demo

January 27, 2024

[1]:
```
# Import code from Asymptotics.sage
load("Asymptotics.sage")
set_random_seed(0)
```

0.1 Fibonacci

[2]:
```
var('x, k')
F = x/(1-x-x^2)
A, B, C, P, U = DiagonalAsymptotics(F.numerator(), F.denominator(),␣
 ↪show_points=True, show_formula=True)

print("The asymptotic formula evaluated at u is")
print(Prettify((A * sqrt(B))(u_=U[0])) * Prettify(C(u_=U[0]))^k)
```

```
Point:  [0.61803]
The dominant asymptotics of G/H are given by:
```

$$\frac{1}{45}\,(2\,u+93)\left(\frac{1}{9}\,u+\frac{17}{3}\right)^{k}$$

```
When u takes the value:
```

$$-36.43769\ldots$$

```
The asymptotic formula evaluated at u is
0.44721*1.61803^k
```

[ ]:
```
```

0.2 Two-Dimensional Lattice Points

[3]:
```
var('x,y,k')
F = 1/(1-x-y)

print("The generating function for the number of lattice paths from (0,0) to␣
 ↪(x,y) is:")
show(LatexExpr(r"\frac{1}{1-x-y}"))
```

```
image_style = '"display: flex; flex-direction:column; width: 100%; padding-left:␣
↪30%; padding-right:30%"'
show(html('<div style={0}> <img src="lattice.png"> </div>'.format(image_style)))


A, B, C, P, U = DiagonalAsymptotics(F.numerator(), F.denominator(),␣
↪show_points=True, show_formula=True)

print("The asymptotic formula evaluated at u is")
show(LatexExpr(r"{2k \choose k} \sim"), Prettify((A * sqrt(B)/
↪sqrt(2*pi))(u_=U[0]))/sqrt(k) * Prettify(C(u_=U[0]))^k)
```

```
The generating function for the number of lattice paths from (0,0) to (x,y) is:
```

$$\frac{1}{1-x-y}$$

```
<div style="display: flex; flex-direction:column; width: 100%; padding-left: 30%;␣
↪padding-right:30%"> <img src="lattice.png"> </div>
```

```
Point:  [0.5, 0.5]
The dominant asymptotics of G/H are given by:
```

$$\frac{4^k}{\sqrt{\pi k}}$$

```
The asymptotic formula evaluated at u is
```

$$\binom{2k}{k} \sim \frac{0.56419 \cdot 4.0^k}{\sqrt{k}}$$

[4]:
```
A, B, C, P, U = GeneralAsymptotics(F.numerator(), F.denominator(), r=[2,1],␣
↪show_points=True, show_formula=True)

print("The asymptotic formula evaluated at u is")
show(LatexExpr(r"{3k \choose k} \sim"), Prettify((A * sqrt(B)/
↪sqrt(2*pi))(u_=U[0]))/sqrt(k) * Prettify(C(u_=U[0]))^k)
```

```
Point:  [0.66667, 0.33333]
The dominant asymptotics of G/H are given by:
```

$$\frac{\sqrt{6}\sqrt{2}\left(\frac{27}{4}\right)^k}{4\sqrt{\pi k}}$$

```
The asymptotic formula evaluated at u is
```

$$\binom{3k}{k} \sim \frac{0.4886 \cdot 6.75^k}{\sqrt{k}}$$

[ ]:

```

```

0.3 Applications

[5]:

```
var('w,x,y,z,k')
F = (x^2*y^2 - x*y + 1)/(1-x-y-x*y+x*y^2+x^2*y-x^2*y^3-x^3*y^2)

print("The sequence alignment problem has generating function:")
show(LatexExpr(r"\frac{x^2y^2 - xy + 1}{1-x-y-xy+xy^2+x^2y-x^2y^3-x^3y^2}"))

image_style = '"display: flex; flex-direction:column; width: 100%; padding-left:␣
↪30%; padding-right:30%"'
show(html('<div style={0}> <img src="dna.png"> </div>'.format(image_style)))


A, B, C, P, U = DiagonalAsymptotics(F.numerator(), F.denominator(),␣
↪show_points=True)

print("The asymptotic formula evaluated at u is")
show(Prettify((A * sqrt(B)/sqrt(2*pi))(u_=U[0]))/sqrt(k) *␣
↪Prettify(C(u_=U[0]))^k)
```

```
The sequence alignment problem has generating function:
```

$$\frac{x^2y^2 - xy + 1}{1 - x - y - xy + xy^2 + x^2y - x^2y^3 - x^3y^2}$$

```
<div style="display: flex; flex-direction:column; width: 100%; padding-left: 30%;␣
↪padding-right:30%"> <img src="dna.png"> </div>
```

```
Point:  [0.47042, 0.47042]
The asymptotic formula evaluated at u is
```

$$\frac{0.53206 \cdot 4.51891^k}{\sqrt{k}}$$

[6]:

```
# Apery 4 variables diagonal asymptotics
var('w,x,y,z,t')
F = 1/(1-w*(1+x)*(1+y)*(1+z)*(1+y+z+y*z+x*y*z))

print("Apery's method of proving the irrationality of ζ(3) involves determining␣
↪the ratio of two sequences, one of which is ")
show(LatexExpr(r"\sum_{k=1}^n{n \choose k}^2{n+k \choose k}^2"))

#print("This sequence has the generating function:")
#show(LatexExpr(r"\frac{1}{1-w(1+x)(1+y)(1+z)(1+y+z+yz+xyz)}"))
```

```
A, B, C, P, U = DiagonalAsymptotics(F.numerator(), F.denominator(), w+t,␣
↪show_points = True, show_formula = False)

print("The asymptotic formula evaluated at u is")
show(Prettify((A * sqrt(B)/(2*pi*sqrt(2*pi)))(u_=U[0]))/(k*sqrt(k)) *␣
↪Prettify(C(u_=U[0]))^k)
```

Apery's method of proving the irrationality of ζ(3) involves determining the ratio of two sequences, one of which is

$$\sum_{k=1}^{n} \binom{n}{k}^2 \binom{n+k}{k}^2$$

```
Point:  [0.02439, 2.41421, 0.70711, 0.70711]
The asymptotic formula evaluated at u is
```

$$\frac{0.22004 \cdot 33.97056^k}{k^{\frac{3}{2}}}$$

```
[ ]:
```