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The approach to investigation of the the regions of self-oscillations

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Abstract. Self-oscillating modes in control systems of computer networks quite negatively affect the characteristics of these networks. The problem of finding the areas of self-oscillations is actual and important as the study of parameters of self-oscillations. Due to the significant nonlinearity of control characteristics, the study of the self-oscillating modes presents certain difficulties. This paper describes the technique of research of self-oscillating modes.

1. Introduction
While modeling technical systems with control it is often required to study their characteristics. Also it is necessary to investigate the influence of system parameters on characteristics. In systems with control there is a parasitic phenomenon as a self-oscillating mode. We have previously carried out studies on the numerical determination of the region of occurrence of self-oscillations [1]. These studies provided information about the presence or absence of self-oscillations. However, the parameters of these self-oscillations the influence of various factors on the cause of self-oscillations was not investigated by as.

In this paper, we propose the research plan that allows us to obtain the parameters of self-oscillations and to investigate various factors of self-oscillations appearance. The proposed method uses the harmonic linearization. In addition, the studies suggest verification of the results obtained using the simulation software.

The section 2 describes the structure of the research. The model under investigation is introduced in the section 3. In the section 4, a brief description of the method of harmonic linearization is given. In the section 5, the harmonic linearization for Random Early Detection (RED) [2, 3, 4, 5] algorithm is described, as also as the verification of the obtained results.

2. Research structure
The general structure of the research is as follows.

2.1. Formulation of the problem

In network systems, in particular in a system with TCP (Transmission Control Protocol) sources and a traffic management module, the phenomenon of global synchronization has been experimentally revealed. In this case, each TCP source decreases and increases the transmission data rate simultaneously with other TCP sources. The effect of global synchronization leads to the inefficient use of bandwidth and the increased data latency. The global synchronization is implemented as a self-oscillating mode in a system consisting of a TCP source and a traffic management module.

One of the tasks of the RED algorithm was to eliminate the global synchronization mode. However, this problem was not solved fundamentally. With some parameters of the RED algorithm, there is a self-oscillating mode in the system. We assume that the form of the drop function has a decisive influence on the occurrence of an self-oscillating mode. We need to obtain phase and parametric portraits of the parameters of system (based on the threshold values of the RED algorithm) to determine the region of self-oscillations occurrence.

2.2. Mathematical model

The model of the system was previously proposed in the works [6, 7] as the system of nonlinear differential equations. The study of the model is complicated by the presence of a nonsmooth function (the drop function) in the equation. This function can be a discontinuous, for example, in the RED algorithm, or it can have the form of a hysteresis, for example, in the Adaptive RED (ARED) algorithm. In addition, there is a delay in the model. All this makes the study the system extremely difficult. We would like to highlight separately the influence of various conditions (such as the nature of the drop function and the delay) on the occurrence of a self-oscillating mode.

2.3. Harmonic linearization

We proposed [8] to linearize the initial model and investigate the behavior of the linearized model. However, the ordinary linearization destroys the information about the oscillating mode and makes the linearized model non-equivalent to the original one. For this type of model, it is possible to apply the so-called harmonic linearization, which allows to carry out a partial linearization of the model and to investigate the occurrence of a self-oscillating mode. Several criteria can be used to find out the presence and nature of the self-oscillating mode. One of them is more convenient to use in order to find out the presence of self-oscillations (Nyquist criterion [9]), others to determine the nature of self-oscillation stability (Hurwitz criterion [10]).

2.4. Verification of results

We suggest to verify the received solutions by using the NS-2 software simulator. This simulator is the reference implementation of network protocols. It is usually assumed that the simulation on NS-2 is equivalent to the behavior of a real system.

3. The RED congestion adaptive control mechanism

The RED algorithm uses a weighted queue length as a factor determining the probability of packet drop. As the average queue length grows, the probability of packets drop also increases (see Eq. (1)). The algorithm uses two threshold values of the average queue length to control the drop function:

$$p(\hat{Q}) = \begin{cases} 0, & 0 < \hat{Q} \leqslant Q_{\min}, \\ \dfrac{\hat{Q} - Q_{\min}}{Q_{\max} - Q_{\min}} p_{\max}, & Q_{\min} < \hat{Q} \leqslant Q_{\max}, \\ 1, & \hat{Q} > Q_{\max}. \end{cases} \qquad (1)$$

Here $p(\hat{Q})$ is the packet drop function (drop probability), $\hat{Q}$ is the exponentially-weighted moving average of the queue size average, $Q_{\min}$ and $Q_{\max}$ are the thresholds for the weighted average of the queue length, $p_{\max}$ is the the maximum level of packet drop.

The RED algorithm is quite effective due to simplicity of its implementation in the network hardware, but it has a number of drawbacks. In particular, for some parameters values there is a steady oscillatory mode in the system, which negatively affects quality of service (QoS) indicators [11, 12, 13]. Unfortunately there are no clear selection criteria for RED parameters values, at which the system does not enter in self-oscillating mode.

To describe the RED algorithm we will use the following continuous model (see [6, 7, 14, 15, 16, 17]) with some simplifying assumptions: the model is written in the moments; the model describes only the phase of congestion avoidance for TCP Reno protocol; in the model the drop is considered only after reception of 3 consistent ACK confirmations.

$$
\begin{cases}
\dot{W}(t) = \dfrac{1}{T(Q,t)} - \dfrac{W(t)W(t - T(Q,t))}{2T(t - T(Q,t))}p(t - T(Q,t)); \\[2mm]
\dot{Q}(t) = \dfrac{W(t)}{T(Q,t)}N(t) - C; \\[2mm]
\dot{\hat{Q}}(t) = -w_q C \hat{Q}(t) + w_q C Q(t).
\end{cases}
\tag{2}
$$

Here the following notation is used: W is the average TCP window size; Q is the average queue size; $\hat{Q}$ is the exponentially weighted moving average (EWMA) of the queue size average; C is the queue service intensity; T is the full round-trip time; $T = T_p + \frac{Q}{C}$, where T_p is the round-trip time for free network (excluding delays in hardware); $\frac{Q}{C}$ is the time which batch spent in the queue; N is the number of TCP sessions; p is the packet drop function.

4. Harmonic linearization method

The method of harmonic linearization is an approximate method. It is used for study of self-oscillation conditions and determination of the parameters of self-oscillations, for the analysis and evaluation of their sustainability, as well as for the study of forced oscillations. Harmonically-linearized system depends on the amplitudes and frequencies of periodic processes. The harmonic linearization differs from the common method of linearization (leading to purely linear expressions) and allows to explore the basic properties of nonlinear systems.

The method of harmonic linearization is used for systems of a certain structure (see Fig. 1). The system consists of linear part H_l and the nonlinear part, which is set by a function $f(x)$. It is generally considered a static nonlinear element.
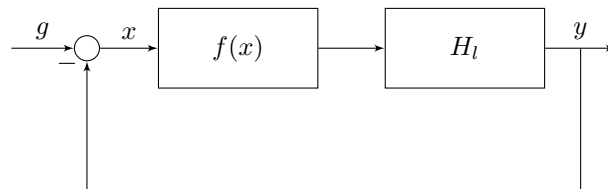

Figure 1. Block structure of the system for the harmonic linearization method

For the harmonic linearization method the free movement mode (input $g(t) = 0$) is assumed. The free harmonic oscillations are applied to the input of the nonlinear element:

$$
x(t) = A\sin(\omega t).
\tag{3}
$$

On the output of the nonlinear element $f(x)$ we get a periodic signal. Let's expand it in a Fourier series.

The linear element is a low-pass filter, that is, when k is increasing the linear elements suppress higher harmonics. We will consider only the first harmonics:

$$f(x) = a_1 \sin(\omega t) + b_1 \cos(\omega t). \tag{4}$$

From (3) one can write:

$$x := A \sin(\omega t);$$
$$\cos(\omega t) = \frac{1}{A\omega} \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{A\omega} \frac{\mathrm{d}}{\mathrm{d}t} x. \tag{5}$$

Then we may rewrite (4) with respect (5):

$$f(x) = \left[\varkappa(A) + \frac{\varkappa'(A)}{\omega} \frac{\mathrm{d}}{\mathrm{d}t} \right] x \xrightarrow{\frac{\mathrm{d}}{\mathrm{d}t} \to \mathrm{i}\omega} \left[\varkappa(A) + \mathrm{i}\varkappa'(A) \right] x = H_{nl}(A)x, \tag{6}$$

where $H_{nl}(A)$ is the approximate transfer function of the nonlinear unit, $\varkappa(A)$ and $\varkappa'(A)$ are the harmonic linearization coefficients:

$$\varkappa(A) = \frac{a_1}{A} = \frac{1}{A\pi} \int_0^{2\pi} f(A\sin(\omega t)) \sin(\omega t) \, \mathrm{d}(\omega t) \,;$$
$$\varkappa'(A) = \frac{b_1}{A} = \frac{1}{A\pi} \int_0^{2\pi} f(A\sin(\omega t)) \cos(\omega t) \, \mathrm{d}(\omega t) \,. \tag{7}$$

After finding the coefficients of harmonic linearization for given nonlinear unit, it is possible to study the parameters of the oscillation mode. The existence of oscillation mode in a nonlinear system corresponds to the determination of oscillating boundary of stability for the linearized system. Then A and ω can be found by using stability criteria of linear systems (Mikhailov, Nyquist–Mikhailov [9], Routh–Hurwitz [10, 18]). Thus, the study of self-oscillation parameters can be done by one of the methods of determining the limits of stability of the linear systems.

4.1. The Nyquist–Mikhailov criterion
The Nyquist-Mikhailov criterion [9, 19] allows to judge about the stability of the open-loop automatic control system by using Nyquist plot (amplitude-phase characteristic) of the open-loop system.

Make the substitutions $\partial_t \to \mathrm{i}\omega$ and $s \to \partial_t \to \mathrm{i}\omega$ in the transfer function H_l and H_{nl}. Undamped sinusoidal oscillations with constant amplitude are determined by passing the amplitude-phase characteristics of the open-loop system through the point $(-1, \mathrm{i}0)$.

Thus we obtain:

$$H_l(\mathrm{i}\omega)H_{nl}(A, \mathrm{i}\omega) = -1. \tag{8}$$

Given by (6) from (8) the equality is obtained:

$$H_l(\mathrm{i}\omega) = -\frac{1}{\varkappa(A) + \mathrm{i}\varkappa'(A)}. \tag{9}$$

The left part of the equation (9) is the amplitude-phase characteristic of the linear unit, and the right part is the inverse of the amplitude-phase characteristic of the first harmonic non-linear level (with opposite sign). And the equation (9) is the equation of balance between the frequency and the amplitude.

4.2. Mikhailov criterion
Let us write the characteristic equation of the conservative system (12) (Fig. 1) with respect of $s \rightarrow \partial_t \rightarrow i\omega$:

$$P_d(i\omega) = 0. \tag{10}$$

In addition, if one explicitly allocates real and complex parts, then the equation (10) can be written in the following form:

$$\begin{aligned} \mathrm{Re}\{P_d(i\omega)\} &= 0, \\ \mathrm{Im}\{P_d(i\omega)\} &= 0. \end{aligned} \tag{11}$$

Thus, the parameters of self-oscillations can be determined from equation (11).

4.3. RouthHurwitz criterion
This criterion is an algebraic criterion of stability [18, 10].

The transfer function of the conservative system (Fig. 1) has the form:

$$H_c(s) = \frac{H_l(s)}{1 + H_l(s)H_{nl}(s)} := \frac{P_n(s)}{P_d(s)}. \tag{12}$$

The equation $P_d(s) = 0$ is the characteristic equation of the system. It can be presented in the polynomial form:

$$P_d(s) := a_0 s^n + a_1 s^{n-1} + \cdots + a_n.$$

Thus, the Hurwitz determinant will look like this [20]:

$$\Delta = \begin{vmatrix} a_1 & a_3 & a_5 & \ldots & 0 \\ a_0 & a_2 & a_4 & \ldots & 0 \\ 0 & a_1 & a_3 & \ldots & 0 \\ 0 & a_0 & a_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & a_n \end{vmatrix}.$$

By the Hurwitz criterion, in order for the dynamical system to be stable, it is necessary and sufficient that all n principal diagonal minors of the Hurwitz determinant be positive, provided that $a_0 > 0$. These minors are called determinants of Hurwitz. For $a_n = 0$, the system is at the boundary of aperiodic stability. If the penultimate Hurwitz determinant (the minor of the Routh–Hurwitz determinant) $\Delta_{n-1} = 0$, then the system is at the boundary of oscillatory stability. Actually, this expression is necessary for us to determine the conditions for the appearance and parameters of self-oscillations.

5. Linearization of the model
To rewrite the model (2) in the block-linear approach we need to linearize it.

We will carry out the linearization near the equilibrium point (the balance point is denoted by f index). At the equilibrium point time derivatives turn to zero, so the system of equations (2) will be as follows:

$$\begin{cases} 0 = \dfrac{1}{T_f} - \dfrac{W_f^2}{2T_f} p_f; \\[2mm] 0 = \dfrac{W_f}{T_f} N_f - C; \\[2mm] 0 = -w_q C \hat{Q}_f + w_q C Q_f. \end{cases} \tag{13}$$
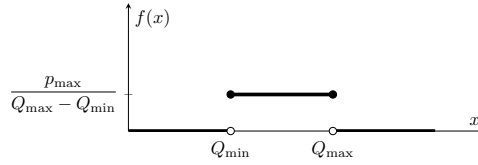
Figure 2. The function P_{RED}

From the system of equations (13) we get the bound equation for the equilibrium values of the variables:

$$
\begin{cases}
p_f = \dfrac{2}{W_f^2}; \\[2mm]
W_f = \dfrac{CT_f}{N_f}; \\[2mm]
\hat{Q}_f = Q_f.
\end{cases}
$$

The variation of the right part (13) for all variables in a neighborhood of the equilibrium point is:

$$
\begin{cases}
\delta W(s) = -\dfrac{1}{s + \frac{N}{CT_f^2}\left(1 + \mathrm{e}^{-sT_f}\right)} \dfrac{C^2 T_f}{2N^2} \mathrm{e}^{-sT_f}\, \delta p(s)\,; \\[4mm]
\delta Q(s) = \dfrac{1}{s + \frac{1}{T_f}} \dfrac{N}{T_f}\, \delta W(s)\,; \\[4mm]
\delta \hat{Q}(s) = \dfrac{1}{1 + \frac{s}{w_q C}}\, \delta Q(s)\,.
\end{cases}
\tag{14}
$$

In addition, let us to linearize the drop function (1):

$$
\delta p(\hat{Q}, t) =
\begin{cases}
0, & 0 < \hat{Q} \leqslant Q_{\min}, \\[2mm]
\dfrac{p_{\max}}{Q_{\max} - Q_{\min}}\, \delta \hat{Q}(t), & Q_{\min} < \hat{Q} \leqslant Q_{\max}, \\[2mm]
0, & \hat{Q} > Q_{\max}.
\end{cases}
\tag{15}
$$

The (15) may be denoted as

$$
\delta p(\hat{Q}, t) = P_{\mathrm{RED}}\, \delta \hat{Q}(t)\,;
$$

$$
P_{\mathrm{RED}} :=
\begin{cases}
0, & 0 < \hat{Q} \leqslant Q_{\min}, \\[2mm]
\dfrac{p_{\max}}{Q_{\max} - Q_{\min}}, & Q_{\min} < \hat{Q} \leqslant Q_{\max}, \\[2mm]
0, & \hat{Q} > Q_{\max}.
\end{cases}
\tag{16}
$$

Considering the formula $\delta\hat{Q}(s)$ from the system of equations (14), we can write out (16) in the following form:

$$
\delta p(s) = P_{\mathrm{RED}} \dfrac{1}{1 + \frac{s}{w_q C}}\, \delta Q(s)\,.
\tag{17}
$$

The function P_{RED} has the form shown in Fig. 2

Based on (14) and (17) the block representation of the linearized RED model (Fig. 3) is constructed.
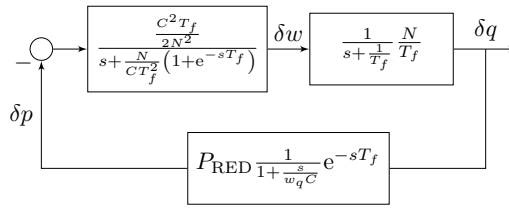
Figure 3. Block representation of the linearized RED model

5.1. Harmonic linearization of the linearized RED model

Let's reduce the block diagram of linearized model (Fig. 3) to the form required for harmonic linearization.

As a static nonlinear function we will use P_{RED} (16). The linear part we get from (14) and (17):

$$H_l = \frac{1}{s + \frac{N}{CT_f^2}\left(1 + \mathrm{e}^{-sT_f}\right)} \frac{C^2 T_f}{2N^2} \mathrm{e}^{-sT_f} \times \frac{1}{s + \frac{1}{T_f}} \frac{N}{T_f} \times \frac{1}{1 + \frac{s}{w_q C}} =$$

$$= \frac{1}{s + \frac{N}{CT_f^2}\left(1 + \mathrm{e}^{-sT_f}\right)} \frac{1}{s + \frac{1}{T_f}} \frac{1}{1 + \frac{s}{w_q C}} \frac{C^2}{2N} \mathrm{e}^{-sT_f}.$$

Let us compute the coefficients of harmonic linearization $\varkappa(A)$ and $\varkappa'(A)$ (7) for the static nonlinearity P_{RED}:

$$\varkappa(A) = \frac{4}{A\pi} \frac{p_{\max}}{Q_{\max} - Q_{\min}} \left(\sqrt{1 - \frac{Q_{\min}^2}{A^2}} - \sqrt{1 - \frac{Q_{\max}^2}{A^2}} \right);$$

$$\varkappa'(A) = \frac{4}{A\pi} \frac{p_{\max}}{Q_{\max} - Q_{\min}} \frac{Q_{\max} - Q_{\min}}{A} = \frac{4p_{\max}}{A^2 \pi}.$$

For the example of the the calculation we have chosen the following parameters: $Q_{\min} = 100$ [packets], $Q_{\max} = 150$ [packets], $p_{\max} = 0.1$, $T_p = 0.0075$ s, $w_q = 0.002$, $C = 2000$ [packets]/s, $N = 60$ (the number of TCP sessions).

As a result we obtained the following values for the amplitude and the cyclic frequency: $A = 1.89$ [packets], $\omega = 16.55 s^{-1}$.

The traffic behavior can be demonstrated by using the standard computer networks simulation software NS-2 [21, 22].

For selected parameters we will get the graph of the oscillations of the instantaneous queue length at a router under RED control (Fig. 4). With the help of fast Fourier transform we will study the results of NS-2 simulation (Fig. 5).

6. Conclusion

The authors proposed the technique for studying the self-oscillating modes of systems with control. The basic stages of the research process are described. The main element of this process is the harmonic linearization.

In the article, we demonstrated the technique for one set of parameters based on our methodology. Accordingly, only one pair of amplitude–frequency values is obtained. The obtained theoretical values are verified by using the NS-2 simulation software. In further research the authors plan to calculate the existence of self-oscillating modes for all threshold values for the RED algorithm. In addition, it is planned to conduct research for different RED-like algorithms.
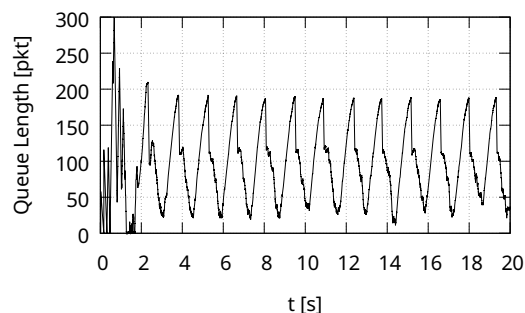
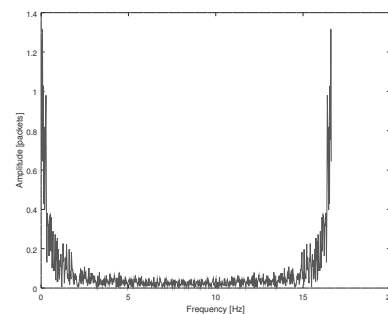**Figure 4.** A router's queue oscillation under RED control

Figure 5. The spectrum of the oscillations

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