Ministry of Education and Science
Russian Federation
National Research Nuclear University MEPhI

6th International conference

"Problems of Mathematical Physics and Mathematical Modelling"

Book of abstracts

(Moscow, NRNU MEPhI, 25–27 May 2017)

Moscow, Russia

УДК 51(06)+53(06)
ББК 22.1г+22.3г
I73

6th International conference "Problems of Mathematical Physics and Mathematical Modelling": Books of abstracts (Moscow, NRNU MEPhI, 25–27 May). / M.B. Kochanov. M.: Moscow, 2017. – 221 p.
ISBN 978-5-7262-2378-0

The book contains abstracts of 6th International conference "Problems of Mathematical Physics and Mathematical Modelling"

The contributions are reproduced directly from the originals presented by the authors

УДК 51(06)+53(06)
ББК 22.1г+22.3г

ISBN 978-5-7262-2378-0

Decision on publication 22.05.2017. Format 60x84 1/8.
Quires 27,75. Circulation 120. Order №76.

National Research Nuclear University MEPhI
Printing house NRNU MEPhI
115409, Moscow, Kashirskoe shosse, 31

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Section "Mathematical modelling"

An approach to invistigate the regions of self-oscillations

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In systems with control, there is such a negative phenomenon as global synchronization, which is expressed in the form of self-oscillations. To eliminate this phenomenon, it is necessary to investigate the effect of system parameters on characteristics. As an implementation of the threshold control system, we investigate the active traffic control module RED, and the cause of the self-oscillation is the type of the reset function. In this paper we consider the application of the block-linear approach to control theory and the application of harmonic linearization.

The work is partially supported by RFBR grants No's 15-07-08795 and 16-07-00556. Also the publication was financially supported by the Ministry of Education and Science of the Russian Federation (the Agreement No 02.A03.21.0008).

1 Application of the method of harmonic linearization

To describe the RED algorithm we will use the following continuous model (see [1, 2, 3]) with some simplifying assumptions: the model is written in the moments; the model describes only the phase of congestion avoidance for TCP Reno protocol; in the model the drop is considered only after reception of 3 consistent ACK confirmations.

The method of harmonic linearization is an approximate method. The harmonic linearization differs from the common method of linearization and allows to explore the basic properties of nonlinear systems.

The method of harmonic linearization is used for systems of a certain structure. The system consists of linear part H_l and the nonlinear part, which is set by function $f(x)$. It is generally considered a static nonlinear element.

In a block-linear approach to control theory the original nonlinear system is linearized and divided into blocks. The blocks are characterized by a transfer function, which connects input and output values. The linearization method is described in the article [2]. We obtain a linearized system:

$$\begin{cases} \delta\dot{W}(t) = \frac{\delta L_W}{\delta W}|_f \delta W(t) + \frac{\delta L_W}{\delta W_T}|_f \delta W(t-T_f) + \frac{\delta L_W}{\delta Q}|_f \delta Q(t) + \frac{\delta L_W}{\delta p}|_f \delta p(t-T_f) = \\ = -\frac{N}{CT_f^2}(\delta W(t) + \delta W(t-T_f)) - \frac{C^2 T_f}{2N^2}\delta p(t-T_f); \\ \delta\dot{Q}(t) = \frac{\delta L_Q}{\delta W}|_f \delta W(t) + \frac{\delta L_Q}{\delta Q}|_f \delta Q(t) = \frac{N}{T_f}\delta W(t) - \frac{1}{T_f}\delta Q(t). \\ \delta\dot{\hat{Q}}(t) = \frac{\delta L_{\hat{Q}}}{\delta \hat{Q}}|_f \delta\hat{Q}(t) + \frac{\delta L_{\hat{Q}}}{\delta Q}|_f \delta Q(t) = -w_q C\delta\hat{Q}(t) + w_q C\delta Q(t). \end{cases} \quad (1)$$

After we linearize the drop function. It will take the following form:

$$\delta p(s) = P_{\mathrm{RED}} \frac{1}{1 + \frac{s}{w_q C}} \delta Q(s). \quad (2)$$

The block representation of the linearized RED model (Fig. 1) is constructed.

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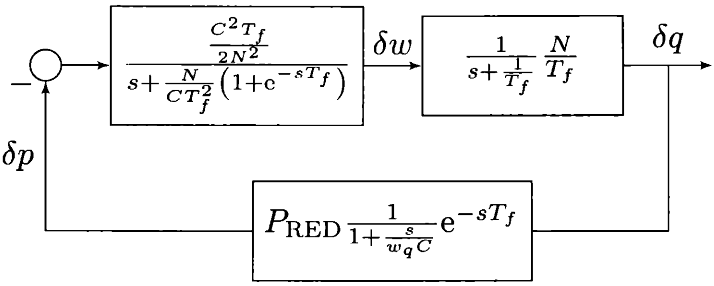

Fig. 1: Block representation of the linearized RED model

To determine the region of self-oscillations, we use the Routh-Hurwitz criterion, and to determine the self-oscillation parameters, use Mikhailov's criterion. This criterion is an algebraic criterion of stability.

The transfer function of the conservative system has the form

$$H_c(s) = \frac{H_l(s)}{1 + H_l(s)H_{nl}(s)} := \frac{P_n(s)}{P_d(s)}. \tag{3}$$

The equation $P_d(s) = 0$ is the characteristic equation of the system. It can be present in the polynomial form:

$$P_d(s) := a_0 s^n + a_1 s^{n-1} + \cdots + a_n. \tag{4}$$

From the coefficients of the characteristic equation one can construct Hurwitz determinant Δ.

By the Hurwitz criterion, in order for the dynamical system to be stable, it is necessary and sufficient that all n principal diagonal minors of the Hurwitz determinant be positive, provided that $a_0 > 0$. These minors are called Hurwitz determinants. The system is located on the boundary of the oscillatory system, if $Delta_{n-1} = 0$. In this case, there are auto-oscillations. Consider the criterion of Mikhailov.

Let us write the characteristic equation of the conservative system (3) with respect of $s \to \delta_t \to i\omega$:

$$P_d(i\omega) = 0. \tag{5}$$

In addition, if one explicitly allocates real and complex parts, then the equation (5) can be written in the following form:

$$\Re P_d(i\omega) = 0, \qquad \Im P_d(i\omega) = 0. \tag{6}$$

Thus, the parameters of self-oscillations can be determined from equation (6). Let us compute the coefficients of harmonic linearization $\varkappa(a)$ and $\varkappa'(a)$ (7)

$$\varkappa(A) = \frac{4}{A\pi}\frac{p_{\max}}{Q_{\max} - Q_{\min}}\left(\sqrt{1 - \frac{Q_{\min}^2}{A^2}} - \sqrt{1 - \frac{Q_{\max}^2}{A^2}}\right); \tag{7}$$

$$\varkappa'(A) = \frac{4}{A\pi}\frac{p_{\max}}{Q_{\max} - Q_{\min}}\frac{Q_{\max} - Q_{\min}}{A} = \frac{4p_{\max}}{A^2\pi}. \tag{8}$$

Let's write the balance equation between frequency and amplitude:

$$\begin{gathered}\frac{1}{I\omega + \frac{N}{CT_f^2}(1 + E^{-I\omega T_f})}\frac{1}{I\omega + \frac{1}{T_f}}\frac{1}{1 + \frac{I\omega}{w_qC}}\frac{C^2}{2N}E^{-I\omega T_f} = \\ = -\frac{A\pi}{4p_{\max}}[\frac{1}{Q_{\max} - Q_{\min}}(\sqrt{1 - \frac{Q_{\min}^2}{A^2}} - \sqrt{1 - \frac{Q_{\max}^2}{A^2}}) + I\frac{1}{A}]^{-1}.\end{gathered} \tag{9}$$

In this paper, harmonic linearization was used to determine the region of origin of self-oscillations and their parameters. The obtained values are verified using the simulation tool NS2.

References

[1] Misra Vishal, Gong Wei-Bo, Towsley Don. Stochastic Differential Equation Modeling and Analysis of TCP-Windowsize Behavior // Proceedings of PERFORMANCE. — 1999. — Vol. 99.

[2] Hollot C. V. V., Misra Vishal, Towsley Don. A Control Theoretic Analysis of RED // Proceedings IEEE INFOCOM 2001. Conference on Computer Communications. Twentieth Annual Joint Conference of the IEEE Computer and Communications Society (Cat. No.01CH37213). — Vol. 3. — IEEE, 2001. — P. 1510–1519. %newblock Combinatorial and Operator Approaches to RED Modeling // Mathematical Modelling and Geometry. — 2015. — Vol. 3, no. 3. — P. 1–18.

[3] Korolkova Anna Vladislavovna, Velieva Tatyana Refatovna, Abaev Pavel Avanesovich et al. Hybrid Simulation Of Active Traffic Management // Proceedings 30th European Conference on Modelling and Simulation. — 2016. — jun. — P. 685–691.

Application of thermal-hydraulic model of RBMK-type reactor fuel channel for correction of experiment-calculated values of power and coolant flow

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The effectiveness and the security of RBMK-type reactor operation depends on the accuracy of the control over reactor's parameters and their limitations. The processing of operational parameters archive helps to adjust different mathematical models and significantly widen their field of use. Pressure differential between common pressure header and steam separator is the sum of calculated pressure differential and friction loss on flow control valve. There is known mathematical software, which allows to adapt such model for each fuel channel using the archive. In this research it is suggested not to replace the regular mechanism with such approach, but to use the adapted mathematical model to calculate corrected values of power and flow, which were measured by regular means. Mathematical expressions and procedures for such approach are given.

The effectiveness and the security of RBMK-type reactor operation depends on the accuracy of the control over reactor's parameters and their limitations. Two of the most important parameters are the coolant flow through fuel channel and the energy emission, or power of the channel. Currently used means of control and algorithms of information processing surely give a reliable solution to the problem [1]. Nevertheless, research of new methods of accuracy increase for control systems is always a problem of interest. At present, such methods may involve using an archive of nuclear reactor operational parameters for a long period of time. In particular, the processing of operational parameters archive helps to adjust different mathematical models and

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