IEOR 160 Final Project

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PROBLEM 1

I. Executive Summary

(i) The posed problem concerned the game scheduling of nine Little League baseball teams throughout their competition season, from March through May. Constraints on possible schedules include those of team availability, day by day playing restrictions, weekly schedule restrictions, and playing time. They are detailed as follows:

1. Each team must play every other team twice during the course of the season
2. Competitions are conducted every week on Monday, Tuesday, Wednesday, Thursday, and Saturday. Only 1 competition occurs for every weekday, while 4 occur on Saturdays.
3. A team can compete at most once during the week and at most once on a Saturday.
4. The game schedule includes holidays and vacations, during which competitions are not to be held.

(ii) The following schedule was formulated using the proposed solution, and specifies the dates on which each pair of teams will play according to the problem restrictions:

March						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
	3 v 5	1 v 2	4 v 8	7 v 9		1 v 5 2 v 7 3 v 4 6 v 9
19	20	21	22	23	24	25
	1 v 4	2 v 9	3 v 7	6 v 8		1 v 6 4 v 5 2 v 8 3 v 7 (2nd)
26	27	28	29	30	31	
	1 v 3	4 v 9	5 v 6			

April

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
						1
						2 v 5 8 v 9 1 v 6 (2nd) 4 v 7
2	3	4	5	6	7	8
	1 v 7	2 v 9 (2nd)	4 v 6	5 v 8		
9	10	11	12	13	14	15
16	17	18	19	20	21	22
	7 v 8	1 v 9	2 v 4	3 v 6		3 v 9 4 v 7 (2nd) 5 v 6 (2nd) 1 v 8
23	24	25	26	27	28	29
	2 v 6	1 v 3 (2nd)	5 v 9	7 v 8 (2nd)		1 v 9 (2nd) 3 v 8 2 v 6 (2nd) 5 v 7
30						

May

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	1	2	3	4	5	6
	1 v 2 (2nd)	3 v 6 (2nd)	4 v 8 (2nd)	5 v 9 (2nd)		2 v 3 4 v 6 (2nd) 5 v 7 (2nd)
7	8	9	10	11	12	13
	4 v 9 (2nd)	6 v 8 (2nd)	1 v 5 (2nd)	2 v 7 (2nd)		6 v 7 1 v 4 (2nd) 3 v 5 (2nd) 8 v 9 (2nd)
14	15	16	17	18	19	20
	2 v 8 (2nd)	3 v 4 (2nd)	6 v 9 (2nd)	1 v 7 (2nd)		2 v 4 (2nd) 5 v 8 (2nd) 3 v 9 (2nd) 6 v 7 (2nd)
21	22	23	24	25	26	27
	1 v 8 (2nd)	7 v 9 (2nd)	2 v 3 (2nd)	4 v 5 (2nd)		
28	29	30	31			
		2 v 5 (2nd)	3 v 8 (2nd)			

II. Formulating the Solution

(i) In order to successfully create this Little League schedule, we have assigned a number, i, to each of the 72 competitions that must take place and another number, j, to each of the 19 available time blocks. Every set of 4 weekday competitions are considered to be one time block, as are the 4 Saturday competitions. The assignments of i and j are detailed below:

Team Match-Ups:

First Round Team Pairings	i	Second Round Team Pairings	i
1 v 2	1	1 v 2	37
1 v 3	2	1 v 3	38
1 v 4	3	1 v 4	39
1 v 5	4	1 v 5	40
1 v 6	5	1 v 6	41
1 v 7	6	1 v 7	42
1 v 8	7	1 v 8	43
1 v 9	8	1 v 9	44
2 v 3	9	2 v 3	45
2 v 4	10	2 v 4	46
2 v 5	11	2 v 5	47
2 v 6	12	2 v 6	48
2 v 7	13	2 v 7	49
2 v 8	14	2 v 8	50
2 v 9	15	2 v 9	51
3 v 4	16	3 v 4	52
3 v 5	17	3 v 5	53
3 v 6	18	3 v 6	54
3 v 7	19	3 v 7	55

3 v 8	20	3 v 8	56
3 v 9	21	3 v 9	57
4 v 5	22	4 v 5	58
4 v 6	23	4 v 6	59
4 v 7	24	4 v 7	60
4 v 8	25	4 v 8	61
4 v 9	26	4 v 9	62
5 v 6	27	5 v 6	63
5 v 7	28	5 v 7	64
5 v 8	29	5 v 8	65
5 v 9	30	5 v 9	66
6 v 7	31	6 v 7	67
6 v 8	32	6 v 8	68
6 v 9	33	6 v 9	69
7 v 8	34	7 v 8	70
7 v 9	35	7 v 9	71
8 v 9	36	8 v 9	72

(ii) The available times for teams to play are split into blocks within which each team may only play once. There are 19 total, and each block is capable of hosting 4 games, with the exception of the final time block which may host 2. All are detailed as follows:

Time Block Chart

j	Time Blocks
1	March 13-16 (Monday - Thursday)
2	March 18 (Saturday)
3	March 20-23 (Monday - Thursday)
4	March 25 (Saturday)

5	March 27-30 (Monday-Thursday)
6	April 1 (Saturday)
7	April 3-6 (Monday-Thursday)
8	April 17-20 (Monday-Thursday)
9	April 22 (Saturday)
10	April 24-27 (Monday-Thursday)
11	April 29 (Saturday)
12	May 1-4 (Monday-Thursday)
13	May 6 (Saturday)
14	May 8-11 (Monday-Thursday)
15	May 13 (Saturday)
16	May 15-18 (Monday-Thursday)
17	May 20 (Saturday)
18	May 22-25 (Monday-Thursday)
19	May 30-31 (Saturday)

(iii) Proposed Solution
The schedule presented in (I) represents the solution. The AMPL code is shown below. **The output solution from the AMPL program is attached as an appendix to this report.

N and G are parameters representing the number of time blocks and the number of games respectively. The set of binary variables x{i,j} are defined to be 1 only if the ith competition is scheduled in the jth time slot and 0 otherwise.

P1.mod

```
param N; #Number of Time Period Blocks
param G; #Number of Games

var x{i in 1..G,j in 1..N} binary;

maximize nothing: 0;

subject to c1{i in 1..G}: sum{j in 1..N} x[i, j]=1;
```

```
subject to c2{j in 1..N}: sum{i in 1..G} x[i, j]<=4;
subject to c3: sum{i in 1..G} x[i,19]<=2;

subject to d1{j in 1..N}:
x[1,j]+x[2,j]+x[3,j]+x[4,j]+x[5,j]+x[6,j]+x[7,j]+x[8,j]+x[37,j]+x[38,j]+x[39,j]+x[40,j]+x[41,j]+x[42,j]+x[43
,j]+x[44,j]<=1;

subject to d2{j in 1..N}:
x[9,j]+x[10,j]+x[11,j]+x[12,j]+x[13,j]+x[14,j]+x[15,j]+x[1,j]+x[45,j]+x[46,j]+x[47,j]+x[48,j]+x[49,j]+x[50
,j]+x[51,j]+x[37,j]<=1;

subject to d3{j in 1..N}:
x[16,j]+x[17,j]+x[18,j]+x[19,j]+x[20,j]+x[21,j]+x[2,j]+x[9,j]+x[52,j]+x[53,j]+x[54,j]+x[55,j]+x[56,j]+x[57
,j]+x[38,j]+x[45,j]<=1;

subject to d4{j in 1..N}:
x[3,j]+x[10,j]+x[16,j]+x[22,j]+x[23,j]+x[24,j]+x[25,j]+x[26,j]+x[39,j]+x[46,j]+x[52,j]+x[58,j]+x[59,j]+x[6
0,j]+x[61,j]+x[62,j]<=1;

subject to d5{j in 1..N}:
x[4,j]+x[11,j]+x[17,j]+x[22,j]+x[27,j]+x[28,j]+x[29,j]+x[30,j]+x[40,j]+x[47,j]+x[53,j]+x[58,j]+x[63,j]+x[6
4,j]+x[65,j]+x[66,j]<=1;

subject to d6{j in 1..N}:
x[5,j]+x[12,j]+x[18,j]+x[23,j]+x[27,j]+x[31,j]+x[32,j]+x[33,j]+x[41,j]+x[48,j]+x[54,j]+x[59,j]+x[63,j]+x[6
7,j]+x[68,j]+x[69,j]<=1;

subject to d7{j in 1..N}:
x[6,j]+x[13,j]+x[19,j]+x[24,j]+x[28,j]+x[31,j]+x[34,j]+x[35,j]+x[42,j]+x[49,j]+x[55,j]+x[60,j]+x[64,j]+x[6
7,j]+x[70,j]+x[71,j]<=1;

subject to d8{j in 1..N}:
x[7,j]+x[14,j]+x[20,j]+x[25,j]+x[29,j]+x[32,j]+x[34,j]+x[36,j]+x[43,j]+x[50,j]+x[56,j]+x[61,j]+x[65,j]+x[6
8,j]+x[70,j]+x[72,j]<=1;

subject to d9{j in 1..N}:
x[8,j]+x[15,j]+x[21,j]+x[26,j]+x[30,j]+x[33,j]+x[35,j]+x[36,j]+x[44,j]+x[51,j]+x[57,j]+x[62,j]+x[66,j]+x[6
9,j]+x[71,j]+x[72,j]<=1;
```

P1.dat

```
param N:= 19;
param G:= 72;
```

Code Comments & Explanation

Constraint "**c1**" effectively ensures that each game is assigned to one and only one time block. It does so by setting the sum across all time period blocks equal to one for each game.

Constraint "**c2**" effectively ensures that in each time period a maximum of four games can be played total. It does so by setting the sum across all games in a time block less than or equal to 4 for each of the possible time blocks.

Constraint "**c3**" effectively ensures that in the period May 30 - May 31 only two games can be played. It does so by summing up across all games in the last time period and setting it less than or equal to 2.

Each constraint "**dk**" effectively ensures that the kth team can only play one game per time period. It does so by summing across all possible games involving one of the nine teams for the specific period and sets it less than or equal to one.

PROBLEM 2

I. Executive Summary

(i) Once again, the given problem concerns the scheduling of a competition season of Little League games. However, it is augmented by the addition of another team, and the consequent splitting of the league into two separate divisions: the American Division, and the National League, both of which contain five teams. In addition to regular season games, the competitions will include playoff games among three top-ranked teams.

The competition season runs from March 17th through May 31st, and additional constraints are as follows:

1. Each team must play every other team in their division twice and teams of the other division once during the course of the season.
2. Competitions are conducted every week on Monday, Tuesday, Wednesday, Thursday, and Saturday. Only 1 competition occurs for every weekday, while 4 occur on Saturdays.
3. A team can compete at most once during the week and at most once on a Saturday.
4. The game schedule includes holidays and vacations, during which competitions are not to be held.

Particular restrictions regarding the playoff games are as follows:

1. The team from each division with the most wins will be a playoff team.
2. The third playoff team will be a wildcard, chosen from whichever team from either division has the most wins behind the first two playoff teams.
3. The playoff games must be played on the end of the regular season, but may not be started on the end of another day.

(ii) The following schedule was formulated using the proposed solution according to the revised constraints, and specifies the dates on which each pair of teams will play according to the problem restrictions:

March

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
	1 v 6	4 v 10	8 v 9	3 v 5		4 v 9 1 v 5 6 v 10 7 v 8
23	24	25	26	27	28	29
	1 v 2	6 v 10 (2nd)	4 v 5	7 v 9		2 v 4 7 v 10 1 v 3 6 v 8
30	31					
	2 v 10					

April

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
		3 v 5 (2nd)	7 v 8 (2nd)	6 v 9		3 v 7 4 v 5 (2nd) 9 v 10 1 v 2 (2nd)
6	7	8	9	10	11	12
	1 v 10	3 v 9	4 v 7	6 v 8 (2nd)		
13	14	15	16	17	18	19
20	21	22	23	24	25	26
	1 v 3 (2nd)	2 v 8	4 v 6	5 v 7		3 v 6 4 v 8 7 v 9 (2nd) 2 v 5
27	28	29	30			
	2 v 6	3 v 10	5 v 8			

May						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
				1 v 4		1 v 9 2 v 5 (2nd) 3 v 4 6 v 7
4	5	6	7	8	9	10
	1 v 8	6 v 9 (2nd)	2 v 3	7 v 10 (2nd)		1 v 5 (2nd) 2 v 9 3 v 8 6 v 7 (2nd)
11	12	13	14	15	16	17
	1 v 7	5 v 6	2 v 4 (2nd)	8 v 10		2 v 7 5 v 10 3 v 4 (2nd) 8 v 9 (2nd)
18	19	20	21	22	23	24
	1 v 4 (2nd)	2 v 3 (2nd)	5 v 9	8 v 10 (2nd)		
25	26	27	28	29	30	31
		9 v 10 (2nd)	Playoff 1	Playoff 2		Playoff 3

II. Formulating the Solution

(i) The following chart details the corresponding team pairings (for both the first and second time each team will face each other, if applicable) and variables. The pairings are represented with a team number (1-10) versus another team (1-10). The variables X[i,j] represent the game to be played (subscript i, numbered 1-68, including playoff games), in one of the 18 time blocks (subscript j, numbered 1-18).

Team Match-Ups:

First Time Teams Play Each Other	i	Second Time Teams Play Each Other	i
1 v 2	1	5 v 10	35
1 v 3	2	6 v 7	36
1 v 4	3	6 v 8	37
1 v 5	4	6 v 9	38
1 v 6	5	6 v 10	39
1 v 7	6	7 v 8	40
1 v 8	7	7 v 9	41

1 v 9	8	7 v 10	42
1 v 10	9	8 v 9	43
2 v 3	10	8 v 10	44
2 v 4	11	9 v 10	45
2 v 5	12	1 v 2	46
2 v 6	13	1 v 3	47
2 v 7	14	1 v 4	48
2 v 8	15	1 v 5	49
2 v 9	16	2 v 3	50
2 v 10	17	2 v 4	51
3 v 4	18	2 v 5	52
3 v 5	19	3 v 4	53
3 v 6	20	3 v 5	54
3 v 7	21	4 v 5	55
3 v 8	22	6 v 7	56
3 v 9	23	6 v 8	57
3 v 10	24	6 v 9	58
4 v 5	25	6 v 10	59
4 v 6	26	7 v 8	60
4 v 7	27	7 v 9	61
4 v 8	28	7 v 10	62
4 v 9	29	8 v 9	63
4 v 10	30	8 v 10	64
5 v 6	31	9 v 10	65
5 v 7	32	Playoff Game 1	66
5 v 8	33	Playoff Game 2	67

5 v 9	34	Playoff Game 3	68

(ii) The available times for teams to play are split into blocks within which each team may only play once. There are 18 total, and each block is capable of hosting 4 games. All are detailed as follows:

Time Period Chart

j in X[i, j]	Block of Dates Corresponding
1	March 17-20 (Monday - Thursday)
2	March 22 (Saturday)
3	March 24-27 (Monday - Thursday)
4	March 29 (Saturday)
5	March 31 - April 3 (Monday-Thursday)
6	April 5 (Saturday)
7	April 7-10 (Monday-Thursday)
8	April 21-24 (Monday-Thursday)
9	April 26 (Saturday)
10	April 28 - May 1 (Monday-Thursday)
11	May 3 (Saturday)
12	May 5-8 (Monday-Thursday)
13	May 10 (Saturday)
14	May 12-15 (Monday-Thursday)
15	May 17 (Saturday)
16	May 19-22 (Monday-Thursday)
17	May 27-29 (Monday-Thursday)
18	May 31 (Saturday)

(iii) Proposed Solution
The schedule presented in (I) represents the solution. The AMPL code is shown below. **The output solution from the AMPL program is attached as an appendix to this report.

Part2.mod

```
param N; #Number of Time Period Blocks
param G; #Number of Games

var x{i in 1..G,j in 1..N} binary;

maximize nothing: 0;

subject to c1{i in 1..G}: sum{j in 1..N} x[i, j]=1;
subject to c2{j in 1..N}: sum{i in 1..G} x[i, j]<=4;

subject to d1{j in 1..N}:
x[1,j]+x[2,j]+x[3,j]+x[4,j]+x[5,j]+x[6,j]+x[7,j]+x[8,j]+x[9,j]+x[46,j]+x[47,j]+x[48,j]+x[49,j]<=1;

subject to d2{j in 1..N}:
x[1,j]+x[10,j]+x[11,j]+x[12,j]+x[13,j]+x[14,j]+x[15,j]+x[16,j]+x[17,j]+x[46,j]+x[50,j]+x[51,j]+x[52,j]<=1;

subject to d3{j in 1..N}:
x[2,j]+x[10,j]+x[18,j]+x[19,j]+x[20,j]+x[21,j]+x[22,j]+x[23,j]+x[24,j]+x[47,j]+x[50,j]+x[53,j]+x[54,j]<=1;

subject to d4{j in 1..N}:
x[3,j]+x[11,j]+x[18,j]+x[25,j]+x[26,j]+x[27,j]+x[28,j]+x[29,j]+x[30,j]+x[48,j]+x[51,j]+x[53,j]+x[55,j]<=1;

subject to d5{j in 1..N}:
x[4,j]+x[12,j]+x[19,j]+x[25,j]+x[31,j]+x[32,j]+x[33,j]+x[34,j]+x[35,j]+x[49,j]+x[52,j]+x[54,j]+x[55,j]<=1;

subject to d6{j in 1..N}:
x[5,j]+x[13,j]+x[20,j]+x[26,j]+x[31,j]+x[36,j]+x[37,j]+x[38,j]+x[39,j]+x[56,j]+x[57,j]+x[58,j]+x[59,j]<=1;

subject to d7{j in 1..N}:
x[6,j]+x[14,j]+x[21,j]+x[27,j]+x[32,j]+x[36,j]+x[40,j]+x[41,j]+x[42,j]+x[56,j]+x[60,j]+x[61,j]+x[62,j]<=1;

subject to d8{j in 1..N}:
x[7,j]+x[15,j]+x[22,j]+x[28,j]+x[33,j]+x[37,j]+x[40,j]+x[43,j]+x[44,j]+x[57,j]+x[60,j]+x[63,j]+x[64,j]<=1;

subject to d9{j in 1..N}:
x[8,j]+x[16,j]+x[23,j]+x[29,j]+x[34,j]+x[38,j]+x[41,j]+x[43,j]+x[45,j]+x[58,j]+x[61,j]+x[63,j]<=1;
```

subject to d10{j in 1..N}:
x[9,j]+x[17,j]+x[24,j]+x[30,j]+x[35,j]+x[39,j]+x[42,j]+x[44,j]+x[45,j]+x[59,j]+x[62,j]+x[64,j]<=1;

Part2.dat
param N:= 16;
param G:= 64;

Code Comments & Explanation

In the data file for part two, parameter N, which is equal to number of time periods is 16. Even though there is technically 18 time periods on the 2014 calendar for games, it is lessened by two in this formulation. Since there are 16 time periods in this formulation with 4 opportunities for games. This mean 64 games can be played in the 16 time periods from the formulation. The purpose of lessening the time periods is that there can be only 1 play off games per day after the regular season, which can not take place on the same day as any other game. After the 16 time periods available days for games are the next Tuesday, Wednesday, Thursday, and Saturday. The last regular season game will take place on Tuesday followed by one playoff game on each of the last three days.

Constraint "**c1**" effectively ensures that each game is assigned to one and only one time block. It does this by summing across all time period blocks to and setting it equal to one.

Constraint "**c2**" effectively ensures that in each time period a maximum of four games can be played total. It does so by summing up across all games in all periods except the last one and setting it less than or equal to 4.

Constraint "**di**" effectively ensure that each team can only play one game per time period. It does so by summing up across all possible games involving one of the nine teams for the specific period and sets it less than or equal to one.