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Research and development company
"Information and networking technologies"

DISTRIBUTED COMPUTER AND COMMUNICATION NETWORKS: CONTROL, COMPUTATION, COMMUNICATIONS (DCCN-2016)

Volume 2
Mathematical Modeling, Simulation and Control Problems

Proceedings
of the Nineteenth International Scientific Conference

Russia, Moscow, 21–25 November 2016

Under the general editorship
of D.Sc. V. M. Vishnevskiy and D.Sc. K. E. Samouylov

Moscow
2016

Федеральное государственное бюджетное учреждение науки
Институт проблем управления им. В. А. Трапезникова
РОССИЙСКОЙ АКАДЕМИИ НАУК
Федеральное государственное автономное
образовательное учреждение высшего образования
«РОССИЙСКИЙ УНИВЕРСИТЕТ ДРУЖБЫ НАРОДОВ»
Национальный исследовательский
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Научно-производственное объединение
«Информационные и сетевые технологии»

РАСПРЕДЕЛЕННЫЕ КОМПЬЮТЕРНЫЕ И ТЕЛЕКОММУНИКАЦИОННЫЕ СЕТИ: УПРАВЛЕНИЕ, ВЫЧИСЛЕНИЕ, СВЯЗЬ (DCCN-2016)

В трех томах

Том 2
Математическое моделирование и задачи управления

Материалы
Девятнадцатой международной научной конференции

Россия, Москва, 21–25 ноября 2016 г.

Под общей редакцией
д.т.н. В.М. Вишневского и д.т.н. К.Е. Самуйлова

Москва
2016

УДК 004.7:004.4.001:621.391:007(063)
ББК 32.973.202:32.968
Р 24

Р 24 **Распределенные компьютерные и телекоммуникационные сети: управление, вычисление, связь (DCCN-2016) = Distributed computer and communication networks: control, computation, communications (DCCN-2016)** : материалы Девятнадцатой международной научной конференции, 21–25 нояб. 2016 г. : в 3 т.; под общ. ред. В. М. Вишневского и К. Е. Самуйлова — М.: РУДН, 2016.
ISBN 978-5-209-07666-7
Т. 2: Математическое моделирование и задачи управления = Mathematical modeling, simulation and control problems. — 300 с. : ил.
ISBN 978-5-209-07668-1 (т. 2)

В научном издании представлены материалы Девятнадцатой международной научной конференции «Распределенные компьютерные и телекоммуникационные сети: управление, вычисление, связь» по следующим направлениям:

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— Математическое моделирование и задачи управления.

Сборник материалов конференции предназначен для научных работников и специалистов в области теории и практики построения компьютерных и телекоммуникационных сетей.

Текст воспроизводится в том виде, в котором представлен авторами

Утверждено к печати Программным комитетом конференции

ISBN 978-5-209-07668-1 (т. 2)
ISBN 978-5-209-07666-7

Contents

Plenary Speakers

Sessional Speakers

UDC 519.614;517.584

Waveguide Modes of a Planar Gradient Optical Waveguide

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Abstract. The mathematical model of light propagation in a planar gradient optical waveguide consists of the Maxwell's equations supplemented by the matter equations and boundary conditions. In the coordinates adapted to the waveguide geometry, the Maxwell's equations are separated into two independent sets for the TE and TM polarizations. For each polarization there are three types of waveguide modes in a regular planar optical waveguide: guided modes, substrate radiation modes, and cover radiation modes. In this work we implement the numerical-analytical calculation of all types of waveguide modes.

For the eigenvalue problem with a piecewise linear-constant potential we used the Airy functions to calculate the cover radiation modes and substrate radiation modes. We took advantage of reducing the initial potential scattering problem (in the case of the continuous spectrum) to the equivalent ones for the Jost functions: the Jost solution from the left for the substrate radiation modes and the Jost solution from the right for the cover radiation modes.

Keywords: waveguide propagation of electromagnetic radiation, equations of waveguide modes of regular waveguide, complete set of modes of a planar waveguide.

1. Introduction

Propagation of monochromatic polarized electromagnetic radiation is described by the vector homogeneous Maxwell's equations [1], the tangential boundary conditions at the interfaces of the waveguide layer with the substrate and the cover, and by asymptotic conditions “at infinity”.

In a Cartesian coordinate system associated with the geometry of the waveguide, the Maxwell's equations, after the separation of variables, split into two linearly independent systems with reduced boundary conditions [1, 2].

2. Statement of the problem

As a result the problem of describing the full set of waveguide modes of regular gradient planar optical waveguide is formulated as an eigenvalue problem (for discrete and continuous spectra) and eigenfunction problem (for classical and generalized functions) of essentially self-adjoint ordinary differential operator of the second order [1, 3]:

$$-p(x)\frac{d}{dx}\left(\frac{1}{p(x)}\frac{d\psi}{dx}(k,x)\right). \tag{1}$$

Here $p(x) = \varepsilon(x)$, $V(x) = -n^2(x)$ is piecewise-continuous (continuous in layers) function, $k^2 = -\beta^2$ is spectral parameter, and

$$\psi_{\mathrm{TE}}(x) = E_y(x), \quad \psi_{\mathrm{TM}}(x) = H_y(x).$$

Let's introduce the auxiliary functions

$$\varphi_{\mathrm{TE}}(x) = \frac{d\varphi_{\mathrm{TE}}}{dx}(x), \quad \varphi_{\mathrm{TM}}(x) = \frac{1}{p(x)}\frac{d\varphi_{\mathrm{TM}}}{dx}(x).$$

Using these functions we can write down reduced boundary conditions at points of discontinuity of the potential, and therefore of the second derivative of the solution:

$$\psi\big|_{x_1-0} = \psi\big|_{x_1+0}, \quad \psi\big|_{x_2-0} = \psi\big|_{x_2+0},$$
$$\varphi\big|_{x_1-0} = \varphi\big|_{x_1+0}, \quad \varphi\big|_{x_2-0} = \varphi\big|_{x_2+0}.$$

Besides, the asymptotic conditions are satisfied

$$\lim_{x\to\pm\infty} |\psi(x)| \leq C_\psi. \tag{2}$$

The spectrum of operator (1)–(2) consists of [4, 5]:

— a finite number of discrete eigenvalues $k_j = i\kappa_j$: $k_j^2 \in \big(\min V(x), \min(V_-, V_+)\big)$ and the corresponding classical eigenfunctions (of guided waveguide modes);

— a single continuous spectrum k_-: $k_-^2 \in (V_-, \infty)$ and corresponding generalized eigenfunctions (substrate radiation modes);

— a single continuous spectrum k_+: $k_+^2 \in (V_+, \infty)$ and corresponding generalized eigenfunctions (cover radiation modes).

For a constructive description of the problem solutions, i.e. eigenfunctions of three types, we shall restrict our consideration to piecewise-linear

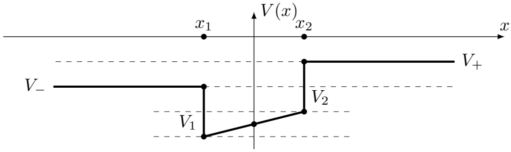

Figure 1. $V(x)$

potential:

$$V(x) = \begin{cases} V_-, & \text{when } x < x_1, \\ ax + b, & \text{when } x_1 < x < x_2, \quad \text{where } a = \frac{V_2 - V_1}{x_2 - x_1},\ b = \frac{V_1 x_2 - V_2 x_1}{x_2 - x_1}, \\ V_+, & \text{when } x > x_2. \end{cases}$$

The function $V(x)$ has the view shown on Fig. 1.

3. The solution for the eigenvalues (of the discrete spectrum) and eigenfunctions (classical) problem

The method of solution is the expansion on the sub-intervals of the general solution in terms of the fundamental system of solutions. To the left and to the right there are decreasing exponential functions in the case of real ε_s, ε_c (due to the asymptotic conditions):

$$\psi_s(k,x) = C_s \exp\{\gamma_s(x - x_1)\},$$
$$\psi_c(k,x) = C_c \exp\{-\gamma_c(x - x_2)\}.$$

In the waveguide layer (with a linear potential in the subdomain) the fundamental system of solutions consists of the functions $Ai(x)$ and $Bi(x)$, such that

$$\psi_f(k,x) = C_1 Ai\left(\frac{a(x - x_2) + b}{(-a)^{2/3}}\right) + C_2 Bi\left(\frac{a(x - x_2) + b}{(-a)^{2/3}}\right).$$

These common solutions in the subdomains form a single particular solution of the problem (1)–(2), therefore, the equalities must be satisfied:

$$\psi_s(k,x_1) = \psi_f(k,x_1), \quad \Phi_s(k,x_1) = \Phi_f(k,x_1),$$
$$\psi_f(k,x_2) = \psi_c(k,x_2), \quad \Phi_f(k,x_2) = \Phi_c(k,x_2).$$

As a result we obtain a homogeneous system of linear algebraic equations for the indefinite coefficients for the TE modes:

$$C_s = C_1 Ai\left(\frac{-ad + b}{(-a)^{2/3}}\right) + C_2 Bi\left(\frac{-ad + b}{(-a)^{2/3}}\right),$$

$$\gamma_s C_s = -C_1(-a)^{1/3}\frac{dAi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right) - C_2(-a)^{1/3}\frac{dBi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right),$$

$$C_1 Ai(0) + C_2 Bi(0) = C_c,$$

$$-C_1(-a)^{1/3}\frac{dAi}{dx}(0) - C_2(-a)^{1/3}\frac{dBi}{dx}(0) = -\gamma_c C_c.$$

The resulting homogeneous system of linear algebraic equations

$$\hat{M}_{\mathrm{TE}}(k)\vec{C}(k) = \vec{0}$$

has a non-trivial solution provided that

$$\det \hat{M}_{\mathrm{TE}}(k) = 0.$$

Solutions of nonlinear transcendental algebraic equation k_j are substituted in SLAE(x) and then this system is solved with respect to $\vec{C}_j = \vec{C}(k_j)$. The obtained coefficients are substituted in the expressions for the fields. The results of calculations are presented in Fig. 2–3.

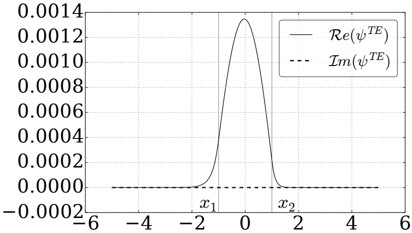

Figure 2. Wave modes TE_1, $n_c = 1.0$, $n_f = 2.15$, $n_s = 1.515$, $\beta^{TE} = 1.6752$, $\beta^{TM} = 1.5955$

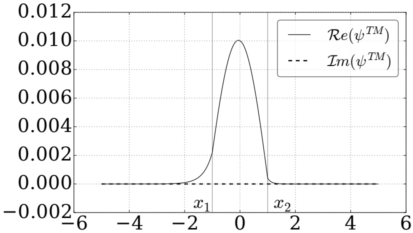

Figure 3. Wave modes TM_1, $n_c = 1.0$, $n_f = 2.15$, $n_s = 1.515$, $\beta^{TE} = 1.6752$,$\beta^{TM} = 1.5955$

4. Calculation of cover radiation modes

Similarly to what was done in [1, 3] for piecewise-constant potentials, let's move from solutions of the problem (1)–(2) satisfying the asymptotic Jost conditions, to the solutions satisfying the "scattering problem" conditions. A one-to-one correspondence between them is set in [1, 3] for the potentials of a more general kind.

So, in the region $(-\infty, x_1)$ the general solutions of equation (1) with constant coefficient V_s are of the form (for TE modes):

$$\psi_c^{\mathrm{TE}}(k,x) = T_-^{\mathrm{TE}}(k)\exp\{-ip_s(x-x_1)\}.$$

In the region (x_2, ∞) the general solutions of equation (1) have the form

$$\psi_c^{\mathrm{TE}}(k,x) = \exp\{-ip_c(x-x_2)\} + R_-^{\mathrm{TE}}(k)\exp\{ip_c(x-x_2)\}.$$

In the region (a, b) the general solutions of equation (1) have the form (for TE modes):

$$\psi_f(k,x) = C_f^1 Ai\left(\frac{a(x-x_2)+b}{(-a)^{2/3}}\right) + C_f^2 Bi\left(\frac{a(x-x_2)+b}{(-a)^{2/3}}\right).$$

Thus, the solutions (for TE modes) are given by sets of amplitude coefficients $(T_-^{\mathrm{TE}}, C_f^1, C_f^2, R_-^{\mathrm{TE}})^T$, satisfying the system of linear algebraic equations:

$$T_-^{\mathrm{TE}}(k) = C_f^1 Ai\left(\frac{-ad+b}{(-a)^{2/3}}\right) + C_f^2 Bi\left(\frac{-ad+b}{(-a)^{2/3}}\right),$$

$$-\frac{p_s}{k_0\mu_s}T_-^{\mathrm{TE}}(k) = -C_f^1(-a)^{1/3}\frac{dAi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right) - C_f^2(-a)^{1/3}\frac{dAi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right),$$

$$C_f^1 Ai(0) + C_f^2 Bi(0) = 1 + R_-^{\mathrm{TE}}(k),$$

$$-C_f^1(-a)^{1/3}\frac{dAi}{dx}(0) - C_f^2(-a)^{1/3}\frac{dBi}{dx}(0) = -\frac{p_c}{k_0\mu_c}\left[1 - R_-^{\mathrm{TE}}(k)\right].$$

The resulting SLAE can be rewritten as:

$$\hat{M}^{\mathrm{TE}}(k)(T_-^{\mathrm{TE}}, C_f^1, C_f^2, R_-^{\mathrm{TE}})^T = \left(0, 0, 1, -\frac{p_c}{k_0\mu_c}\right)^T,$$

so that the solution exists for any $k^2 \in (V_c, \infty)$ and is unique up to a complex factor (Fig. 4–5).

5. Calculation of substrate radiation modes

Solutions have different view for different values of spectral parameter k from the two spectral subregions $k^2 \in (V_s, V_c)$ and $k^2 \in (V_c, \infty)$. But for both regions the solution, as in the case of guided modes, is constructed by matching at the boundaries of the general solutions of equation (1) in the regions of the argument $(-\infty, x_1)$, (x_1, x_2) and (x_2, ∞).

In the region $(-\infty, x_1)$ the general solutions of equation (1) with a spectral parameter $k^2 \in (V_s, V_c)$ have the form:

$$\psi_s^{\mathrm{TE}}(k,x) = \exp\{ip_s(k)(x-x_1)\} + R_+^{\mathrm{TE}}(k)\exp\{ip_s(k)(x-x_1)\}.$$

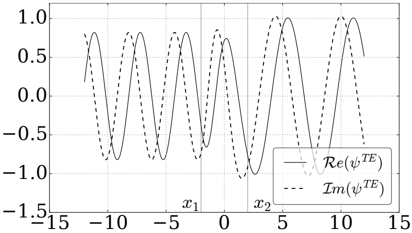

Figure 4. $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = 0.250$

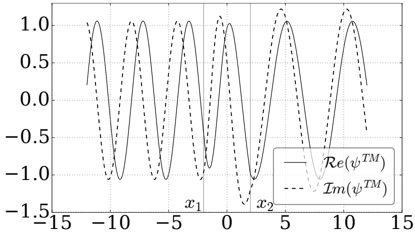

Figure 5. $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = 0.250$

In the region (x_1, x_2) the general solutions of equation (1.9) with a spectral parameter $k^2 \in (V_s, V_c)$ have the form:

$$\psi_f(k,x) = C_f^1 Ai\left(\frac{a(x-x_2)+b}{(-a)^{2/3}}\right) + C_f^2 Bi\left(\frac{a(x-x_2)+b}{(-a)^{2/3}}\right).$$

In the region (x_2, ∞) the general solutions of equation (1.9) with a spectral parameter $k^2 \in (V_s, V_c)$ have the form (by virtue of the asymptotic decay at infinity):

$$\psi_s^{\mathrm{TE}}(k,x) = A_c \exp\{-\gamma_c(x-x_2)\}.$$

Thus, the solutions (for TE modes) are given by sets of amplitude coefficients $(R_+^{\mathrm{TE}}, C_f^1, C_f^2, A_c)^T$ satisfying the system of linear algebraic equations:

$$1 + R_+^{\mathrm{TE}}(k) = C_f^1 Ai\left(\frac{-ad+b}{(-a)^{2/3}}\right) + C_f^2 Bi\left(\frac{-ad+b}{(-a)^{2/3}}\right),$$

$$\frac{p_s}{k_0\mu_s}\left[1 - R_+^{\mathrm{TE}}(k)\right] = -C_f^1(-a)^{1/3}\frac{dAi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right) - C_f^2(-a)^{1/3}\frac{dAi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right),$$

$$C_f^1 Ai(0) + C_f^2 Bi(0) = A_c,$$

$$-C_f^1(-a)^{1/3}\frac{dAi}{dx}(0) - C_f^2(-a)^{1/3}\frac{dBi}{dx}(0) = -\frac{\gamma_c}{ik_0\mu_c}A_c.$$

The resulting SLAE can be rewritten as:

$$\hat{M}^{\mathrm{TE}}(k)(R_+^{\mathrm{TE}}, C_f^1, C_f^2, A_c)^T = \left(-1, -\frac{p_c}{k_0\mu_s}, 0, 0\right)^T,$$

so that there exists a solution for any $k^2 \in (V_s, V_c)$ and it is unique up to a complex multiplier (Fig. 6–7).

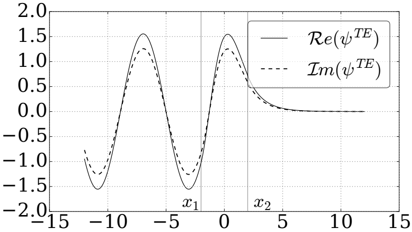

Figure 6. $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = -1.648$.

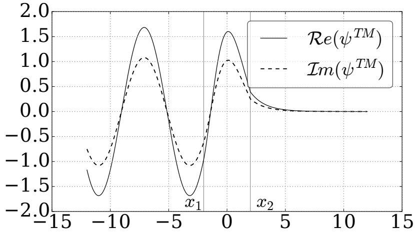

Figure 7. $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = -1.648$.

For the spectral parameter k from the region $k^2 \in (V_c, \infty)$, in the coordinate regions $(-\infty, x_1)$ and (x_1, x_2) common solutions have the same form as in the case $k^2 \in (V_s, V_c)$, and in the region (x_2, ∞), they take the form:

$$\psi_s^{\mathrm{TE}}(k, x) = T_+^{\mathrm{TE}}(k) \exp\{ip_c(k)(x - x_2)\}.$$

Consequently, the second pair of boundary equations at the point $x = x_2$ for TE modes take the form:

$$C_f^1 Ai(0) + C_f^2 Bi(0) = T_+^{\mathrm{TE}}(k),$$

$$-C_f^1(-a)^{1/3} \frac{dAi}{dx}(0) - C_f^2(-a)^{1/3} \frac{dBi}{dx}(0) = \frac{p_c(k)}{k_0 \mu_c} T_+^{\mathrm{TE}}(k).$$

The resulting SLAE can be rewritten as:

$$\hat{M}^{\mathrm{TE}}(k)(R_+^{\mathrm{TE}}, C_f^1, C_f^2, T_+^{\mathrm{TE}})^T = \left(-1, -\frac{p_s}{k_0 \mu_s}, 0, 0\right)^T,$$

so that there exists a solution for any $k^2 \in (V_c, \infty)$ and it is unique up to a complex multiplier (Fig. 8–9).

6. Conclusion

This paper presents the numerical implementations on a computer of square-integrable eigenfunctions corresponding to discrete spectrum $k_j = i\kappa_j$ for a piecewise-linear potential $V(x)$ (for the gradient waveguide). The present study also shows the numerical implementations on a computer of the cover radiation modes and substrate radiation modes. For modeling these modes, the problems of scattering on the potential $V(x)$ of Jost functions equivalent to the original problem in the case of the continuous spectrum were used: the problems of scattering on the left for the substrate radiation modes and the problems of scattering on the right for the cover radiation modes.

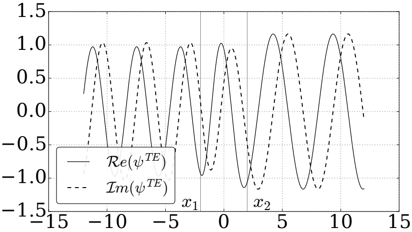

Figure 8. $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = 0.500$

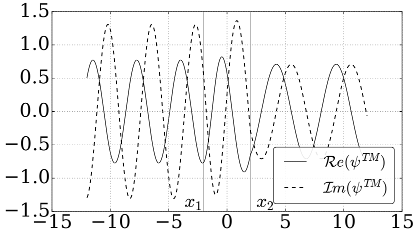

Figure 9. $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = 0.500$.

Acknowledgments

The work was partially supported by RFBF grants No 14-01-00628, No 15-07-08795, No 16-07-00556.

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