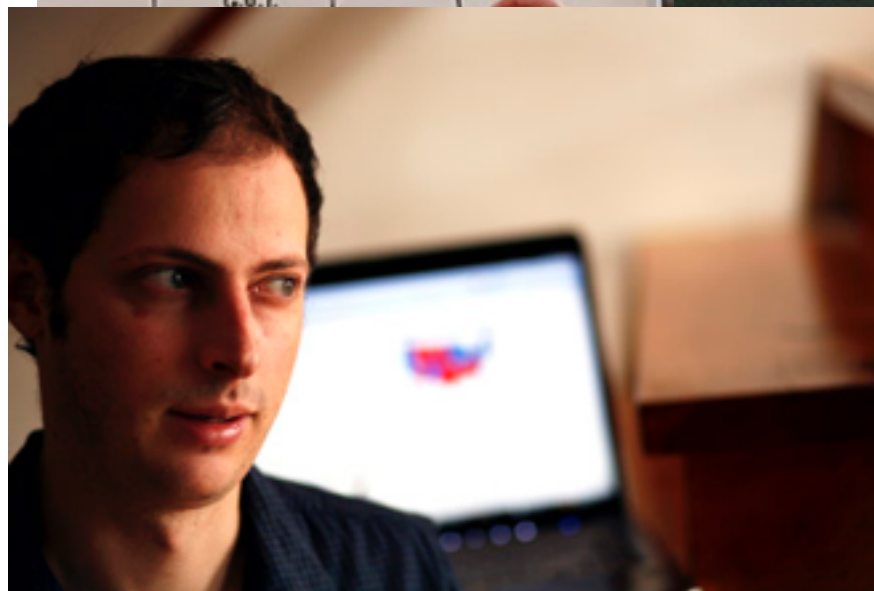
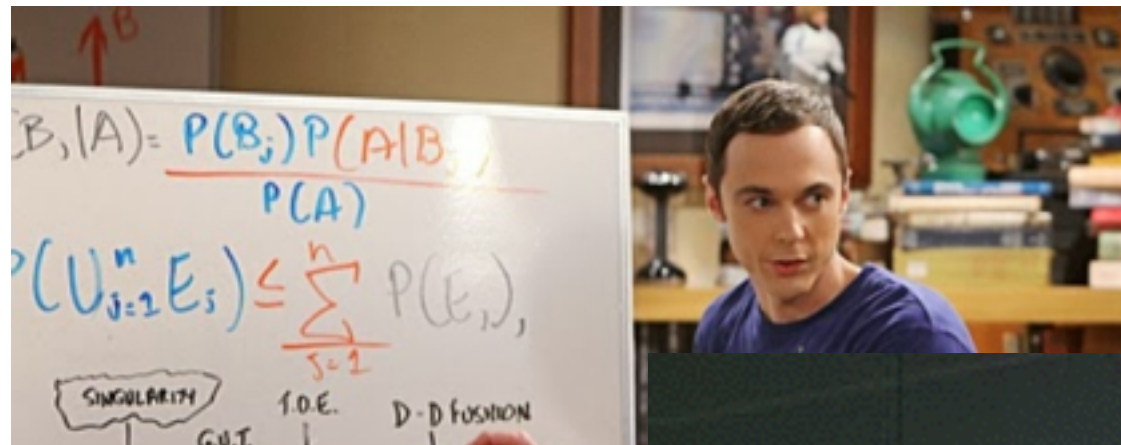


Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$



T. Bayes.

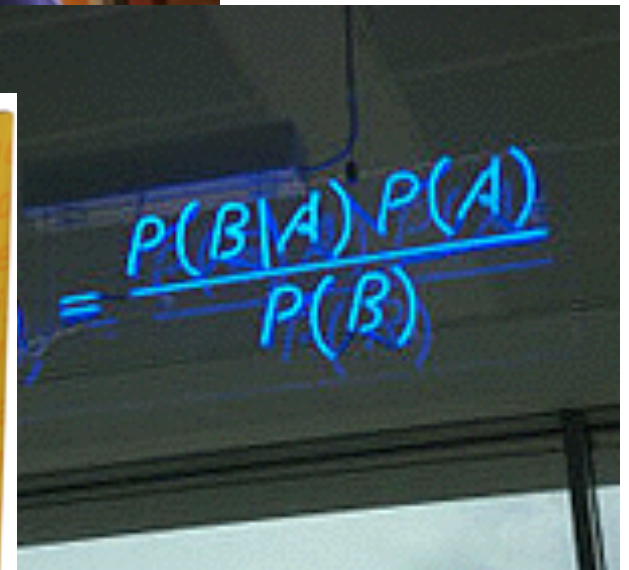
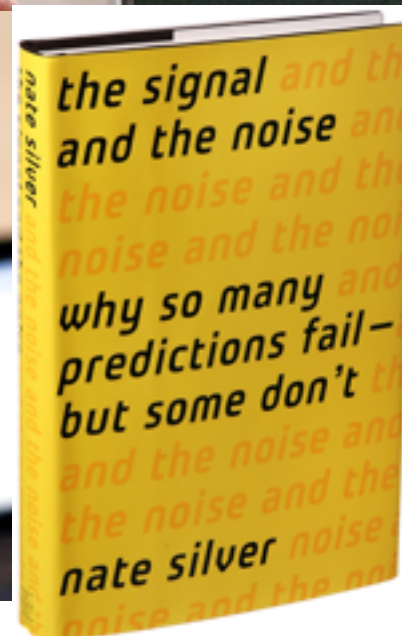
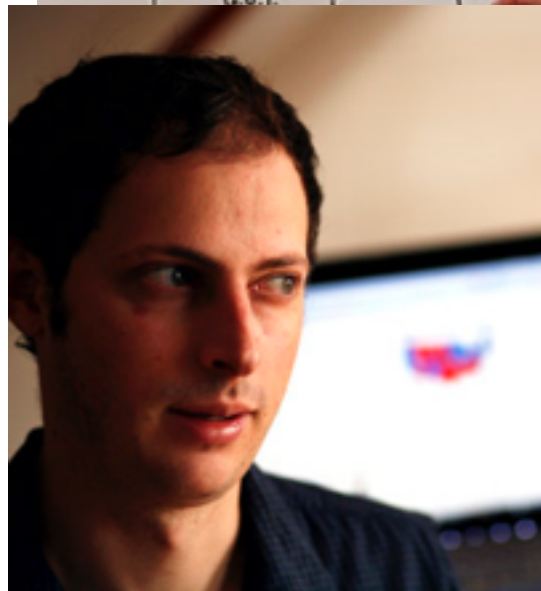
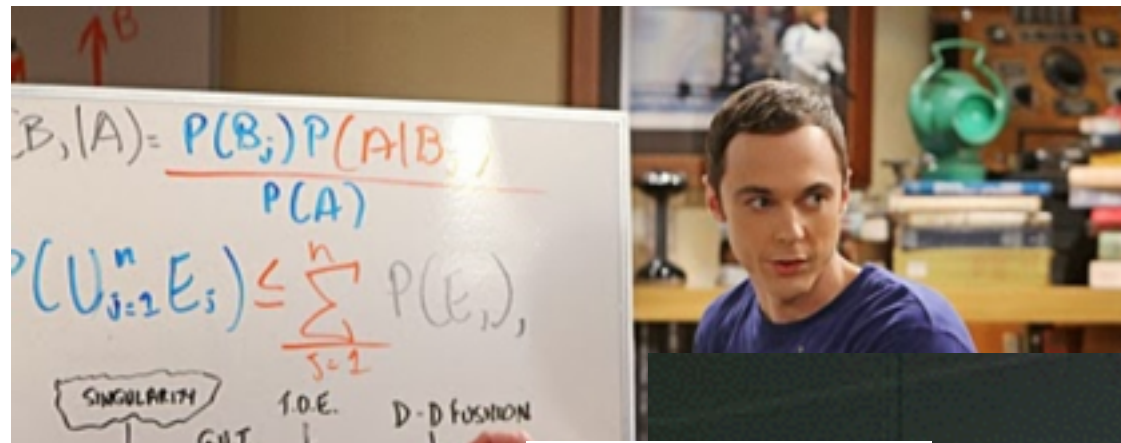


Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$



T. Bayes.



Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$



T. Bayes.

Rev. Thomas Bayes
1701-1761

“An Essay towards solving a
Problem in the Doctrine of Chances”
published in 1763 (Richard Price)

Binomial with a uniform prior on p

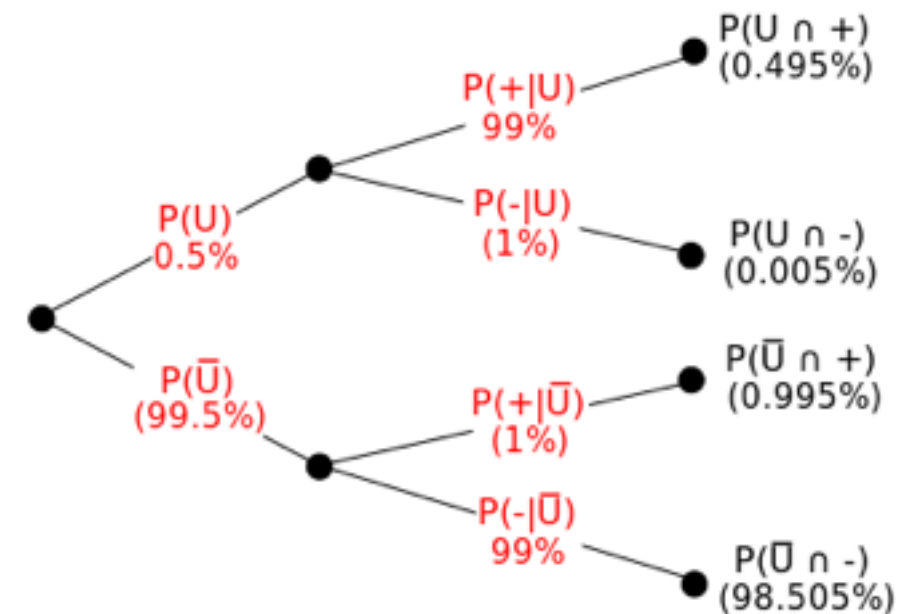
Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Also used with frequentist probabilities!

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Suppose a drug test is 99% sensitive and 99% specific. That is, the test will produce 99% true positive results for drug users and 99% true negative results for non-drug users. Suppose that 0.5% of people are users of the drug. If a randomly selected individual tests positive, what is the probability that he is a user?



$$\begin{aligned}
 P(\text{User} \mid +) &= \frac{P(+ \mid \text{User})P(\text{User})}{P(+ \mid \text{User})P(\text{User}) + P(+ \mid \text{Non-user})P(\text{Non-user})} \\
 &= \frac{0.99 \times 0.005}{0.99 \times 0.005 + 0.01 \times 0.995} \\
 &\approx 33.2\%
 \end{aligned}$$

Bayes Theorem

Posterior

Likelihood

Prior

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

Normalizing Constant
(Marginal Likelihood)

The diagram illustrates the components of Bayes' Theorem. The equation $P(H|D) = \frac{P(D|H)P(H)}{P(D)}$ is centered. A red arrow points from the word 'Posterior' to the term $P(H|D)$ in the numerator. A blue arrow points from the word 'Likelihood' to the term $P(D|H)$ in the numerator. A green arrow points from the word 'Prior' to the term $P(H)$ in the numerator. An orange arrow points from the text 'Normalizing Constant (Marginal Likelihood)' to the term $P(D)$ in the denominator. The terms in the equation are color-coded to match their respective labels: $P(H|D)$ is red, $P(D|H)$ is blue, $P(H)$ is green, and $P(D)$ is orange.