International Conference on
Mathematical Modeling and Computational Physics

MMCP 2015

Book of Abstracts

July 13 – 17, 2015
Stará Lesná, Slovakia

Joint Institute for Nuclear Research, Dubna
Institute of Experimental Physics SAS, Košice
Slovak Physical Society
University of Pavol Jozef Šafárik, Košice
Technical University, Košice
IFIN-HH, Bucharest, Romania

International Conference on

Mathematical Modeling and Computational Physics

MMCP 2015

Book of Abstracts

July 13 – 17, 2015

Stará Lesná, Slovakia

Joint Institute for Nuclear Research, Dubna

Institute of Experimental Physics SAS, Košice

Slovak Physical Society

University of Pavol Jozef Šafárik, Košice

Technical University, Košice

IFIN-HH, Bucharest, Romania

International Conference on Mathematical Modeling and Computational Physics (MMCP 2015). Book of Abstracts. FEE&I TU Košice, 2015, 96 p.

Conference Chairmen:

V. V. Korenkov (LIT JINR) and G. Semanišin (FS UPJŠ Košice)

Organizing Committee

Chairmen: Gh. Adam (JINR Dubna, IFIN-HH Bucharest) and M. Hnatič (UPJŠ, JINR Dubna, IEP SAS Košice)

S.–A. Adam (JINR, IFIN-HH), J. Buša (TU) – Scientific Secretary, J. Buša Jr. (TU), E. Hayryan (JINR), P. Kopčanský (IEP), G. Kozáková (IEP), D. V. Podgainy (JINR) – Scientific secretary, M. Reiffers (PU Prešov, IEP), O. Yu. Rumyantseva (JINR). E. N. Rusakovich (JINR), O. I. Streltsova (JINR), T. A. Strizh (JINR), P. V. Zrelov (JINR), M. I. Zuev (JINR)

International Program Committee

Gh. Adam (JINR), B. N. Chetverushkin (Russia), A. B. Degtyarev (Russia), S. Dimova (Bulgaria), M. Dulea (Romania), Yu. G. Evtushenko (Russia), A. Feher (Slovakia), K. Flachbart (Slovakia), V. Friese (Germany), V. Gerdt(JINR), U. H. E. Hansmann (USA), J. Honkonen (Finland), Ch.-K. Hu (Taiwan), V. V. Ivanov (JINR), T. Jarlborg (Switzerland), I. Kisel (Germany), N. Kolkovska (Bulgaria), P. Kopčanský (Slovakia), N. A. Kudryashov (Russia), V. D. Lakhno (Russia), R. Lazarov (USA), V. S. Melezhik (JINR), N. A. Rusakovich (JINR), H. Safouhi (Canada), S. Scott (North Ireland), L. A. Sevastianov (Russia), T. A. Strizh (JINR), P. N. Vabishchevich (Russia), P. Zinterhof (Austria), P. V. Zrelov (JINR)

ISBN 978-80-553-2156-1

Typeset by pdfLaTeX

Copyright © Ján Buša, Michal Hnatič, 2015

Operator Approach to the One-Step Process Master Equation[1]

M. Hnatich[†], E. Eferina[‡], A. Korolkova[‡], D. Kulyabov[‡,§], L. Sevastyanov[†,‡]

[†] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Joliot-Curie 6, Dubna, Moscow region, 141980, Russia*
[‡] *Department of Applied Probability and Informatics, Peoples' Friendship University of Russia, Miklukho-Maklaya str. 6, Moscow, 117198, Russia,*
[§] *Laboratory of Information Technologies, Joint Institute for Nuclear Research, Joliot-Curie 6, Dubna, Moscow region, 141980, Russia*
E-mail: hnatic@saske.sk, eg.eferina@gmail.com, akorolkova@sci.pfu.edu.ru, yamadharma@gmail.com, leonid.sevast@gmail.com

Background. Presentation of probability as an intrinsic property of the nature leads researchers to switch from deterministic to stochastic description of the phenomena. On the basis of the ideology of N. G. van Kampen and C. W. Gardiner the procedure of stochastization of one-step process was formulated. It allows to write down the master equation based on the type of of the kinetic equations (equations of interactions) and assumptions about the nature of the process (which may not necessarily process of birth – death). The kinetics of the interaction has recently attracted attention because it often occurs in the physical, chemical, technical, biological, environmental, economic, and sociological systems. However, there are no general methods for the direct study of this equation. The expansion of the equation in a formal Taylor series (so called Kramers-Moyal's expansion) is used in the procedure of stochastization of one-step processes. It is also possible to apply system size expansion (van Kampen's expansion). Leaving in the expansion terms up to second order we can get the Fokker–Planck equation, and after the Langevin equation. It should be clearly understood that these equations are approximate recording of the master equation.

Purpose. However, this does not remove the need for the study of the master equation. Moreover, that the power series produced during the decomposition of the master equation may be divergent (for example, in spatial models). This makes it impossible to apply the classical perturbation theory.

Method. It is proposed to use quantum field perturbation theory for the statistical systems (so-called method Doi). The perturbation series are treated in the spirit of the Feynman path integral, where the Green's functions of the perturbed Liouville operator of the master equation are propagators. For more convenience of selection of the perturbed and unperturbed parts of the

[1]This work is partially supported by RFBR grants No's 14-01-00628 and 15-07-08795.

Liouville operator and to obtain the explicit form of the Green function of the master equation we need to rewrite the equation in the occupation number representation (Fock state).

RESULTS. This work is a methodological material that describes the principles of master equation solution based on methods of quantum field perturbation theory. The feature presentation is that it is designed for non-specialists in quantum field theory. This example uses Verhulst model because of its simplicity and clarity (the first order equation, is independent of the spatial variables, however, contains non-linearity).

CONCLUSIONS. The described method allows to solve directly the master equation and obtain the model parameters, such as the moments and the probability density distribution.