已知某加密芯片通过密钥对数据进行相关运算，假设无密钥、密钥为1和密钥为0的出现服从马尔科夫特性(请注意以下一步转移概率矩阵与课堂讲述可能不同)，当前密钥为1下一密钥为1、0的或者无密钥的概率分别是0.5，0.2，0.3，当前密钥为0下一密钥为1、0的或者无密钥的概率分别是0、0.7、0.3，当前无密钥下一密钥为1、0的或者无密钥的概率分别是0.4、0.2、0.4。通过能耗收集工具观测该芯片，发现采用密钥1进行运算时，以0.9的概率观测到下图标记为1的宽波形，0.1的概率观测到下图标记为0的窄波形，当采用密钥0进行运算时，以0.8的概率观测到窄波形，以0.2的概率观测到宽波形，当无密钥时，以概率0.6观测到宽波形，以概率0.4观测到窄波形。回答下述问题

1.请分析该HMM的三要素

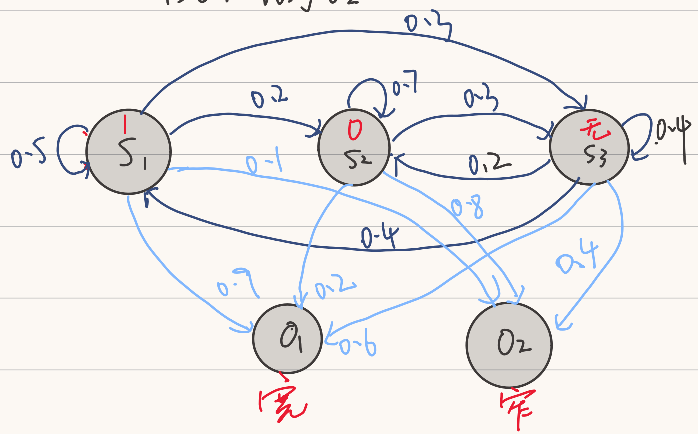
2.若初始状态为 (1,0,0)，请给出一条可能的观测序列

3.若观测序列为O=(宽，窄，宽)，初始状态为(0.2,0.4,0.4)，求该观测序列出现的概率

4.若观测到序列O=(宽，窄，宽)，用近似算法求最大概率出现的隐藏状态序列I(通常初始状态取平稳状态)

5. 若观测到序列O=(宽，窄，宽)，用Viterbi算法求最大概率出现的隐藏状态序列I(通常初始状态取平稳状态)

解：分别将密钥为1，密钥为0和无密钥记为 S_1, S_2, S_3，宽波形记为 O_1，窄波形记为 O_2



1. $\mu=(A, B, \pi)$

$$A=\begin{pmatrix} 0.5 & 0 & 0.4 \\ 0.2 & 0.7 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{pmatrix} \qquad B=\begin{pmatrix} 0.9 & 0.2 & 0.6 \\ 0.1 & 0.8 & 0.4 \end{pmatrix}$$

π为平稳分布，已知一步转移概率矩阵 A

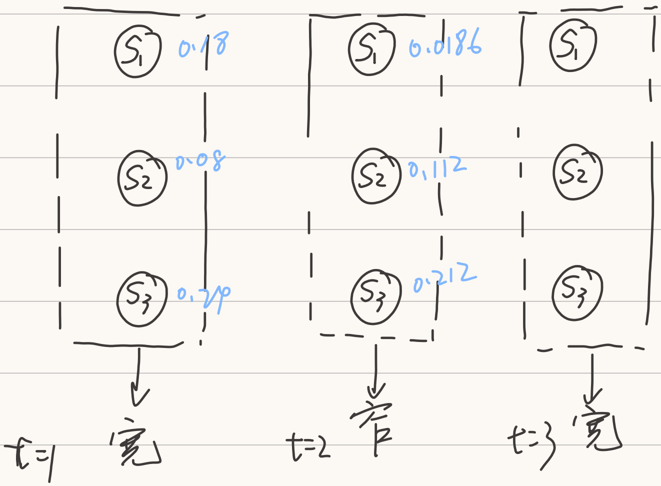
则有 $\begin{cases} Ax = X \\ \delta(x) = \sum_{i=1}^{n} x_i = 1 \end{cases}$ $\quad \begin{pmatrix} 0.5 & 0 & 0.4 \\ 0.2 & 0.7 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

又 $\delta(x)=1$ 即 $x_1+x_2+x_3=1$ 解得 $x = \begin{pmatrix} 4/15 \\ 2/5 \\ 1/3 \end{pmatrix}$

即 $\pi = \begin{pmatrix} 4/15 \\ 2/5 \\ 1/3 \end{pmatrix}$

2、 O = (宽, 窄, 宽)

3.



时刻1 $\quad \delta_1 = \pi_1 b_{o_1 1} = 0.2 \times 0.9 = 0.18$

$\delta_2 = \pi_2 b_{o_1 2} = 0.4 \times 0.2 = 0.08$

$\delta_3 = \pi_3 b o_{13} = 0.4\times0.6 = 0.24$

时刻2 $\delta_2(1) = (0.18\times0.5 + 0.24\times0.4)\,0.1 = 0.0186$

$\delta_2(2) = (0.18\times0.2 + 0.08\times0.7 + 0.24\times0.2)\,0.8 = 0.112$

$\delta_2(3) = (0.18\times0.3 + 0.08\times0.3 + 0.24\times0.4)\,0.4 = 0.212$

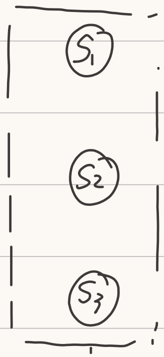
时刻3. $\delta_3(1) = (0.0186\times0.5 + 0.212\times0.4)\,0.9 = 0.08469$

$\delta_3(2) = (0.0186\times0.2 + 0.112\times0.7 + 0.212\times0.2)\,0.2 = 0.024904$

$\delta_3(3) = (0.0186\times0.3 + 0.112\times0.3 + 0.212\times0.4)\,0.6 = 0.074388$

$\therefore P = 0.08469 + 0.024904 + 0.074388 = 0.183982$

4.



平稳. $\left(\frac{4}{15}, \frac{2}{5}, \frac{1}{3}\right)$

当T=1时，观测到O1=宽，则对于

S_1, $\pi_1 b o_{11} = \frac{4}{15}\times0.9 = 0.24$

S_2, $\pi_1 b o_{12} = \frac{2}{5}\times0.2 = 0.08$

S_3, $\pi_1 b o_{13} = \frac{1}{3}\times0.6 = 0.2$

$\therefore \arg\max_{i} \varphi_1(i) = 1$，$\varphi_1(i) = 0.24$，$i=1$

$T=2$时，观测到 o_2=窄，则有

① $\varphi_{T-1}(1)\, a_{11}\, b_{窄} = 0.24 \times 0.5 \times 0.1 =$

✓② $\varphi_{T-1}(1)\, a_{12}\, b_{窄} = 0.24 \times 0.2 \times 0.8 = 0.0384$

③ $\varphi_{T-2}(1)\, a_{13}\, b_{窄} = 0.24 \times 0.3 \times 0.4 =$

$T=3$时，观测到 o_3=宽，则有

① $\varphi_{T-1}(2)\, a_{21}\, b_{宽} = 0.0384 \times 0 = 0$

② $\varphi_{T-1}(2)\, a_{22}\, b_{宽} = 0.0384 \times 0.7 \times 0.2 =$

✓③ $\varphi_{T-2}(2)\, a_{23}\, b_{宽} = 0.0384 \times 0.3 \times 0.6 = 0.006912$.

故最大概率序列为 (1, 2, 3)

5. 当 $T_1=1$时，$\delta_1(i) = \pi_i b_{o_1 i}$　$\psi_1(i)=0$

$T=2$，$\delta_2(1) = \max\{0.24 \times 0.5, 0.08 \times 0, 0.2 \times 0.4\} \times 0.1$

$= 0.012$

$\delta_2(2) = \max\{0.24 \times 0.2, 0.08 \times 0.7, 0.2 \times 0.2\} \times 0.8$

$= 0.0448$

$\delta_2(3) = \max\{0.24 \times 0.3, 0.08 \times 0.3, 0.2 \times 0.4\} \times 0.4$

$= 0.032$

S_1 0.24

S_2 0.08

S_3 0.2

当T=3时. ① $\delta_3(1) = \max\{0.012\times0.5,\ 0.032\times0.4\}\times0.9$

$= 0.01152$

$\delta_3(2) = \max\{0.012\times0.2,\ 0.0448\times0.7,\ 0.032\times0.2\}\times0.2$

$= 0.006272$

$\delta_3(3) = \max\{0.012\times0.3,\ \overset{0.01344}{0.0448\times0.3},\ 0.032\times0.4\}\times0.4$

$= 0.005376$

$(3,\ 3,\ 1)$