INSTITUTE OF EXPERIMENTAL PHYSICS
SLOVAK ACADEMY OF SCIENCES

THE 15th SMALL TRIANGLE MEETING

on Theoretical Physics

October 27–30, 2013

Stará Lesná

PREFACE

This proceedings comprises the talks presented at the 15^{th} *SMALL TRIANGLE MEETING on theoretical physics* conference, which was held in Stará Lesná, on October 27–30, 2013.

This year, it was already the 15^{th} STM conference, which is organized annually since 1999. The aim of the conference was to serve as a forum for meeting between theoretical and experimental physicists from Ukraine, Russia, Finland, Hungary, Czech Republic and Slovakia, where scientists from different research areas of physics met together. This provided an ideal opportunity to exchange knowledge, ideas and experiences. We believe that it helps us in our future work and that we find joint tasks in the following scientific collaboration. It was the fourth time that this conference was organized under auspieces of Centre of Excellence of Slovak Academy of Sciences named NANOFLUID as well as in frame of scientific activities of project of structural funds of European Union "Cooperative phenomena and phase transitions in nanosystems with perspective of utilization in technology and biomedicine". This conference significantly contributes to scientific activities of above mentioned projects in theoretical physics as well as in experimental physics. The scientific programme presented at this year's meeting covered the research areas from solid state physics, through nonlinear dynamical systems, atomic, nuclear, high energy physics to biophysics. The final programme included 21 oral presentations. We would like to thank the authors for their cooperation and we are looking forward for the following STM meeting.

We also included to the proceedings the article devoted to the philosophical aspects and methodology in physics.

Michal Hnatič and Peter Kopčanský

The 15th International workshop on theoretical physics Small Triangle Meeting, October 27 – 30, 2013, Hotel Academia, Stará Lesná

Program Committee

Michal Hnatič, Faculty of Science, P. J. Šafárik University, Košice
& Institute of Experimental Physics, SAS, Košice, Slovakia
& BLTP, Joint institute for Nuclear Research, Dubna, Russia
Peter Kopčanský, Institute of Experimental Physics, SAS, Košice, Slovakia
Ján Buša, Faculty of Electrical Engineering and Informatics, TU, Košice, Slovakia
Juha Honkonen, National Defence University, Helsinki, Finland
Volodymyr Lazur, Faculty of Physics, Uzhgorod National University, Ukraine
Alexei Gladyshev, BLTP, Joint institute for Nuclear Research, Dubna, Russia

List of participants

Serge Bondarenko, JINR, Dubna, Russian Federation
Vladimir Brekhovskikh, NRC Kurchatov Institute, Moscow, Russian Federation
Nándor Éber, ISSPO, WRCP HAS, Budapest, Hungary
Ján Fuksa , Czech Technical University & JINR, Dubna, Russian Federation
Alexei Gladyshev, JINR, Dubna, Russian Federation
Vladimir Gorev, NRC Kurchatov Institute, Moscow, Russian Federation
Ivan Haysak, UzhNU, Uzhgorod, Ukraine
Michal Hnatič, UPJŠ & IEP SAS, Košice, & Slovakia & JINR, Dubna, Russia
Juha Honkonen, HU, Helsinki, Finland
Viktor Khmara, UzhNU, Uzhgorod, Ukraine
Mikhailo Khoma, UzhNU, Uzhgorod, Ukraine
Peter Kopčanský, IEP SAS, Košice, Slovakia
Gabriela Kozáková, IEP SAS, Košice, Slovakia
Mikhail Kompaniets, Sankt Petersburg State University, Russia
Olexander Kovalchuk, IP, NASU,Kyiv, Ukraine
Tibor Kožár, IEP SAS, Košice, Slovakia
Vladimír Kravčak , FEE&I TU, Slovakia
Vladimír Lisý, FEE&I TU, Slovakia
Robert Lompay, UzhNU, Uzhgorod, Ukraine
Tomáš Lučivjanský, IEP SAS, Košice, Slovakia
Lukáš Mižšin, P.J. Šafarik University, Košice, Slovakia
Ivan Nebola (UzhNU, Ushgorod)
Michal Pudlák, IEP SAS, Košice, Slovakia
Olexander Reity, UzhNU, Uzhgorod, Ukraine
Leonid Sevastyanov, Peoples' Friendship University of Russia, Moscow, Russia
Milan Timko, IEP SAS, Košice, Slovakia
Tibor Tóth-Katona , WRCP HAS, Budapest, Hungary
Janka Tóthová, FEE&I TU, Slovakia
Peter Zalom, IEP SAS, Košice, Slovakia

Contents

A Naive Geometrization of Maxwell's Equations

D. S. Kulyabov[1], A. V. Korolkova[1], L. A. Sevastyanov[1,2]

[1] *Telecommunication System Department*
Peoples' Friendship University of Russia
Miklukho-Maklaya str., 6, Moscow, 117198, Russia

[2] *Bogoliubov Laboratory of Theoretical Physics*
Joint Institute for Nuclear Research
Joliot-Curie 6, 141980 Dubna, Moscow region, Russia

Abstract

For research in the field of transformation optics and for the calculation of optically inhomogeneous lenses the method of geometrization of the Maxwell equations seems to be perspective. The basic idea is to transform the coefficients of material equations, namely the dielectric permittivity and magnetic permeability in the effective geometry of space-time (besides the vacuum Maxwell equations). This allows us to solve the direct and inverse problems, that is, to find the permittivity and magnetic permeability for a given effective geometry (paths of rays), as well as finding an effective geometry on the dielectric permittivity and magnetic permeability. The most popular naive geometrization was proposed by J. Plebanski. Under certain limitations it is quite good for solving relevant problems. It should be noted that in his paper only the resulting formulas and exclusively for Cartesian coordinate systems are given. In our work we conducted a detailed derivation of formulas for the naive geometrization of Maxwell's equations, and these formulas are written for an arbitrary curvilinear coordinate system. This work is a step toward building a complete covariant geometrization of the macroscopic Maxwell's equations.

1 Introduction

Differential geometry was an important language of physics XX-th century. Basic elements of it where developed within the general relativity theory. There is a desire to use its power in other areas of physics, in particular in the optics.

The first attempts to apply the methods of differential geometry in electrodynamics should be attributed to publications I. E. Tamm [1, 2, 3]. In 1960 E. Plebansky proposed method of geometrization of the constitutive equations of the electromagnetic field, which became classic [4, 5, 6, 7]. All subsequent works, either used it or tried to correct a little, without changing ideology [8]. Unfortunately Plebansky gives no deriving formulas. Ideology is not expressed explicitly too.

For applying and deepening geometrization of material equations the authors have restored the ideology and specific Plebanski's calculations.

In paragraph 2 are the main relations for the Maxwell's equations in curvilinear coordinates (for more detailed discussion the reader can be refer to other authors articles). In paragraph 3 are presented actual calculations on Plebanski geometrization.

Notations and conventions We prosecuted provides basic notation and conventions used in the article.

1. We will use the notation of abstract indices [9]. In this notation tensor as a complete object is denoted merely by an index (eg, x^i). Its components are designated by underlined indices (e.g., $x^{\underline{i}}$).
2. We will adhere to the following agreements . Greek indices (α, β) will refer to the four-dimensional space , in component form it looks like: $\underline{\alpha} = \overline{0,3}$. Latin indices from the middle of the alphabet (i, j, k) will refer to the three-dimensional space , in the component form it looks like: $\underline{i} = \overline{1,3}$.
3. The comma in the index denotes partial derivative with respect to corresponding coordinate ($f_{,i} := \partial_i f$); semicolon denotes covariant derivative ($f_{;i} := \nabla_i f$).
4. Antisymmetrization is denoted by straight brackets.

2 Maxwell's equations in curvilinear coordinates

Here are the basics of Maxwell's equations in curvilinear coordinates. A more detailed description is given in articles [11, 12, 13, 14].

Let's write the Maxwell equations with the help of electromagnetic field tensors $F_{\alpha\beta}$ and $G_{\alpha\beta}$ [15, 16] in CGS symmetrical system [10]:

$$\nabla_\alpha F_{\beta\gamma} + \nabla_\beta F_{\gamma\alpha} + \nabla_\gamma F_{\alpha\beta} = F_{[\alpha\beta;\gamma]} = 0, \tag{1}$$

$$\nabla_\alpha G^{\alpha\beta} = \frac{4\pi}{c} j^\beta, \tag{2}$$

where the tensors $F_{\alpha\beta}$ and $G^{\alpha\beta}$ have the following components

$$F_{\underline{\alpha\beta}} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B^3 & B^2 \\ -E_2 & B^3 & 0 & -B^1 \\ -E_3 & -B^2 & B^1 & 0 \end{pmatrix}, \qquad G^{\underline{\alpha\beta}} = \begin{pmatrix} 0 & -D^1 & -D^2 & -D^3 \\ D^1 & 0 & -H_3 & H_2 \\ D^2 & H_3 & 0 & -H_1 \\ D^3 & -H_2 & H_1 & 0 \end{pmatrix}, \tag{3}$$

$E_{\underline{i}}$, $H_{\underline{i}}$ are components of electric and magnetic fields intensity vectors; $D^{\underline{i}}$, $B^{\underline{i}}$ are components of vectors of electric and magnetic induction.

It is also useful to introduce the tensor ${}^*F^{\alpha\beta}$ dual conjugated to $F_{\alpha\beta}$

$$ {}^*F^{\alpha\beta} = \frac{1}{2} e^{\alpha\beta\gamma\delta} F_{\gamma\delta}, \tag{4} $$

where $e^{\alpha\beta\gamma\delta}$ is the alternating tensor, $\varepsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita symbol:

$$ e_{\underline{\alpha\beta\gamma\delta}} = \sqrt{-g}\varepsilon_{\underline{\alpha\beta\gamma\delta}}, \quad e^{\underline{\alpha\beta\gamma\delta}} = -\frac{1}{\sqrt{-g}}\varepsilon^{\underline{\alpha\beta\gamma\delta}}. \tag{5} $$

Similarly, we can write

$$ {}^*G_{\alpha\beta} = \frac{1}{2} e_{\alpha\beta\gamma\delta} G^{\gamma\delta}. \tag{6} $$

The components of these dual tensors are given (by the expressions) as follows

$$ {}^*F^{\underline{\alpha\beta}} = \frac{1}{\sqrt{-g}} \begin{pmatrix} 0 & -B^1 & -B^2 & -B^3 \\ B^1 & 0 & E_3 & -E_2 \\ B^2 & -E_3 & 0 & E_1 \\ B^3 & E_2 & -E_1 & 0 \end{pmatrix}, {}^*G_{\underline{\alpha\beta}} = \sqrt{-g} \begin{pmatrix} 0 & H_1 & H_2 & H_3 \\ -H_1 & 0 & D^3 & -D^2 \\ -H_2 & -D^3 & 0 & D^1 \\ -H_3 & D^2 & -D^1 & 0 \end{pmatrix}. \tag{7} $$

With dual tensor (4) the equation (1) may be rewritten in a simpler form:

$$ \nabla_\alpha {}^*F^{\alpha\beta} = 0. \tag{8} $$

Next, we will explain why, for the purposes of implementation of the Plebanski's program, we prefer to write equations in the form (1) rather the in simpler form (8).

Maxwell's equations in a medium In the presence of the medium a group of equations Maxwell containing bound charges is changed, namely equation (2). Amongst the Maxwell's equations (1) and (2) must be added the constitutive relations between tensors $G^{\alpha\beta}$ and $F^{\alpha\beta}$. When introduced additional assumptions about the linearity of the environment and immobility substances they may be written in three-dimensional form as follows:

$$ D^i = \varepsilon^{ij} E_j, \qquad B^i = \mu^{ij} H_j, 3 \tag{9} $$

where ε^{ij} and μ^{ij} are the permittivity and permeability tensors. In four-dimensional form relation (9) takes the following form:

$$ G^{\alpha\beta} = \lambda^{\alpha\beta}_{\gamma\delta} F^{\gamma\delta}, \quad \lambda^{\alpha\beta}_{\gamma\delta} = \lambda^{[\alpha\beta]}_{[\gamma\delta]}, \tag{10} $$

here tensor $\lambda^{\alpha\beta}_{\gamma\delta}$ is containing information on the permittivity and permeability, as well as electro-magnetic coupling.

Assuming that the vacuum permittivity and permeability are view:

$$ \varepsilon^{ij} := \delta^{ij}, \qquad \mu^{ij} := \delta^{ij}, \tag{11} $$

we find that the vacuum constraint equations (9) and (10) take view

$$ D^i = E^i, \qquad B^i = H^i, \qquad G^{\alpha\beta} = F^{\alpha\beta}. \tag{12} $$

3 Formal geometrization of material Maxwell's equations

Plebanski has offered the elementary geometrization of Maxwell's equations [4, 5]. However, in the original article the final formula are immediately given, and the principles and methods for their preparation remain obscure. The authors tried to explicitly describe technique that we believe Plebanski used and to perform calculations in detail.

The basic ideas of Plebanski's geometrization are as follows:

1. We write the Maxwell's equations in a medium in Minkowski space.

2. We write the vacuum Maxwell's equations in the effective Riemann space.

3. We equate the corresponding terms of both equations.

As a result, we obtain an expression of the permittivity and permeability in terms of geometric objects.

Before attempting the program of Plebanski let us recall some auxiliary relations.

3.1 Auxiliary relations

Differential Bianchi identity Note that the equation (1) can be written as

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = F_{[\alpha\beta,\gamma]} = 0. \tag{13}$$

Really:

$$\nabla_\alpha F_{\beta\gamma} + \nabla_\beta F_{\gamma\alpha} + \nabla_\gamma F_{\alpha\beta} =$$
$$= \partial_\alpha F_{\beta\gamma} - \Gamma^\delta_{\alpha\beta} F_{\delta\gamma} - \Gamma^\delta_{\alpha\gamma} F_{\beta\delta} + \partial_\beta F_{\gamma\alpha} - \Gamma^\delta_{\beta\gamma} F_{\delta\alpha} - \Gamma^\delta_{\beta\alpha} F_{\gamma\delta} + \partial_\gamma F_{\alpha\beta} - \Gamma^\delta_{\gamma\alpha} F_{\delta\beta} - \Gamma^\delta_{\gamma\beta} F_{\alpha\delta}. \tag{14}$$

Taking into account the anti-symmetry of the tensor $F_{\alpha\beta}$ and the symmetry in the lower indices of Christoffel symbols $\Gamma^\delta_{\alpha\beta}$, we obtain (13).

The resulting equation can be written form-invariant by in any coordinate system. Therefore, we will use the equation (13) instead of the more commonly used equation (8).

The metric tensor relations In addition, we need simple relations for the metric tensor.

$$g_{\alpha\delta} g^{\delta\beta} = \delta^\beta_\alpha \tag{15}$$

The relation (15) leads to the following special relations:

$$g_{0\delta} g^{\delta i} = g_{00} g^{0i} + g_{0k} g^{ki} = \delta^i_0 = 0, \tag{16}$$
$$g_{i\delta} g^{\delta j} = g_{i0} g^{0j} + g_{ik} g^{kj} = \delta^j_i. \tag{17}$$

Let equation (16) be rewritten in form

$$g^{0i} = -\frac{1}{g_{00}} g_{0k} g^{ki}. \tag{18}$$

Substituting equation (18) in equation (17), we obtain:

$$\left(g_{ik} - \frac{1}{g_{00}} g_{0i} g_{0k} \right) g^{kj} = \delta_i^j. \tag{19}$$

This relation will be used later to simplify the final equations.

3.2 Geometrization in Cartesian coordinates

Let us write the Maxwell's equations in a medium in Cartesian coordinates with the metric tensor $\eta_{\alpha\beta} = \operatorname{diag}(1, -1, -1, -1)$:

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0, \qquad \partial_\alpha G^{\alpha\beta} = \frac{4\pi}{c} j^\beta. \tag{20}$$

Now we write the vacuum Maxwell's equations in effective Riemann space with the metric tensor $g_{\alpha\beta}$ (their related values mark on by the tilde):

$$\partial_\alpha \tilde{F}_{\beta\gamma} + \partial_\beta \tilde{F}_{\gamma\alpha} + \partial_\gamma \tilde{F}_{\alpha\beta} = 0, \qquad \frac{1}{\sqrt{-g}} \partial_\alpha \left(\sqrt{-g} \tilde{G}^{\alpha\beta} \right) = \frac{4\pi}{c} \tilde{j}^\beta. \tag{21}$$

In a vacuum, the following relation is true (see (12)):

$$\tilde{F}_{\alpha\beta} = \tilde{G}_{\alpha\beta}. \tag{22}$$

Raising the indices in (22), we obtain

$$\tilde{F}^{\alpha\beta} = g^{\alpha\gamma} g^{\beta\delta} \tilde{G}_{\gamma\delta}. \tag{23}$$

Comparing term by term (20) and (21), and taking into account the (23) we obtain:

$$F_{\alpha\beta} = \tilde{F}_{\alpha\beta}, \qquad j^\alpha = \sqrt{-g} \tilde{j}^\alpha, \tag{24}$$

$$G^{\alpha\beta} = \sqrt{-g} g^{\alpha\gamma} g^{\beta\delta} F_{\gamma\delta}. \tag{25}$$

Equations (25) are actually 4-dimensional geometrized constitutive relations (10), we were seeking for. Following the Plebanski's program we must obtain the explicit form for the 3-dimensional constitutive relations (9).

Electric displacement field Let us rewrite (25) in the form of:

$$F_{\alpha\beta} = \frac{1}{\sqrt{-g}} g_{\alpha\gamma} g_{\beta\delta} G^{\gamma\delta} \tag{26}$$

and look for the value components of $F_{0\underline{i}}$, taking into account relations (3):

$$F_{0i} = E_i = \frac{1}{\sqrt{-g}} g_{0\gamma} g_{i\delta} G^{\gamma\delta} = \frac{1}{\sqrt{-g}} \left(g_{0j} g_{i0} G^{j0} + g_{00} g_{ij} G^{0j} \right) + \frac{1}{\sqrt{-g}} g_{0j} g_{ik} G^{jk} = \\ = \frac{1}{\sqrt{-g}} g_{00} \left(\frac{1}{g_{00}} g_{0j} g_{i0} - g_{ij} \right) D^j - \frac{1}{\sqrt{-g}} g_{0j} g_{ik} \varepsilon^{jkl} H_l. \quad (27)$$

For induction components D^i we apply the relation (19) and obtain

$$D^i = -\frac{\sqrt{-g}}{g_{00}} g^{ij} E_j + \frac{1}{g_{00}} \varepsilon^{ijk} g_{j0} H_k. \quad (28)$$

From (28) we can formally deduce the expression for the permittivity:

$$\varepsilon^{ij} = -\frac{\sqrt{-g}}{g_{00}} g^{ij}. \quad (29)$$

In this sense the second term in (28) needs further clarification.

Magnetic induction To obtain expressions for the magnetic induction we will use tensors (7). Moving down the indices of ${}^*G^{\alpha\beta}$ in terms of (25) and applying relations (4), (6) we obtain

$${}^*G_{\alpha\beta} = \sqrt{-g} g_{\alpha\gamma} g_{\beta\delta} {}^*F^{\gamma\delta}. \quad (30)$$

We will seek for the value components of ${}^*G_{0\underline{i}}$:

$${}^*G_{0i} = \sqrt{-g} H_i = \sqrt{-g} g_{0\gamma} g_{i\delta} {}^*F^{\gamma\delta} = \\ = \sqrt{-g} \left(g_{0j} g_{i0} {}^*F^{j0} + g_{00} g_{ij} {}^*F^{0j} \right) + \sqrt{-g} g_{0j} g_{ik} {}^*F^{jk} = \\ = \sqrt{-g} g_{00} \left(\frac{1}{g_{00}} g_{0j} g_{i0} - g_{ij} \right) \frac{1}{\sqrt{-g}} B^j - \sqrt{-g} g_{0j} g_{ik} \varepsilon^{jkl} \frac{1}{\sqrt{-g}} E_l. \quad (31)$$

Applying the relation (19) we obtain for B^i the following expression:

$$B^i = -\frac{\sqrt{-g}}{g_{00}} g^{ij} H_j - \frac{1}{g_{00}} \varepsilon^{ijk} g_{j0} E_k. \quad (32)$$

From (32) we can formally write the expression for permeability:

$$\mu^{ij} = -\frac{\sqrt{-g}}{g_{00}} g^{ij}. \quad (33)$$

Thus geometrized constitutive relations in Cartesian coordinates are as follows:

$$\begin{gathered} D^i = \varepsilon^{ij} E_j + \varepsilon^{ijk} w_j H_k, \qquad B^i = \mu^{ij} H_j - \varepsilon^{ijk} w_j E_k, \\ \varepsilon^{ij} = -\frac{\sqrt{-g}}{g_{00}} g^{ij}, \qquad \mu^{ij} = -\frac{\sqrt{-g}}{g_{00}} g^{ij}, \qquad w_i = \frac{g_{i0}}{g_{00}}. \end{gathered} \quad (34)$$

These equations were given in the original paper [4]. Thus, we can assume that we have performed our task.

3.3 Geometrization in curvilinear coordinates

We now extend the scope of the formulas obtained to the Maxwell's equations in arbitrary curvilinear coordinates. Suppose that this space is defined by the metric tensor $\gamma_{\alpha\beta}$. Then the system (20) takes the following form:

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0, \qquad \frac{1}{\sqrt{-\gamma}} \partial_\alpha \left(\sqrt{-\gamma} G^{\alpha\beta} \right) = \frac{4\pi}{c} j^\beta. \tag{35}$$

Further, repeating the steps for the effective Riemann space (21), (22), (23), we obtain the analogues to (24) and (25) as follows

$$F_{\alpha\beta} = \tilde{F}_{\alpha\beta}, \qquad j^\alpha = \frac{\sqrt{-g}}{\sqrt{-\gamma}} \tilde{j}^\alpha, \tag{36}$$

$$G^{\alpha\beta} = \frac{\sqrt{-g}}{\sqrt{-\gamma}} g^{\alpha\gamma} g^{\beta\delta} F_{\gamma\delta}. \tag{37}$$

Electric displacement field We write the expression (37) as:

$$F_{\alpha\beta} = \frac{\sqrt{-\gamma}}{\sqrt{-g}} g_{\alpha\gamma} g_{\beta\delta} G^{\gamma\delta}. \tag{38}$$

Arguing similarly to (27), we obtain for the components of the electric displacement field D^i the relation:

$$D^i = -\frac{\sqrt{-g}}{\sqrt{-\gamma}} \frac{1}{g_{00}} g^{ij} E_j + \frac{1}{g_{00}} \varepsilon^{ijk} g_{j0} H_k, \tag{39}$$

and the expression for the permittivity takes the form:

$$\varepsilon^{ij} = -\frac{\sqrt{-g}}{\sqrt{-\gamma}} \frac{1}{g_{00}} g^{ij}. \tag{40}$$

Magnetic induction Let's rewrite (30) taking into account (37)

$$^*G_{\alpha\beta} = \frac{\sqrt{-g}}{\sqrt{\gamma}} g_{\alpha\gamma} g_{\beta\delta} {}^*F^{\gamma\delta}. \tag{41}$$

By analogy with (31), (32) and (33) we obtain relations for the B^i:

$$B^i = -\frac{\sqrt{-g}}{\sqrt{\gamma}} \frac{1}{g_{00}} g^{ij} H_j - \frac{1}{g_{00}} \varepsilon^{ijk} g_{j0} E_k, \tag{42}$$

and permeability takes the form

$$\mu^{ij} = -\frac{\sqrt{-g}}{\sqrt{-\gamma}} \frac{1}{g_{00}} g^{ij}. \tag{43}$$

Thus geometrized constitutive relations in curvilinear coordinates with the metric tensor $\gamma_{\alpha\beta}$ are of the following form:

$$\begin{gathered} D^i = \varepsilon^{ij} E_j + \varepsilon^{ijk} w_j H_k, \qquad B^i = \mu^{ij} H_j - \varepsilon^{ijk} w_j E_k, \\ \varepsilon^{ij} = -\frac{\sqrt{-g}}{\sqrt{-\gamma}} \frac{1}{g_{00}} g^{ij}, \qquad \mu^{ij} = -\frac{\sqrt{-g}}{\sqrt{-\gamma}} \frac{1}{g_{00}} g^{ij}, \qquad w_i = \frac{g_{i0}}{g_{00}}. \end{gathered} \tag{44}$$

4 Conclusions

The authors have restored the program and calculations of Plebanski. This approach to geometrization appears inconclusive. But this does not prevent on to use this method for the calculations in the transformational optics.

References

[1] Tamm, I. E. *Russian Journal of Physical and Chemical Society. Part physical* **56**, 248–262 (1924). In Russian.

[2] Tamm, I. E. *Russian Journal of Physical and Chemical Society. Part physical* **57**, 209–240 (1925). In Russian.

[3] Tamm, I. E. & Mandelstam, L. I. *Mathematische Annalen* **95**, 154–160 (1925).

[4] Plebanski, J. *Physical Review* **118**, 1396–1408 (1960).

[5] Felice, F. *General Relativity and Gravitation* **2**, 347–357 (1971).

[6] Leonhardt, U., Philbin, T. G. & Haugh, N. 1–19 (2008). ArXiv: `0607418v2`.

[7] Leonhardt, U. & Philbin, T. In *Progress in Optics*, vol. 53, 69–152 (2009). ArXiv: `0805.4778v2`.

[8] Thompson, R. T., Cummer, S. a. & Frauendiener, J. *Journal of Optics* **13**, 024008 (2011).

[9] Penrose, R. & Rindler, W. *Spinors and Space-Time: Two-Spinor Calculus and Relativistic Fields*, vol. 1 (Cambridge University Press, 1984).

[10] Sivukhin, D. V. *Soviet Physics Uspekhi* **22**, 834–836 (1979).

[11] Kulyabov, D. S., Korolkova, A. V. & Korolkov, V. I. *Bulletin of Peoples' Friendship University of Russia. Series "Mathematics. Information Sciences. Physics"* 96–106 (2012). ArXiv: `1211.6590`.

[12] Korol'kova, A. V., Kulyabov, D. S. & Sevast'yanov, L. A. *Programming and Computer Software* **39**, 135–142 (2013).

[13] Kulyabov, D. S. In *Mathematical Modeling and Computational Physics*, 120 (JINR, Dubna, 2013).

[14] Kulyabov, D. S. & Nemchaninova, N. A. *Bulletin of Peoples' Friendship University of Russia. Series Mathematics. Information Sciences. Physics* 172–179 (2011). In Russian.

[15] Minkowski, H. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse* 53–111 (1908).

[16] Stratton, J. A. *Electromagnetic Theory* (MGH, 1941).

Title: The 15$^{\text{th}}$ Small Triangle Meeting
Publisher: The Institute of Experimental Physics SAS, Košice, Slovakia
Editors: Ján Buša, Michal Hnatič, and Peter Kopčanský
Page number: 170
Copies number: 60
Edition: The first
TEXnical set up: Ján Buša
Publishing house: EQUILIBRIA, s.r.o., Letná 42, 040 01 Košice, Slovakia

ISBN 978-80-8143-141-8