Field calculation for the horn waveguide transition in the single-mode approximation of the cross-sections method

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ABSTRACT

We investigate the waveguide propagation of polarized monochromatic light in a smoothly irregular transition between two regular planar dielectric waveguides. The single-mode approximation of the cross-sections method is used. The smooth evolution of the electromagnetic field propagating mode is calculated. The calculation is performed using the regularized stable numerical method.

Keywords: open waveguides, irregular waveguides, guided modes, cross-section method, stable numerical method

1. INTRODUCTION. PHYSICAL SETTING OF THE PROBLEM

We consider the solution of the direct problem, i.e., the mode field calculation, in an integrated optical waveguide transition of the horn type. We study the waveguide propagation of the polarized monochromatic light in the smoothly irregular transition between two regular planar dielectric waveguides (Fig. 1). The propagating electromagnetic radiation satisfied the homogeneous Maxwell equations (in Gaussian system of units)[1–5]

$$\mathrm{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}; \quad \mathrm{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}; \tag{1}$$

where c is the velocity of light in free space, and the isotropic constitutive equations

$$\mathbf{D} = \varepsilon\mathbf{E}, \quad \mathbf{B} = \mu\mathbf{H}. \tag{2}$$

For piecewise constant ε, μ Eqs (1) and (2) are valid in the domains of ε, μ continuity, and at the discontinuity surfaces (interlayer interfaces) the tangential boundary conditions are additionally used[1–5]

$$\tilde{\mathbf{H}}^{\tau}\Big|_1 = \tilde{\mathbf{H}}^{\tau}\Big|_2, \quad \tilde{\mathbf{E}}^{\tau}\Big|_1 = \tilde{\mathbf{E}}^{\tau}\Big|_2 \tag{3}$$

The uniqueness of the direct problem solution is provided by the asymptotic conditions[1]

$$\left\|\tilde{\mathbf{E}}^{\tau}\right\|\Big|_{x\to\pm\infty} < +\infty, \quad \left\|\tilde{\mathbf{H}}^{\tau}\right\|\Big|_{x\to\pm\infty} < +\infty. \tag{4}$$

The aim of the paper is to study the variation of the guided mode electromagnetic field in the direction of propagation (along the Oz axis). The left-hand part of Fig. 1 where $z \leq 0$ shows the planar waveguide formed by the waveguide layer with the refractive index n_f and the thickness h_I. The right-hand part of the Figure at $z \geq z_1$ shows the planar waveguide formed by two waveguide layers, one having the refractive index n_f and the thickness d_0 and the other having the refractive index n_l and the thickness h ($n_j^2 = \varepsilon_j\mu_j$). The waveguide layers lie on the substrate with the refractive index n_s, and are covered from above by the coating layer with the refractive index. Between the planar waveguides there is a waveguide transition. At first ($0 \leq z \leq z_0$) it is homogeneous, and then ($z_0 \leq z \leq z_1$) inhomogeneous.

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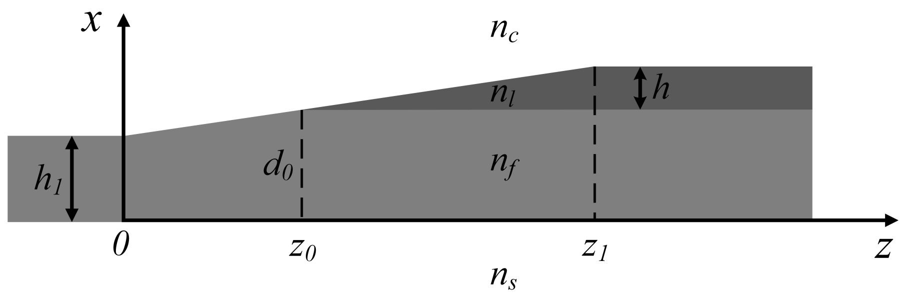

Figure 1. Waveguide transition of the horn type

2. THE CROSS-SECTION METHOD FOR INTEGRATED OPTICAL WAVEGUIDE

To describe the waveguide propagation of monochromatic light in a smoothly irregular integrated optical waveguide the cross-section method[6] is often used. The method is based on the adiabatic approximation of the asymptotic expansion of locally plane waves, simplified by two additional assumptions: 1) In the derivatives of the adiabatic waveguide modes only the zero-order contributions are taken into account. 2) At the points of irregular boundaries instead of tangent planes their approximation by horizontal projections is used to formulate the boundary conditions.[7–10]

The use of the second assumption reduces the list of the complete system of modes used in the method to the system of modes of the regular reference waveguide.[11–13] The first assumption leads to the appearance of a system of ordinary differential equation for the expansion coefficients of the general solution in the method of cross sections.[6]

However, in spite of the simplicity of the cross sections method, the construction of the electromagnetic field evolution even for an individual mode propagating along the nonuniform waveguide transition is a somewhat difficult problem. The difficulties are related to the variable growth rate of the phase deceleration coefficient with the transition thickness and other specific features of the behavior of modal dispersion curves.

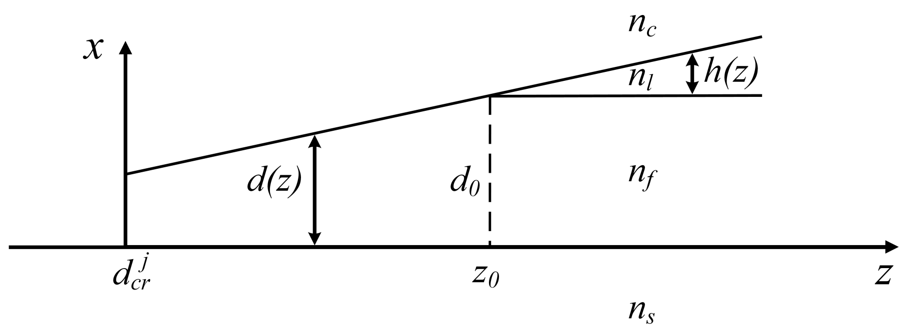

Figure 2. Nonuniform waveguide transition: the dependence of thicknesses upon the coordinate

Therefore, we concentrate our efforts at calculating the electromagnetic field of a separate guided mode of the regular reference waveguide.

The x-distributions of the fields of $TE-$ and $TM-$polarized modes of a regular waveguide satisfy the equations:[2–5]

$$\frac{d^2 E_y}{dx^2} + {k_0}^2 \left(\varepsilon\mu - \beta^2(z)\right) E_y\left(x\right) = 0 \qquad \varepsilon \frac{d}{dx}\left(\frac{1}{\varepsilon}\frac{dH_y}{dx}\right) + {k_0}^2 \left(\varepsilon\mu - \beta^2(z)\right) H_y\left(x\right) = 0 \tag{5}$$

where $\beta(z)$ is the phase deceleration coefficient, $k_0 = 2\pi/\lambda$ is the wavenumber, and λ is the radiation wavelength.

The solution of Eqs.5 with piecewise constant coefficients is expanded over the fundamental solutions $E_y^j(x;z) = A_1^j\psi_1^j(x;z) + A_2^j\psi_2^j(x;z)$ in the layers with matching at the interlayer boundaries. As a result, we get the problem of determining the eigenvalues (equal to zero) and the eigenvectors, i.e., the set of linear homogeneous algebraic equations

$$\hat{M}\left(\beta^j(z)\right)\vec{A}\left(\beta^j(z)\right) = \vec{0}, \tag{6}$$

nontrivially solvable under the condition that

$$\det \hat{M}\left(\beta^j(z)\right) = 0. \tag{7}$$

After the calculation of the Roots of Eq. 7 we solve Eq. 6 by minimizing the functional

$$\left\|\hat{M}\left(\beta_k^j\right)\vec{A}_k\left(\beta_k^j\right)\right\|^2 + \alpha_1\left\|\vec{A}_k\left(\beta_k^j\right) - \vec{A}_{k-1}\left(\beta_{k-1}^j\right)\right\| + \alpha_2\left(\left\|\vec{A}_k\left(\beta_k^j\right)\right\|^2 - 1\right)^2 \tag{8}$$

The solutions $\vec{A}_k\left(\beta_k^j\right)$ are determined to the complex factor $e^{i\varphi}$, so we add to Eq. 8 the condition minimizing the angle φ. Thus we get the smooth variation of the field $E_y^j(x;z)$ as a function of the parameter z.

3. HOMOGENEOUS TRANSITION (THREE-LAYER SMOOTHLY IRREGULAR WAVEGUIDE)

The interfaces between the layers of the three-layer planar waveguide cross the Ox axis at the point $x = a_1$ between the substrate and the waveguide layer and at the point $x = a_2$ between the waveguide layer and the cover layer. Below we restrict ourselves to considering the TE-mode, when at the boundaries (Fig. 1) the following boundary conditions are satisfied: $E_{ys}(a_1) = E_{yf}(a_1)$ and $H_{zs}(a_1) = H_{zf}(a_1)$ at $x = a_1$; $E_{yf}(a_2) = E_{yc}(a_2)$ and $H_{zf}(a_2) = H_{zc}(a_2)$ at $x = a_2$ ($a_2 - a_1 = d$).

The expansion of the solution for E_y in the above subdomains with constant refractive indices (with the asymptotic boundary conditions taken into account) has the form:[2–5]

$$\begin{aligned} E_y^s &= A_s \exp\{\gamma_s(x - a_1)\} \\ E_y^f &= A_f^+ \exp\{i\chi_f(x - a_2)\} + A_f^- \exp\{-i\chi_f(x - a_2)\} \\ E_y^c &= A_c \exp\{-\gamma_c(x - a_2)\} \end{aligned} \tag{9}$$

Here $\gamma_s = k_0\sqrt{\beta^2 - n_s^2}$, $\chi_f = k_0\sqrt{n_f^2 - \beta^2}$, $\gamma_c = k_0\sqrt{\beta^2 - n_c^2}$. Then the expressions for the component H_z take the form:

$$\begin{aligned} H_z^s &= A_s\frac{\gamma_s}{ik_0\mu_s}\exp\{\gamma_s(x - a_1)\} \\ H_z^f &= \frac{i\chi_f}{ik_0\mu_f}\Big(A_f^+ \exp\{i\chi_f(x - a_2)\} - A_f^- \exp\{-i\chi_f(x - a_2)\}\Big) \\ H_z^c &= A_c\frac{-\gamma_c}{ik_0\mu_c}\exp\{-\gamma_c(x - a_2)\} \end{aligned} \tag{10}$$

Substituting the expressions (9)-(10) into the boundary conditions, we arrive at the set of linear algebraic equations for the amplitude coefficients $A_j^{\pm}$.

$$\begin{aligned} A_s^+ &= A_f^+ + A_f^- \\ \frac{\gamma_s^j}{ik_0}A_s^+ &= \frac{\chi_1^j}{k_0}\left(A_f^+ - A_f^-\right) \end{aligned} \tag{11}$$

$$\begin{aligned} &A_f^+ \exp\left\{i\chi_f^j d(z)\right\} + A_f^- \exp\left\{-i\chi_f^j d(z)\right\} = A_c^- \exp\left\{-\gamma_c^j d(z)\right\} \\ &\frac{\chi_f^j}{k_0}\left(A_f^+ \exp\left\{i\chi_f^j d(z)\right\} - A_f^- \exp\left\{-i\chi_f^j d(z)\right\}\right) = -\frac{\gamma_c^j}{ik_0}A_c^- \exp\left\{-\gamma_c^j d(z)\right\} \end{aligned} \tag{12}$$

4. STABLE ALGORITHM IN THE HOMOGENEOUS TRANSITION AND THE RESULTS OF MODE FIELDS CALCULATION

At each individual point $z = const$ of the cross-section of a homogeneous transition the x-distributions of the fields $E_y(x; z)$ for the guided modes of the waveguide are easily calculated according to Eqs. (9) and (10), expressing the fields in each subinterval with the constant refractive index, and the boundary conditions (11)-(12) at the boundaries between subintervals. The calculated fields $E_y(x; z_1)$ and $E_y(x; z_2)$ at close points z_1 and $z_2 = z_1 + \Delta z$ can be essentially different, whereas in the course of propagation of the mode at a very small distance Δz the field cannot change strongly. To provide the smoothness of the calculated field $E_y(x; z)$ along ,z the procedure of determining the parameters $\vec{A}_k$ of the field $E_y(x; z_k)$ is completed with the penalty $\left\|\vec{A}_k - \vec{A}_{k-1}\right\|^2$, where the parameter $\vec{A}_{k-1}$ corresponds to the field $E_y(x; z_{k-1})$ at the point $z_{k-1} = z_k - \Delta z$. Besides that let us assume that the L_2-norm $\|E_y(x; z_k)\|^2 = \int\limits_{-\infty}^{\infty} |E_y(x; z_k)|^2 dx$ of the field $E_y(x; z_k)$ is conserved. To calculate the norm we use the formula

$$\begin{gathered}\int\limits_{-\infty}^{+\infty} E_y(x)\,\overline{E_y(x)}\,dx = \frac{1}{2}\frac{|A_s|^2}{\gamma_s} + \frac{1}{2}\frac{|A_c|^2}{\gamma_c} + \left|A_f^+\right|^2 (a_2 - a_1) + \left|A_f^-\right|^2 (a_2 - a_1) + \\ + i\frac{1}{2}\frac{A_f^+\overline{A_f^-}}{\chi_f}\left(-1 + \exp\left\{-2i\chi_f(a_2 - a_1)\right\}\right) + i\frac{1}{2}\frac{\overline{A_f^+}A_f^-}{\chi_f}\left(1 - \exp\left\{2i\chi_f(a_2 - a_1)\right\}\right)\end{gathered} \tag{13}$$

A more detailed expression for it can be derived in the following way. Let us introduce the notations $\gamma_s = \gamma'_s + i\gamma''_s; \gamma_c = \gamma'_c + i\gamma''_c; \chi_s = \chi'_s + i\chi''_s$ and calculate the integral of the function

$$E_y = \begin{cases} E_y^s = A_s \exp\left\{\gamma_s(x - a_1)\right\} & -\infty < x < a_1 \\ E_y^s = A_f^+ \exp\left\{i\chi_f(x - a_2)\right\} + A_f^- \exp\left\{-i\chi_f(x - a_2)\right\} & a_1 < x < a_2 \\ E_y^s = A_c \exp\left\{-\gamma_c(x - a_2)\right\} & a_2 < x < \infty \end{cases} \tag{14}$$

over each of subintervals.

In the interval $-\infty < x < a_1$ for $\gamma'_s \neq 0$:

$$\int\limits_{-\infty}^{a_1} E_y \bar{E}_y dx = \int\limits_{-\infty}^{a_1} |A_s|^2 e^{2\gamma'_s(x - a_1)} dx = \frac{|A_s|^2}{2\gamma'_s}(1 - 0) = \frac{|A_s|^2}{2\gamma'_s} \tag{15}$$

In the interval $a_2 < x < \infty$ for $\gamma'_c \neq 0$:

$$\int\limits_{a_2}^{\infty} E_y \bar{E}_y dx = \int\limits_{a_2}^{\infty} |A_c|^2 e^{-2\gamma'_c(x - a_2)} dx = \frac{|A_c|^2}{-2\gamma'_c}(0 - 1) = \frac{|A_c|^2}{2\gamma'_c} \tag{16}$$

In the interval $a_1 < x < a_2$ with the notations $A_f^+ = \left|A_f^+\right| e^{i\varphi_+}$, $A_f^- = \left|A_f^-\right| e^{i\varphi_-}$ taken into account

$$\begin{gathered}\int\limits_{a_1}^{a_2} E_y \bar{E}_y dx = \int\limits_{a_1}^{a_2} \left[\left|A_f^+\right| e^{i\varphi_+} e^{i\chi_f(x - a_2)} + \left|A_f^-\right| e^{i\varphi_-} e^{-i\chi_f(x - a_2)}\right] \times \\ \times \overline{\left[\left|A_f^+\right| e^{i\varphi_+} e^{i\chi_f(x - a_2)} + \left|A_f^-\right| e^{i\varphi_-} e^{-i\chi_f(x - a_2)}\right]}\, dx = \\ = \int\limits_{a_1}^{a_2} \left[\left|A_f^+\right|^2 e^{-2\chi''_f(x - a_2)} + \left|A_f^-\right|^2 e^{2\chi''_f(x - a_2)}\right] dx + \\ + \int\limits_{a_1}^{a_2} \left|A_f^+\right| \left|A_f^-\right| \left[e^{i(\varphi_+ - \varphi_- + 2\chi'_f(x - a_2))} + e^{-i(\varphi_+ - \varphi_- + 2\chi'_f(x - a_2))}\right] dx\end{gathered} \tag{17}$$

Then we distinguish the cases of zero and nonzero imaginary part $\chi_f = {\chi'}_f + i{\chi''}_f$.

$$\begin{gathered}{\chi''}_f = 0 \quad \Rightarrow \int\limits_{a_1}^{a_2} \left[\left| A_f^+ \right|^2 e^{-2{\chi''}_f (x-a_2)} + \left| A_f^- \right|^2 e^{2{\chi''}_f (x-a_2)} \right] \, dx = \\ = \int\limits_{a_1}^{a_2} \left[\left| A_f^+ \right|^2 e^0 + \left| A_f^- \right|^2 e^0 \right] \, dx = \left(\left| A_f^+ \right|^2 + \left| A_f^- \right|^2 \right) (a_2 - a_1)\end{gathered} \tag{18}$$

$$\begin{gathered}{\chi''}_f \neq 0 \quad \Rightarrow \int\limits_{a_1}^{a_2} \left[\left| A_f^+ \right|^2 e^{-2{\chi''}_f (x-a_2)} + \left| A_f^- \right|^2 e^{2{\chi''}_f (x-a_2)} \right] \, dx = \\ = -\frac{\left| A_f^+ \right|^2}{2{\chi''}_f} \left(1 - e^{2{\chi''}_f (a_2-a_1)} \right) + \frac{\left| A_f^- \right|^2}{2{\chi''}_f} \left(1 - e^{-2{\chi''}_f (a_2-a_1)} \right)\end{gathered} \tag{19}$$

The rest part of the integral is calculated under the assumption that ${\chi'}_f \neq 0$.

$$\begin{aligned}&{\chi'}_f \neq 0 \quad \Rightarrow \int\limits_{a_1}^{a_2} \left| A_f^+ \right| \left| A_f^- \right| \left[e^{i(\varphi_+ - \varphi_- + 2{\chi'}_f (x-a_2))} + e^{-i(\varphi_+ - \varphi_- + 2{\chi'}_f (x-a_2))} \right] \, dx = \\ &= 2 \left| A_f^+ \right| \left| A_f^- \right| / {\chi'}_f * \int\limits_{a_1}^{a_2} \cos(\varphi_+ - \varphi_- + 2{\chi'}_f (x - a_2)) \, dx = \\ &= 2 \left| A_f^+ \right| \left| A_f^- \right| / {\chi'}_f * \left[\sin(\varphi_+ - \varphi_- + 2{\chi'}_f (a_1 - a_2)) - \sin(\varphi_+ - \varphi_-) \right] = \\ &= \frac{2}{{\chi'}_f} \left| A_f^+ \right| \left| A_f^- \right| \sin({\chi'}_f (a_2 - a_1)) \cos(\varphi_+ - \varphi_- - 2{\chi'}_f (a_2 - a_1))\end{aligned} \tag{20}$$

When the eigenvectors are calculated over the field of complex numbers, they are determined to a complex factor $\exp\{i\varphi\}$. To eliminate this uncertainty (of the orientation of the vector $\vec{A}_k$ in its one-dimensional space), we add the penalty $\left\| \vec{A}_k \exp\{i\varphi_k\} - \vec{A}_{k-1} \right\|^2$ to the functional (8) at each point z_k of the waveguide transition.

Using this regularization of the algorithm for calculating the x-distribution of the field $E_y(x; z_k)$ of the guided mode at each point z_k, we get the stable calculation of two-dimensional (in x, z) surfaces $\operatorname{Re} E_y$ and $\operatorname{Im} E_y$.

The nonzero solutions of Eq. (7) for the $j-$th TE mode exist only in the domain $z \geq d_{cr}^j$, where $\left(d_{cr}^j\right)^{TE} = \frac{\arctan\left({}^{\gamma_c(n_s)}/_{\chi_f(n_s)} \right) + j\pi}{\chi_f(n_s)}$, $j = 0, ..., N$. Therefore, we calculate the field $E_y^j(x; z)$ for the waveguide layer thickness $d(z)$ varying from d_{cr}^j to d_0.

Consider a few particular examples. Let at the wavelength $\lambda = 0.5\mu$ the refractive index be $n_s = 1.47$ for the substrate, $n_f = 1.565$ for the waveguide layer, and $n_c = 1.0$ for the covering layer (air). All the three layers are dielectric, so that their permeability is $\mu_s = \mu_f = \mu_c = 1.0$.

For calculating the field of TE mode we calculate $d_{cr}^0 = 0.328548448735762\,\lambda$ and the dispersion curve $\beta^0(d),\ d \in (d_{cr}^0, d_0]$. For each calculated value of $\beta^0(d(z_k))$ we calculate $\vec{A}_k$ by minimizing the functional (8). The results of the calculations are presented in Fig. 3.

For calculating the field of TE_1 mode we find $d_{cr}^1 = 1.2597181879094\,\lambda$ and the dispersion curve $\beta^1(d),\ d \in (d_{cr}^1, d_1]$. For each calculated value of $\beta^1(d(z_k))$ we calculate $\vec{A}_k$ by the minimization of the functional (8). The results of the calculations are presented in Fig. 4.

For the field of TE_2 mode we calculate $d_{cr}^2 = 2.19088792708303\,\lambda$ and the dispersion curve $\beta^2(d),\ d \in (d_{cr}^2, d_2]$. For each calculated value of $\beta^2(d(z_k))$ we calculate $\vec{A}_k$ by the minimization of the functional (8). The results of the calculations are presented in Fig.5.

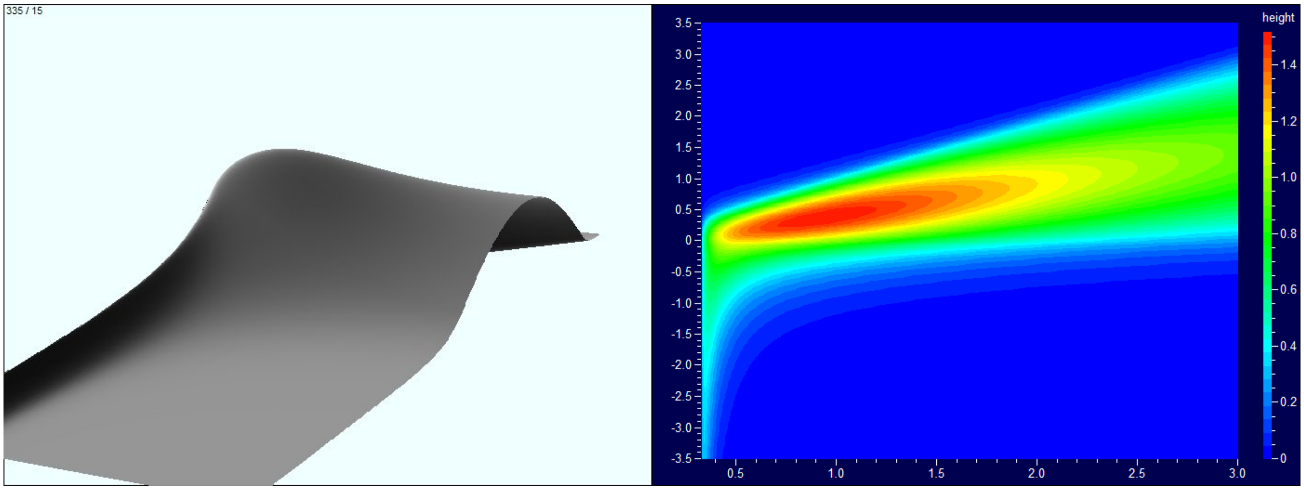

Figure 3. The distribution of the TE_0 mode field over the thickness $d(z)$ (along the ordinate axis) depending on the coordinate z (along the abscissa) in the homogeneous waveguide transition

5. INHOMOGENEOUS TRANSITION (FOUR-LAYER SMOOTHLY IRREGULAR WAVEGUIDE)

The boundaries between separating the layers of the four-layer planar waveguide cross the Ox axis at the point $x = a_1$ between the substrate and the first waveguide layer, at the point $x = a_2$ between the first waveguide layer and the second one, and at the point $x = a_3$ between the second waveguide layer and the covering one. At these boundaries for the mode the following boundary conditions are to be valid: $E_{ys}(a_1) = E_{yf}(a_1)$ and $H_{zs}(a_1) = H_{zf}(a_1)$ at $x = a_1$, $E_{y1}(a_2) = E_{y2}(a_2)$ and $H_{z1}(a_2) = H_{z2}(a_2)$ at $x = a_2$, $E_{y2}(a_3) = E_{yc}(a_3)$ and $H_{z2}(a_3) = H_{zc}(a_3)$ at $x = a_3$ ($a_3 - a_2 = h$).

The expansion of the solution for E_y in the above subdomains of uniform refractive index, with the asymptotic boundary conditions taken into account has the form:

$$\begin{aligned}
E_y^s &= A_s \exp\left\{\gamma_s (x - a_1)\right\} \\
E_y^f &= A_f^+ \exp\left\{i\chi_f (x - a_2)\right\} + A_f^- \exp\left\{-i\chi_f (x - a_2)\right\} \\
E_y^l &= A_l^+ \exp\left\{i\chi_l (x - a_2 - h)\right\} + A_l^- \exp\left\{-i\chi_l (x - a_2 - h)\right\} \\
E_y^c &= A_c \exp\left\{-\gamma_c (x - a_2 - h)\right\}
\end{aligned} \tag{21}$$

Here $\gamma_s = k_0\sqrt{\beta^2 - n_s^2}$, $\chi_f = k_0\sqrt{n_f^2 - \beta^2}$, $\chi_l = k_0\sqrt{n_l^2 - \beta^2}$, $\gamma_c = k_0\sqrt{\beta^2 - n_c^2}$.

Then the expressions for the component H_z take the form:

$$\begin{aligned}
H_z^s &= A_s \frac{\gamma_s}{ik_0\mu_s} \exp\left\{\gamma_s (x - a_1)\right\} \\
H_z^f &= \frac{i\chi_f}{ik_0\mu_f} \left(A_f^+ \exp\left\{i\chi_f (x - a_2)\right\} - A_f^- \exp\left\{-i\chi_f (x - a_2)\right\}\right) \\
H_z^l &= \frac{i\chi_l}{ik_0\mu_l} \left(A_l^+ \exp\left\{i\chi_l (x - a_2 - h)\right\} - A_l^- \exp\left\{-i\chi_l (x - a_2 - h)\right\}\right) \\
H_z^c &= A_c \frac{-\gamma_c}{ik_0\mu_c} \exp\left\{-\gamma_c (x - a_2 - h)\right\}
\end{aligned} \tag{22}$$

Substituting the expressions (21)-(22) into the boundary conditions, we get the set of linear algebraic equations for the amplitude coefficients $A_j^{\pm}$.

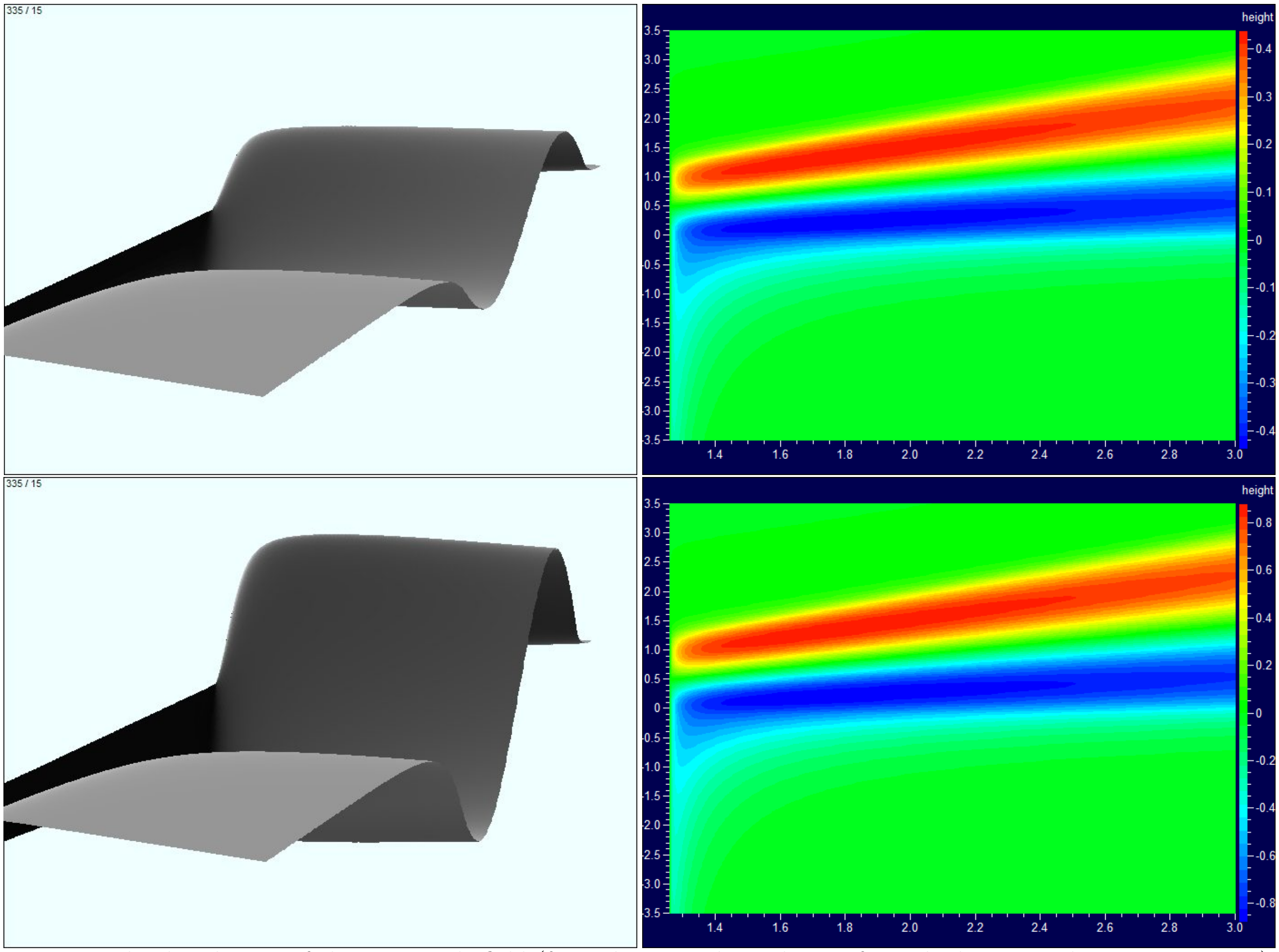

Figure 4. The distribution of the TE_1 mode field (from the top - the real part, from the bottom - the imaginary part) over the thickness $d(z)$ (along the ordinate axis) depending on the coordinate z (along the abscissa) in the homogeneous waveguide transition

$$\begin{aligned} &A_s^+ \exp\left\{-\gamma_s^j d\right\} = A_f^+ \exp\left\{-i\chi_f^j d\right\} + A_f^- \exp\left\{i\chi_f^j d\right\} \\ &\frac{\gamma_s^j}{ik_0} A_s^+ \exp\left\{-\gamma_s^j d\right\} = \frac{\chi_1^j}{k_0}\left(A_f^+ \exp\left\{-i\chi_f^j d\right\} - A_f^- \exp\left\{i\chi_f^j d\right\}\right) \end{aligned} \tag{23}$$

$$\begin{aligned} &A_f^+ + A_f^- = A_l^+ + A_l^- \\ &\frac{\chi_f^j}{k_0}\left(A_f^+ - A_f^-\right) = \frac{\chi_l^j}{k_0}\left(A_l^+ - A_l^-\right) \end{aligned} \tag{24}$$

$$\begin{aligned} &A_l^+ \exp\left\{i\chi_l^j h\left(z\right)\right\} + A_l^- \exp\left\{-i\chi_l^j h\left(z\right)\right\} = A_c^- \exp\left\{-\gamma_c^j h\left(z\right)\right\} \\ &\frac{\chi_l^j}{k_0}\left(A_l^+ \exp\left\{i\chi_l^j h\left(z\right)\right\} - A_l^- \exp\left\{-i\chi_l^j h\left(z\right)\right\}\right) = -\frac{\gamma_c^j}{ik_0} A_c^- \exp\left\{-\gamma_c^j h\left(z\right)\right\} \end{aligned} \tag{25}$$

As a result, we arrived at the homogeneous system of linear algebraic equations (SLAE) with the matrix $\mathbf{M}_{TE}^{\perp 6}(\beta)$ with respect to the unknowns A_s^+, A_f^+, A_f^-, A_l^+, A_l^-, A_c^-, the solutions of which give us the values of the unknown amplitude coefficients. The homogeneous SLAE are nontrivially solvable under the condition that their determinants are zero. These conditions determine the dependence of the phase deceleration coefficients β of the mode upon the thickness of the waveguide layers $d = a_2 - a_1$ and $h = a_3 - a_2$.

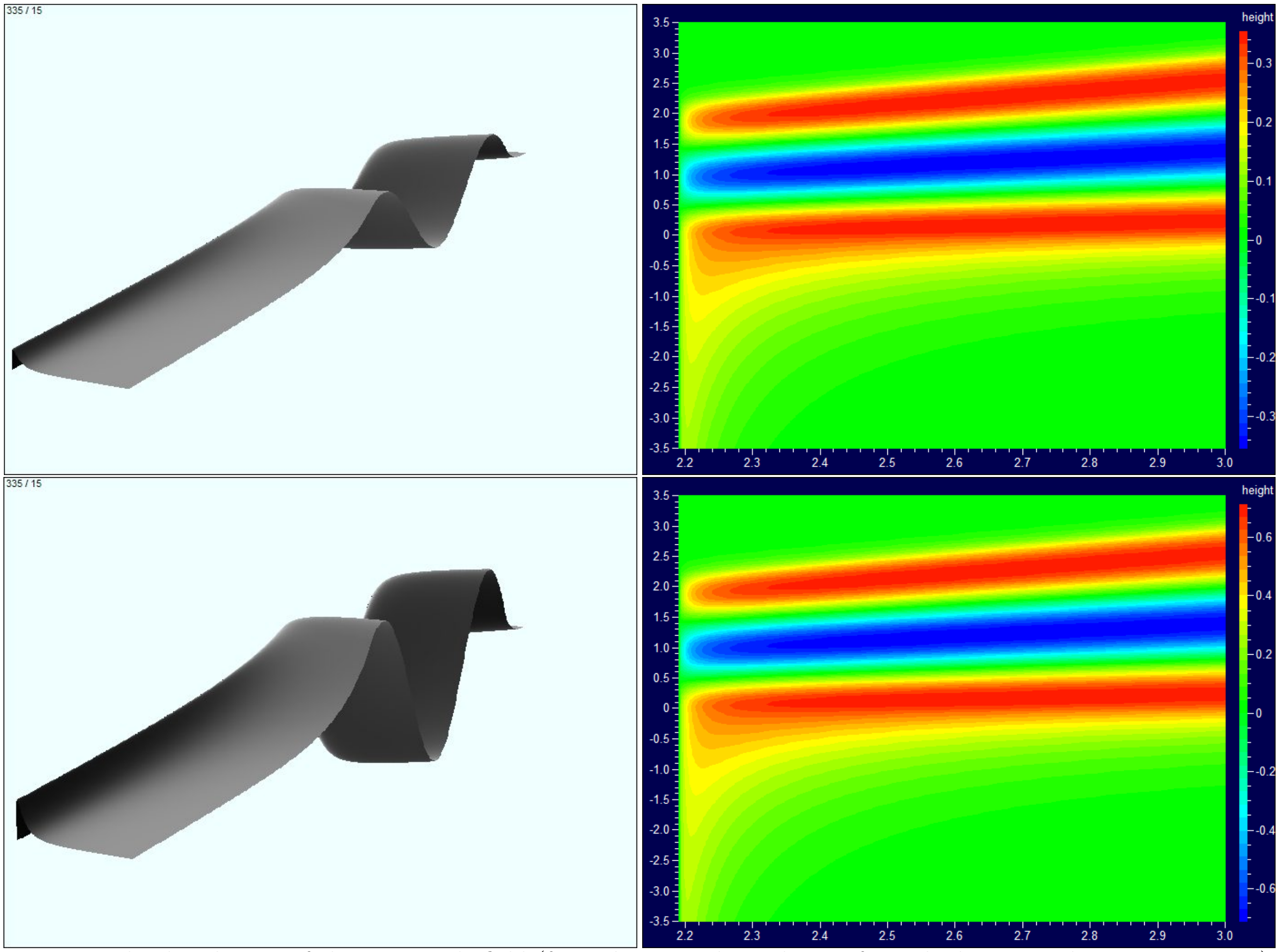

Figure 5. The distribution of the TE_2 mode field (from the top - the real part, from the bottom - the imaginary part) over the thickness $d(z)$ (along the ordinate axis) depending on the coordinate z (along the abscissa) in the homogeneous waveguide transition

6. STABLE ALGORITHM FOR THE INHOMOGENEOUS TRANSITION AND THE RESULTS OF THE MODE FIELDS CALCULATION

In the inhomogeneous transition in the case when the refractive index of the second waveguide layer n_l significantly exceeds the refractive index of the first waveguide layer n_f the interval $(z_1 < z < z_2)$ appears, it which the derivative of the dispersion curve (see Fig. 6) increases much faster than in the domains lying to the right and to the left from the interval $[z_1, z_2]$.

In the above interval the direction of the vector $\vec{A}_k$ with respect to the vector $\vec{A}_{k-1}$ in the linear space of the eigenvector search begins to change faster. The rotation through the angle φ_k as compared to the angle φ_{k-1} is also accelerated, the increase rate for the increment $\Delta z = z_k - z_{k-1}$ being unchanged. This fact requires additional constructions when implementing the algorithm of stable calculation of the two-dimensional distribution of the field $E_y(x; z)$ depending on the coordinates (x, z).

For normalizing the function $E_y(x; z)$ on the axis Ox in the four-layer waveguide transition one should use

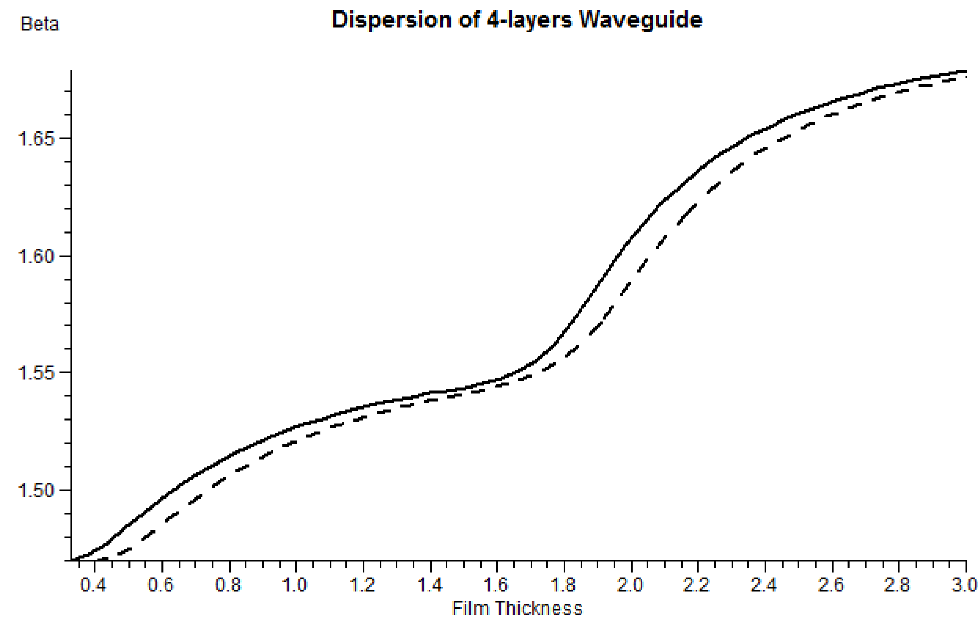

Figure 6. Plot of dispersion curves for TE_0 and TM_0 modes in the inhomogeneous four-layer waveguide transition.

the formulae

$$\begin{aligned}\int_{-\infty}^{+\infty} E_y(x)\,\overline{E_y(x)}\,dx = {} & \frac{1}{2}\frac{|A_s|^2}{\gamma_s} + \frac{1}{2}\frac{|A_c|^2}{\gamma_c \exp\{2\gamma_c h\}} + \left|A_f^+\right|^2 (a_2 - a_1) + \\ & + \left|A_f^-\right|^2 (a_2 - a_1) + \left|A_l^+\right|^2 h + \left|A_l^-\right|^2 h + \\ & + i\frac{1}{2}\frac{A_f^+ \overline{A_f^-}}{\chi_f}\left(-1 + \exp\{-2i\chi_f (a_2 - a_1)\}\right) + i\frac{1}{2}\frac{\overline{A_f^+} A_f^-}{\chi_f}\left(1 - \exp\{2i\chi_f (a_2 - a_1)\}\right) + \\ & + i\frac{1}{2}\frac{A_l^+ \overline{A_l^-}}{\chi_l}\left(-1 + \exp\{-2i\chi_l h\}\right) + i\frac{1}{2}\frac{\overline{A_l^+} A_l^-}{\chi_l}\left(1 - \exp\{2i\chi_l h\}\right)\end{aligned} \tag{26}$$

and the appropriate generalizations of the formulae for the case of complex amplitudes over the four-layer waveguide.

The regularization of the algorithm for the calculation of the x-distribution of the field $E_y(x; z_k)$ of the guided TE mode at each point z_k allows the stable calculation of two-dimensional (in x, z) surfaces $\operatorname{Re} E_y$ and $\operatorname{Im} E_y$. Let us present the profiles of these surfaces and the corresponding spectrograms for the mode TE_0 in the inhomogeneous waveguide transition for different relations between n_l and n_f. First, let us consider the case when the refractive index of the second waveguide layer $n_l = 1.57$ slightly exceeds that of the first layer $n_f = 1.565$, the rest parameters of the waveguide being the same as in the previous example. The results of the calculations are presented in Fig. 7.

When the refractive index of the second waveguide layer $n_l = 1.6$ significantly exceeds that of the first waveguide layer $n_f = 1.565$, the rest parameters of the waveguide being the same as in the previous example, the results of the calculations are presented in Fig. 8.

7. CONCLUSION

In the integrated optical waveguide transition of the horn type the TE and TM polarized waves, which are eigenmodes of the regular waveguide, propagate without hybridization. This fact allows one to describe the

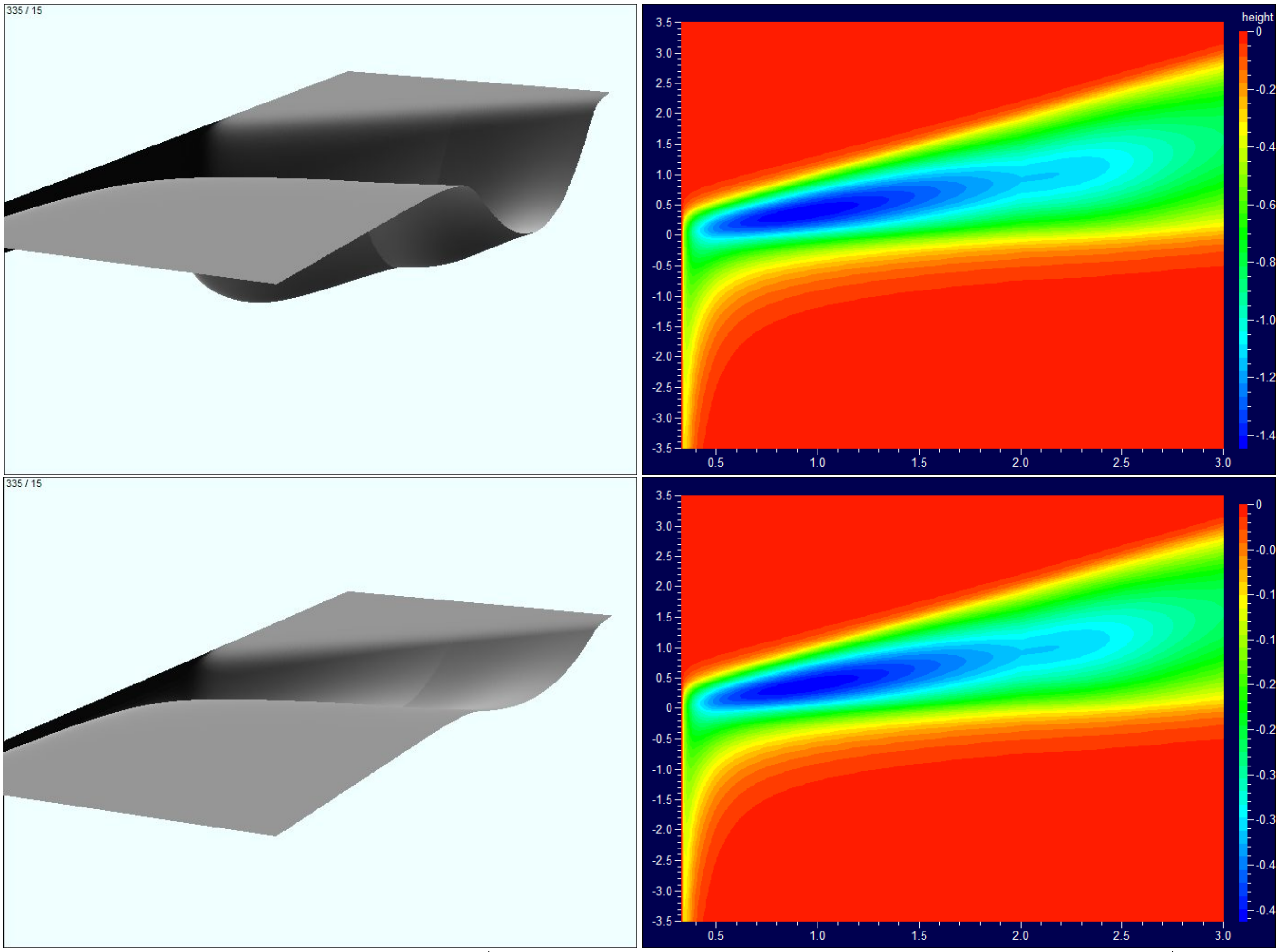

Figure 7. Field distribution for the TE_0 mode (from the top - the real part, from the bottom - the imaginary part) over the thickness $d(z)$ (along the ordinate axis) depending on the coordinate z (along the abscissa) in the homogeneous waveguide transition at $n_l \approx n_f$

propagation of the polarized monochromatic electromagnetic radiation in the horn type transition by means of Helmholtz equations using the method of cross sections. The method is based on the adiabatic approximation for the asymptotic expansion of locally plane waves that reduces the problem to a set of ordinary differential equations for the general solution expansion coefficients in the cross section method.

However, for all the simplicity of the cross sections method, the investigation of even a single-mode electromagnetic field evolution in the course of propagation through an irregular waveguide transition is associated with certain difficulties. They are related to the variable rate of the growth of the phase deceleration factor as a function of the transition thickness, to other features of the mode dispersion curve behavior, and to the variation of the appropriate solutions with thickness.

In the paper, the guided propagation of polarized monochromatic light in a smoothly irregular transition between two planar regular dielectric waveguides is studied. We consider the problem of calculating the mode field in the integrated optical waveguide transition of the horn type.

Within the single-mode approximation of the method of cross sections, the transverse distribution of the electromagnetic field of the deformed mode is calculated as the field of the regular reference waveguide in each individual cross section of the transition. In the paper the numerical solutions for the field smoothly varying in the process of propagation is obtained using the stable method based on Tikhonov regularization.

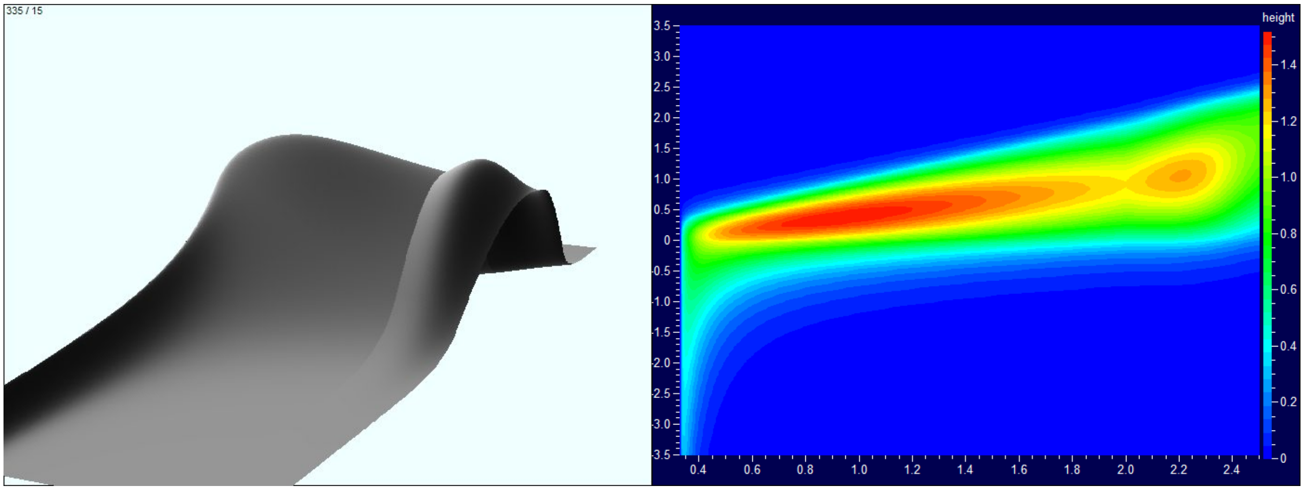

Figure 8. Field distribution for the TE_0 mode over the thickness $d(z)$ (along the ordinate axis) depending on the coordinate z (along the abscissa) in the inhomogeneous waveguide transition at $n_l > n_f$

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