# Linear Programming
# Final Paper

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# INTRODUCTION

Nike's Running department is making a commercial to advertise the release of their new Track Shoe. They want to show off their new shoe my staging a four by four hundred relay where one team will be wearing all Nikes, one team will be wearing Adidas, one team will be wearing Asics, one team will be wearing Reebok, and one team will be wearing Mizuno. The Nike team will be made up of all Nike Sponsored athletes, which Nike has all around the world.

# PROBLEM

Nike will hold the relay in Beaverton, Oregon at their world headquarters. Nike wants to build a four by four hundred relay team with several requirements including popularity, race time, gender, and ethnicity.

Nike has a total of 30 athletes around the world from 8 different countries: United States, Jamaica, Australia, Uganda, China, Kenya, United Kingdom, and Canada. Seven of the athletes are women and twenty-three of the athletes are men. Each athlete has a popularity rating from 1 to 10. The 400 meter times of each runner are listed, as well as their age and cost of flight to Nike's headquarters in Beaverton, Oregon.

# STATEMENT

Nike wants to make a great commercial and show off their shoe for the cheapest price possible. Therefore their goal is to minimize the cost of hosting the athletes and flying their athletes out to their headquarters in Beaverton, Oregon. They want to do this while having one female on the relay team, one athlete from each country on the relay team, and a certain level of popularity with the general public.

# METHOD

We will solve this problem by using a concept of linear programming.

*Decision variables*

| Variable | Description |
|---|---|
| $X_i =$ Google Docs is not very convenient with typing equations. After we finalize the content I'll fix all of that Word | Specifically this problem involves Binary Integer Programming w the decision variables (X_i) correspond to athletes one through thi Each decision variable is either a 0 or 1. |

*Parameters*

| Parameter | Description |
|---|---|
| $T_i$ | 400 meter time for athlete i |
| $P_i \in \{1, 2, ..., 10\}$ | Popularity score for athlete i, 1 denoting highest popularity, 10 lov |
| $F_i$ | Cost of flight ticket for athlete i to travel to Oregon |
| $R = 400$ | Standard hotel room cost per day for all athletes |
| $D_i$ | Number of days athlete i will stay in the hotel if he/she were chose |

A detailed table of the above information is presented in the appendix.

*Objective Function*

The objective is to minimize the cost of flying and housing the athletes during the production of the commercial. Therefore, we seek to minimize the product of total cost per athlete and the decision variable.

$$min \sum_{i=1}^{30} (F_i + R \cdot D_i) \cdot X_i$$

*Constraints*

In order to ensure that there are exactly four athletes on the relay team, one constraint is that the sum of all decision variables is equal to four.

$$\sum_{i=(1,...,30)} X_i = 4$$

In order to make sure that the relay team is quick enough the decision variable of each athlete multiplied by the 400 meter time of each athlete is less than or equal to 220 seconds. Where the time of each athlete's 400 meter time is represented by $T_i$.

$$\sum_{i=(1,...,30)} X_i * T_i <= 220$$

Each popularity score of each athlete ranges from 1 as the highest to 10 at the lowest. In order to ensure that the team is on average popular enough among the general public we will multiply each decision variable of each athlete by their individual popularity score and divide the product by four in order to make it less than or equal to a desired number. Each athlete's popularity score is represented by the symbol $P_i$. In this example we set the upper bound for the popularity score to be 7.

$$\left(\sum_{i=(1,...,30)} X_i * P_i\right) / 4 <= 7$$

Nike would also like for there to be at least one female on the relay team so that they can show off the female version of their shoes. In order to ensure this, we sum up all the decision variable of all thirty athletes and set it greater than or equal to at least one.

$$\sum_{i=(\text{set of women})} X_i >= 1$$

Nike would also to showcase its athletes around the world by having the four members of the relay from four different countries around the world. This would be guaranteed by one constraint for each of the eight countries. These constraints would sum up the decision variable of all the athletes belonging to each country and make the sum less than or equal to one.

$$\sum_{i \ni \text{set } j} X_i <= 1 \quad \forall \ j=(\text{all } X_i \text{ in country } j)$$

Nike wants at least one member of the relay team to be from the United States. To guarantee this we

sum all the decision variable of athletes from the United States and set equal to one. Nike also wants one member to be from a country in Africa. To guarantee this we sum up the decision variables from athletes from any country within Africa and setting it equal to one.

$$\sum_{i \ni \text{set } j} X_i = 1 \quad \forall \ j=(X_i \text{ in US or African Countries})$$

# APPENDIX

*Complete Chart of Parameters*

| Country | Name of Runner | Gender | 400 m time | Cost of flight | Popularity | Hotel Room Cost | Nights Spent |
|---|---|---|---|---|---|---|---|
| USA (Chicago) | A | M | 50 | 250 | 5 | 400 | 1 |
| Kenya | B | F | 54 | 1300 | 2 | 400 | 2 |
| Australia | C | F | 53 | 1500 | 1 | 400 | 4 |
| Uganda | D | M | 51 | 1200 | 6 | 400 | 3 |
| Kenya | E | M | 48 | 1300 | 10 | 400 | 3 |
| Jamaica | F | M | 46 | 600 | 7 | 400 | 2 |
| USA (Los Angeles) | G | M | 47 | 130 | 1 | 400 | 1 |
| Jamaica | H | M | 51 | 600 | 6 | 400 | 2 |
| Australia | I | M | 47 | 1500 | 4 | 400 | 4 |
| USA (Dallas) | J | M | 48 | 160 | 9 | 400 | 1 |
| United Kingdom | K | M | 46 | 1400 | 10 | 400 | 2 |
| USA (Raleigh) | L | M | 51 | 300 | 8 | 400 | 1 |
| United Kingdom | M | M | 47 | 1400 | 9 | 400 | 2 |
| Australia | N | M | 50 | 1500 | 10 | 400 | 4 |
| USA (Miami) | O | M | 47 | 250 | 3 | 400 | 1 |
| China | P | M | 48 | 300 | 5 | 400 | 4 |
| USA (Denver) | Q | M | 46 | 140 | 6 | 400 | 1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Jamaica | R | M | 50 | 600 | 1 | 400 | 2 |
| Canada (Quebec) | S | M | 51 | 300 | 9 | 400 | 1 |
| Jamaica | T | F | 55 | 600 | 5 | 400 | 2 |
| Australia | U | F | 56 | 1500 | 5 | 400 | 4 |
| Uganda | V | F | 52 | 1200 | 6 | 400 | 3 |
| Kenya | W | M | 48 | 1300 | 8 | 400 | 2 |
| Canada (Toronto) | X | M | 46 | 400 | 1 | 400 | 1 |
| USA (Berkeley) | Y | M | 47 | 100 | 8 | 400 | 1 |
| Jamaica | Z | F | 58 | 600 | 2 | 400 | 2 |
| USA (Baltimore) | AB | M | 48 | 175 | 1 | 400 | 1 |
| Canada (Vancouver) | AC | M | 51 | 250 | 3 | 400 | 1 |
| United Kingdom | AD | M | 48 | 1400 | 10 | 400 | 2 |
| United Kingdom | AE | F | 55 | 1400 | 4 | 400 | 2 |