Ministry of Education and Science
Russian Federation
National Research Nuclear University MEPhI

6th International conference

“Problems of Mathematical Physics and Mathematical Modelling”

Book of abstracts

(Moscow, NRNU MEPhI, 25–27 May 2017)

Moscow, Russia

УДК 51(06)+53(06)
ББК 22.1г+22.3г
I73

6th International conference “Problems of Mathematical Physics and Mathematical Modelling”: Books of abstracts (Moscow, NRNU MEPhI, 25–27 May). / M.B. Kochanov. M.: Moscow, 2017. – 221 p.
ISBN 978-5-7262-2378-0

The book contains abstracts of 6th International conference “Problems of Mathematical Physics and Mathematical Modelling”

The contributions are reproduced directly from the originals presented by the authors

УДК 51(06)+53(06)
ББК 22.1г+22.3г

ISBN 978-5-7262-2378-0

Decision on publication 22.05.2017. Format 60x84 1/8.
Quires 27,75. Circulation 120. Order №76.

National Research Nuclear University MEPhI
Printing house NRNU MEPhI
115409, Moscow, Kashirskoe shosse, 31

Co-chairs

V.V. Kozlov, Member of the Russian Academy of Science, Russia

N.A. Kudryshov, Head of department of Applied Mathematics NRNU MEPhI, Russia

O.V. Nagornov, Head of department of Higher Mathematics of NRNU MEPhI, Russias

Program committee

V.V. Kozlov, Member of the Russian Academy of Science, Russia

N.A. Kudryshov, Head of department of Applied Mathematics NRNU MEPhI, Russia

O.V. Nagornov, Head of department of Pure Mathematics of NRNU MEPhI, Russia

N. Pogorelov, University of Alabama in Huntsville, USA

A.V. Aksenov, Lomonosov Moscow State University, Russia

S.Yu. Misyurin, Director of Institute of Cyber Intelligence Systems NRNU MEPhI, Russia

V. Arakelyan, Institut National des Sciences Appliques, France

P. Chardonne, University of Savoy, France

Y. Efendiev, Director of Institute for Scientific Computation, Texas A&M University, USA

S.A. Kaschenko, Yaroslavl State University, Russia

A.V. Kryanev, NRNU MEPhI, Russia

R. Lazarov, Texas A&M University, USA

A.D. Polyanin, IPMech RAS, Russia

O.I. Vinogradova, Lomonosov Moscow State University, Russia

R. Conte, Condensed Matter Laboratory, CEA Saclay, France

P. Bedrikovetsky, University of Adelaide, Australia

Section “Mathematical modelling”

where φ is an auxiliary variable.

We show that this interacting system is dynamically stable to periodic drives with finite frequency and amplitude. Lyapunov stability analysis for this system was studied.

In this work the system (7) was investigated and Lyapunov stability analysis for an one-body generalization of the Kapitza pendulum was studied.

References

[1] Stephenson A. On induced stability. Philosophical Magazine Series, 6, 1908; 15: 233-6.

[2] Stephenson A. On a new type of dynamical stability. Mem Proc Manch Lit Phil Soc 1908; 52 (8): 1-10.

[3] Kapitza P.L. Dynamic stability of a pendulum with oscillating point of suspension. Sov. Phys. JETP 1951; 21: 588-597 (in Russian).

[4] Kapitza P.L. A pendulum with vibrating point of suspension. Usp Phys Nauk 1951; 44: 7-20 (in Russian).

[5] Levi M. Geometry of Kapitza's potential. Nonlinearity 1998; 11: 1365-8.

[6] Bardin B.S., Markeyev A.P. On the stability of equilibrium of a pendulum with vertical oscillations of its suspension point. J. Appl. Math. Mech. 1995; 59: 879-86.

[7] Ovseyevich A.I. The stability of an inverted pendulum when there are rapid random oscillations of the suspension point. J. Appl. Math. Mech. 2006; 70: 761-8.

[8] Neishtadt A.I., Shenga K. Bifurcations of phase portraits of pendulum with vibrating suspension point. https://arxiv.org/abs/1605.09448 (29 Sep. 2016).

Implementation of diagram technique for statistical systems in Sympy

E.G. Eferina[1,a) , D.S. Kulyabov[1,2,b)]
[1] *Peoples' Friendship University of Russia (RUDN University)*
[2] *Joint Institute for Nuclear Research*

During development of various methods for randomization of one-step processes the attention was focused on obtaining the stochastic equations in the Langevin's form, since this form is most usual in the construction and study of models of one-step processes. However, the partial differential equations (master equation and the Fokker–Planck equation) can provide richer description of the model to researchers. It is proposed to use a help of perturbation theory in the framework of quantum field theory to study these equations. This method is based on the Feynman diagram technique that, in turn, is a universal and suitable for any models. For this purpose a methodology is introduced and an analytical software framework is constructed to represent the main kinetic equation in the operator form in the terms

[a)] Email: eg.eferina@gmail.com
[b)] Email: kulyabov_ds@rudn.university

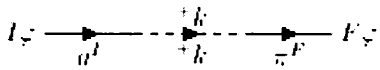

Fig. 1: Forward interaction (operator approach)

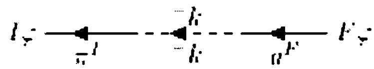

Fig. 2: Backward interaction (operator approach)

of Fock representation. Additionally the developed framework allows to generate feynman diagrams and to obtain model equations using them. SymPy library is employed as a symbolic calculations engine.

The work is partially supported by RFBR grants No's 14-01-00628, 15-07-08795 and 16-07-00556. Also the publication was financially supported by the Ministry of Education and Science of the Russian Federation (the Agreement No 02.A03.21.0008).

1 Diagram technique

Initially the models are described by the interaction schemes and studied using two approaches: combinatorial and operator ones. In the combinatorial approach, all operations are performed in the space of states of the system, so we deal with a particular system throughout manipulations with the model [1]. For the operator approach we can abstract from the specific implementation of the system under study. We are working with abstract operators. We return to the state space only at the end of the calculations. In addition, we choose a particular algebra operator on the basis of symmetry of the problem.

Therefore, the diagrammatic technique should be determined for all of them. We will write the scheme of interaction in the formalism of diagrams. Each of scheme corresponds to a pair of diagrams for forward and backward interaction respectively.

Each line is attributed to a certain factor (depending on the the approach chosen). The resulting expression is obtained by multiplying these factors.

We could obtain the Liouville operator using interaction diagrams in the operator approach [2]. Let us assign the corresponding factor for each line. The Liouville operator is obtained as the normal ordered product of factors. We use the following factors for each type of line (fig. 1).

- Incoming line. This line corresponds to the disappearance of one entity from the system. Therefore, it corresponds to the annihilation operator a. It is clear that the line with combined capacity I corresponds to the operator a^I.
- Outgoing line. This line corresponds to the emergence of one entity in the system. Therefore, it corresponds to the creation operator π. It is clear that the line with combined capacity F corresponds to the operator π^F.
- Line of interaction. This line corresponds to the ratio of the interaction intensity.

For the combinatorial approach we get the master equation in the representation of the state vectors.

We use the following factors for each type of line (fig. 3).

- Incoming line. If all lines correspond to different state vectors, the factor of each line is the corresponding state vector. If there are several lines corresponding to the same state

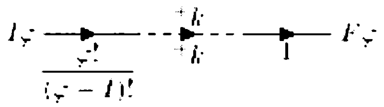

Fig. 3: Forward interaction (combinatorial approach)

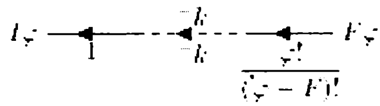

Fig. 4: Backward interaction (combinatorial approach)

```
import graph_state
g = graph_state.GraphState.from_str(
"123||||:>_>_>:10_kplus_0_0||||:Iphi_a^I_pi^F_Fphi")
```

Fig. 5: The diagram (fig. 1) in the B. G. Nickel notation

vector, the first line corresponds to the actual state vector (φ), the second line corresponds to the value of $\varphi - 1$ (as the first line has reduced the number of entities of this type in the system by one), and so further. That is, for a combined line factor can be written as follows:

$$\frac{\varphi!}{(\varphi - I)!}. \tag{1}$$

- Outgoing line do not give multiplicative contribution. It serves to obtain the step coefficient r:

$$r = F - I. \tag{2}$$

- Line of interaction. This line corresponds to the ratio of the interaction intensity.

To implement the diagram technique there are several libraries used: SymPy, Graph-tool and GraphState. The framework contains rules for diagram generation and rules to obtain model equations. A user can set initial parameters in both of terms: kinetic schemes and diagrams. Graph-tool library allows to obtain diagrams in familiar representation but output for large number of diagrams is very verbose. So we have written a script to transform these diagrams to latex notation and this allows to provide output result as pictures.

For further diagram processing GraphState library is used that's based on BG Nickel notation. Independently to a library choice, that describes diagrams in terms of graphs as a result we obtain a set of diagrams and corresponding model equations.

The diagram technique has been developed on the basis of Feynman diagrams and appears as very easy and versatile to use with previous approaches: the combinatorial and the operator since the schematic notation is determined by a set of rules, and they can be formulated for a particular approach, herewith methods of calculation of schemes remain unchanged.

References

[1] A. V. Demidova, D. S. Kulyabov, The Introduction of an Agreed Term in the Equation of Stochastic Population Model, Bulletin of Peoples Friendship University of Russia. Series Mathematics. Information Sciences. Physics (3) (2012) 69 – 78.

[2] E. G. Eferina, A. V. Korolkova, D. S. Kulyabov, L. A. Sevastyanov, Quantum Field Theory Approach to the Analysis of One-step Model, Bulletin of Peoples' Friendship University of Russia. Series "Mathematics. Information Sciences. Physics" (3) (2015) 46 – 59.