Vladimir M. Vishnevskiy
Konstantin E. Samouylov
Dmitry V. Kozyrev (Eds.)

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Distributed Computer and Communication Networks

19th International Conference, DCCN 2016
Moscow, Russia, November 21–25, 2016
Revised Selected Papers

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Editors
Vladimir M. Vishnevskiy
V.A. Trapeznikov Institute of Control Sciences
Russian Academy of Sciences
Moscow
Russia

Konstantin E. Samouylov
RUDN University
Moscow
Russia

Dmitry V. Kozyrev
V.A. Trapeznikov Institute of Control Sciences
Russian Academy of Sciences
Moscow
Russia

and

RUDN University
Moscow
Russia

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Preface

This volume contains a collection of revised selected full-text papers presented at the 19th International Conference on Distributed Computer and Communication Networks (DCCN 2016), held in Moscow, Russia, November 21–25, 2016.

The conference is a continuation of traditional international conferences of the DCCN series, which took place in Bulgaria (Sofia, 1995, 2005, 2006, 2008, 2009, 2014), Israel (Tel Aviv, 1996, 1997, 1999, 2001), and Russia (Moscow, 1998, 2000, 2003, 2007, 2010, 2011, 2013, 2015) in the past 19 years. The main idea of the conference is to provide a platform and forum for researchers and developers from academia and industry from various countries working in the area of theory and applications of distributed computer and communication networks, mathematical modeling, methods of control and optimization of distributed systems, by offering them a unique opportunity to share their views, discuss prospective developments, and pursue collaborations in this area. The content of this volume is related to the following subjects:

1. Computer and communication networks architecture optimization
2. Control in computer and communication networks
3. Performance and QoS/QoE evaluation in wireless networks
4. Analytical modeling and simulation of next-generation communications systems
5. Queuing theory and reliability theory applications in computer networks
6. Wireless 4G/5G networks, cm- and mm-wave radio technologies
7. RFID technology and its application in intellectual transportation networks
8. Internet of Things, wearables, and applications of distributed information systems
9. Probabilistic and statistical models in information systems
10. Mathematical modeling of high-tech systems
11. Mathematical modeling and control problems
12. Distributed and cloud computing systems, big data analytics

The DCCN 2016 conference gathered 208 submissions from authors from 20 different countries. From these, 141 high-quality papers in English were accepted and presented during the conference, 56 of which were recommended by session chairs and selected by the Program Committee for the Springer proceedings.

All the papers selected for the proceedings are given in the form presented by the authors. These papers are of interest to everyone working in the field of computer and communication networks.

We thank all the authors for their interest in DCCN, the members of the Program Committee for their contributions, and the reviewers for their peer-reviewing efforts.

November 2016

Vladimir M. Vishnevskiy
Konstantin E. Samouylov

Organization

DCCN 2016 was jointly organized by the Russian Academy of Sciences (RAS), the V. A. Trapeznikov Institute of Control Sciences of RAS (ICS RAS), the Peoples' Friendship University of Russia (RUDN), the National Research Tomsk State University, and the Institute of Information and Communication Technologies of Bulgarian Academy of Sciences (IICT BAS).

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Contents

Computer and Communication Networks

Numerical and Analytical Modeling of Guided Modes of a Planar Gradient Waveguide

Edik Ayrjan[1,4], Migran Gevorkyan[2], Dmitry Kulyabov[1,2], Konstantin Lovetskiy[2], Nikolai Nikolaev[2], Anton Sevastianov[2], Leonid Sevastianov[2,3](✉), and Eugeny Laneev[2]

[1] Laboratory of Information Technologies, Joint Institute for Nuclear Research, 6 Joliot-Curie, Dubna, Moscow Region 141980, Russia
ayrjan@jinr.ru, dharma@sci.pfu.edu.ru

[2] RUDN University (Peoples' Friendship University of Russia), 6 Miklukho-Maklaya Str., Moscow 117198, Russia
mngevorkyan@sci.pfu.edu.ru, lovetskiy@gmail.com, alsevastyanov@gmail.com, nnikolaev@sci.pfu.edu.ru, leonid.sevast@gmail.com

[3] Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 6 Joliot-Curie, Dubna, Moscow Region 141980, Russia

[4] Yerevan Physics Institute, 2 Alikhanian Brothers St., 375036 Yerevan, Armenia

Abstract. The mathematical model of light propagation in a planar gradient optical waveguide consists of the Maxwell's equations supplemented by the matter equations and boundary conditions. In the coordinates adapted to the waveguide geometry, the Maxwell's equations are separated into two independent sets for the TE and TM polarizations. For each there are three types of waveguide modes in a regular planar optical waveguide: guided modes, substrate radiation modes, and cover radiation modes. We implemented in our work the numerical-analytical calculation of typical representatives of all the classes of waveguide modes.

In this paper we consider the case of a linear profile of planar gradient waveguide, which allows for the most complete analytical description of the solution for the electromagnetic field of the waveguide modes. Namely, in each layer we are looking for a solution by expansion in the fundamental system of solutions of the reduced equations for the particular polarizations and subsequent matching them at the boundaries of the waveguide layer.

The problem on eigenvalues (discrete spectrum) and eigenvectors is solved in the way that first we numerically calculate (approximately, with double precision) eigenvalues, then numerically and analytically—eigenvectors. Our modelling method for the radiation modes consists in

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reducing the initial potential scattering problem (in the case of the continuous spectrum) to the equivalent ones for the Jost functions: the Jost solution from the left for the substrate radiation modes and the Jost solution from the right for the cover radiation modes.

Keywords: Waveguide propagation of electromagnetic radiation · Equations of waveguide modes of regular waveguide · Numerical-analytical modelling

1 Introduction

Waveguide propagation of polarized light is widely used in engineering, optoelectronics and nanophotonics [4,15,26]. Most of the integrated optical waveguide structures are formed on the basis of thin-film planar waveguides [2,9,14] and contain all sorts of waveguide transitions [3,7,8,10,11,20–22] with gradient planar waveguides [1,4,15,16,25–27]. In this connection the analysis of the propagation of guided and radiation waveguide modes in gradient waveguides is of particular interest. Some works [5,6,17] are devoted to finding approximate solutions of the electromagnetic field of the waveguide modes under the assumption of a given analytical behavior of the transverse distribution of refractive index in the waveguide layer. In other works [12,13,18,19], this study is carried out initially by approximate numerical methods.

In this paper we consider the case of a linear profile of planar gradient waveguide, which allows for the most complete analytical description of the solution for the electromagnetic field of the waveguide modes. Namely, in each layer we are looking for a solution by expansion in the fundamental system of solutions of the reduced equations for the particular polarizations and subsequent matching them at the boundaries of the waveguide layer.

Propagation of monochromatic polarized electromagnetic radiation is described by the system of vector homogeneous Maxwell's equations [2,9,14]:

$$\operatorname{rot}\boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t}, \quad \operatorname{rot}\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t}. \tag{1}$$

When there is a waveguide propagation of the radiation, at the interfaces of the waveguide layer with the substrate and the cover (see Fig. 1) the tangential boundary conditions are satisfied [2,9,14]:

$$\boldsymbol{E}^{\tau}\Big|_1 = \boldsymbol{E}^{\tau}\Big|_2, \quad \boldsymbol{H}^{\tau}\Big|_1 = \boldsymbol{H}^{\tau}\Big|_2. \tag{2}$$

And asymptotic conditions "at infinity" (at an infinite distance from the waveguide layer):

$$\lim_{x\to\pm\infty}\left|\boldsymbol{E}(x,y,z)\right| \le C_E, \quad \lim_{x\to\pm\infty}\left|\boldsymbol{H}(x,y,z)\right| \le C_H. \tag{3}$$

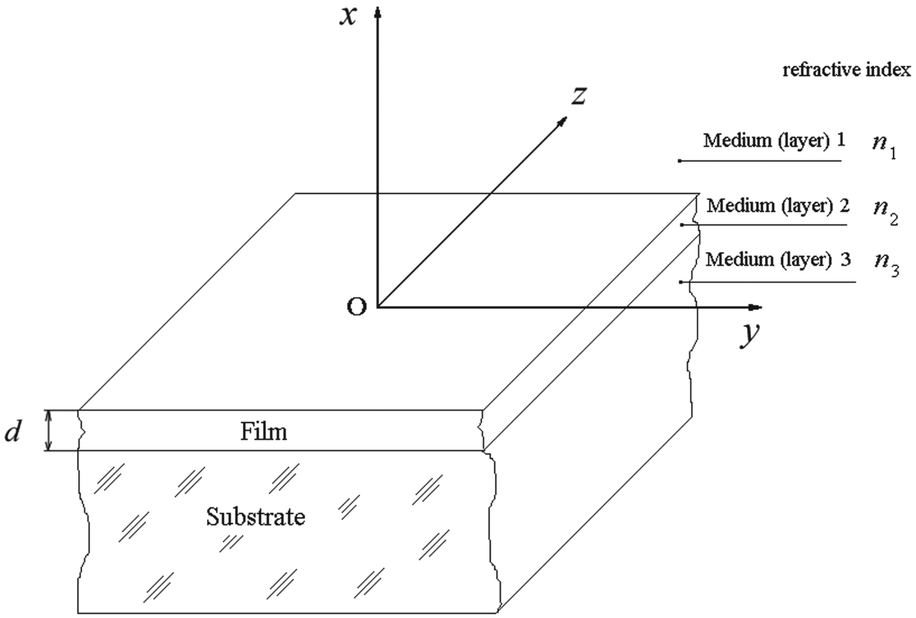

Fig. 1. Scheme of a flat three-layer dielectric waveguide. Waveguide is formed by media 1–3. The figure indications are: 1 is a framing medium or cover layer (air) with refractive index n_c; 2 is a waveguide layer (film) with a refractive index n_f; 3 is a substrate with refractive index n_s; d is the thickness of the waveguide layer. Film and substrate are homogeneous in the x and z directions, the substrate is usually much thicker than the film

In a Cartesian coordinate system associated with the geometry of the waveguide (see Fig. 1), the Maxwells equations, after the separation of variables, split into two linearly independent systems, which take the form [4,15]:

$$\frac{d^2 E_y}{dx^2} + k_0^2\left(\varepsilon\mu - \beta^2\right)E_y(x) = 0, \quad H_z = \frac{1}{ik_0\mu}\frac{dE_y}{dx}, \quad H_x = -\frac{\beta}{\mu}E_y, \tag{4}$$

$$\varepsilon\frac{d}{dx}\left(\frac{1}{\varepsilon}\frac{dH_y}{dx}\right) + k_0^2\left(\varepsilon\mu - \beta^2\right)H_y(x) = 0, \quad E_z = \frac{1}{ik_0\varepsilon}\frac{\partial H_y}{\partial x}, \quad E_x = \frac{\beta}{\varepsilon}H_y. \tag{5}$$

Here the invariance of the process in the direction Oy is taken into account: $\frac{\partial}{\partial y} = 0$.

The boundary conditions (2) are reduced to the following conditions: conditions for TE modes

$$E_y\Big|_1 = E_y\Big|_2, \quad H_z\Big|_1 = H_z\Big|_2, \tag{6}$$

and the boundary conditions for TM modes

$$H_y\Big|_1 = H_y\Big|_2, \quad E_z\Big|_1 = E_z\Big|_2. \tag{7}$$

Asymptotic conditions (3) are reduced to the following conditions:

$$\lim_{x\to\pm\infty}\left|\boldsymbol{E}(x)\right| \le C_E, \quad \lim_{x\to\pm\infty}\left|\boldsymbol{H}(x)\right| \le C_H. \tag{8}$$

2 Statement of the Problem

Thus the problem of describing the full set of waveguide modes of regular gradient planar optical waveguide is formulated as an eigenvalue problem (for discrete and continuous spectra) and eigenfunction problem (for classical and generalized functions) of essentially self-adjoint ordinary differential operator of the second order [9,14]:

$$-p(x)\frac{d}{dx}\left(\frac{1}{p(x)}\frac{d\psi}{dx}(k,x)\right). \tag{9}$$

Here $p(x) = \varepsilon(x)$, $V(x) = -n^2(x)$ is piecewise-continuous (continuous in layers) function, $k^2 = -\beta^2$ is spectral parameter, and

$$\psi_{\mathrm{TE}}(x) = E_y(x), \quad \psi_{(\mathrm{TM}}(x) = H_y(x). \tag{10}$$

The function $V(x)$ has the view shown in Fig. 2.

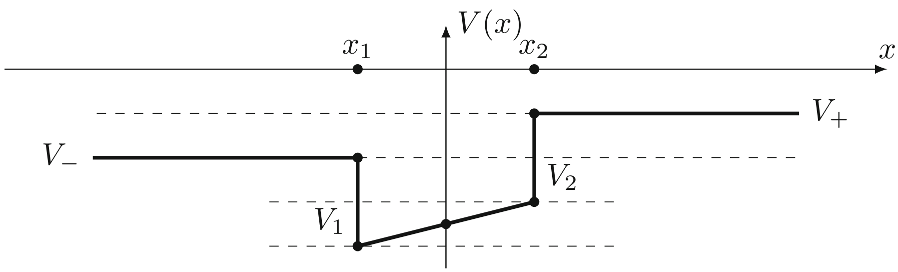

Fig. 2. The potential $V(x)$ graph

Lets introduce the auxiliary functions

$$\varphi_{\mathrm{TE}}(x) = \frac{d\varphi_{\mathrm{TE}}}{dx}(x), \quad \varphi_{\mathrm{TM}}(x) = \frac{1}{p(x)}\frac{d\varphi_{\mathrm{TM}}}{dx}(x). \tag{11}$$

Using these functions we can write down reduced boundary conditions at points of discontinuity of the potential, and therefore of the second derivative of the solution:

$$\psi\big|_{x_1-0} = \psi\big|_{x_1+0}, \quad \psi\big|_{x_2-0} = \psi\big|_{x_2+0}, \tag{12}$$

$$\varphi\big|_{x_1-0} = \varphi\big|_{x_1+0}, \quad \varphi\big|_{x_2-0} = \varphi\big|_{x_2+0}. \tag{13}$$

Besides, the asymptotic conditions are satisfied

$$|\psi(x)|_{x\to\pm\infty} \le C^{\pm}. \tag{14}$$

The spectrum of operator (9)–(14) consists of [23,24]:

- a finite number of discrete eigenvalues $k_j = i\kappa_j$: $k_j^2 \in \big(\min V(x), \min(V_-, V_+)\big)$ and the corresponding classical eigenfunctions (of guided waveguide modes);
- a single continuous spectrum k_-: $k_-^2 \in (V_-, \infty)$ and corresponding generalized eigenfunctions (substrate radiation modes);
- a single continuous spectrum k_+: $k_+^2 \in (V_+, \infty)$ and corresponding generalized eigenfunctions (cover radiation modes).

For a constructive description of the problem solutions, i.e. eigenfunctions of three types, we shall restrict our consideration to piecewise-linear potential:

$$V(x) = \begin{cases} V_-, & \text{when } x < x_1, \\ ax + b, & \text{when } x_1 < x < x_2, \\ V_+, & \text{when } x > x_2. \end{cases} \quad \text{where } a = \tfrac{V_2 - V_1}{x_2 - x_1},\ b = \tfrac{V_1 x_2 - V_2 x_1}{x_2 - x_1}, \tag{15}$$

3 The Solution to the Problem on Eigenvalues (of the Discrete Spectrum) and Eigenfunctions (Classical)

The method of solution is the expansion on the sub-intervals of the general solution in terms of the fundamental system of solutions. To the left and to the right there are decreasing exponential functions in the case of real ε_s, ε_c (due to the asymptotic conditions):

$$\psi_s(k, x) = C_s \exp\{\gamma_s (x - x_1)\}, \tag{16}$$

$$\psi_c(k, x) = C_c \exp\{-\gamma_c (x - x_2)\}. \tag{17}$$

In the waveguide layer (with a linear potential in the subdomain) the fundamental system of solutions consists of the functions $Ai(x)$ and $Bi(x)$, such that

$$\psi_f(k, x) = C_1 Ai\left(\frac{a(x - x_2) + b}{(-a)^{2/3}}\right) + C_2 Bi\left(\frac{a(x - x_2) + b}{(-a)^{2/3}}\right). \tag{18}$$

These common solutions in the subdomains form a single particular solution of the problem (9)–(14), therefore, the equalities must be satisfied:

$$\psi_s(k, x_1) = \psi_f(k, x_1), \qquad \Phi_s(k, x_1) = \Phi_f(k, x_1), \tag{19}$$

$$\psi_f(k, x_2) = \psi_c(k, x_2), \qquad \Phi_f(k, x_2) = \Phi_c(k, x_2). \tag{20}$$

Thus we obtain a homogeneous system of linear algebraic equations for the indefinite coefficients of the expansion of common solutions in terms of the fundamental systems of solutions, which for the TE modes has the view:

$$C_s = C_1 Ai\left(\frac{-ad + b}{(-a)^{2/3}}\right) + C_2 Bi\left(\frac{-ad + b}{(-a)^{2/3}}\right), \tag{21}$$

$$\gamma_s C_s = -C_1 (-a)^{1/3} \frac{dAi}{dx}\left(\frac{-ad + b}{(-a)^{2/3}}\right) - C_2 (-a)^{1/3} \frac{dBi}{dx}\left(\frac{-ad + b}{(-a)^{2/3}}\right), \tag{22}$$

$$C_1 Ai(0) + C_2 Bi(0) = C_c, \tag{23}$$

$$- C_1(-a)^{1/3}\frac{dAi}{dx}(0) - C_2(-a)^{1/3}\frac{dBi}{dx}(0) = -\gamma_c C_c. \tag{24}$$

The resulting homogeneous system of linear algebraic equations

$$\hat{M}_{\mathrm{TE}}(k)\boldsymbol{C}(k) = \mathbf{0} \tag{25}$$

has a non-trivial solution provided that

$$\det \hat{M}_{\mathrm{TE}}(k) = 0. \tag{26}$$

Solutions of nonlinear transcendental algebraic equation k_j are substituted in SLAE(x) and then this system is solved with respect to $\boldsymbol{C}_j = \boldsymbol{C}(k_j)$. The obtained coefficients are substituted in the expressions for the fields. The results of calculations are presented in Figs. 3, 4, 5 and 6.

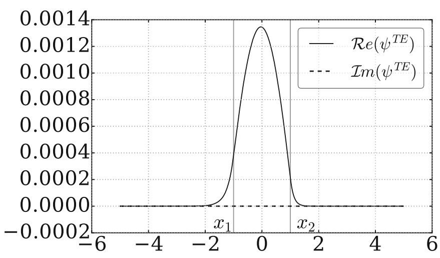

Fig. 3. Waveguide mode TE_0, $n_c = 1.0$, $n_f = 2.15$, $n_s = 1.515$, $\beta^{TE} = 1.6752$

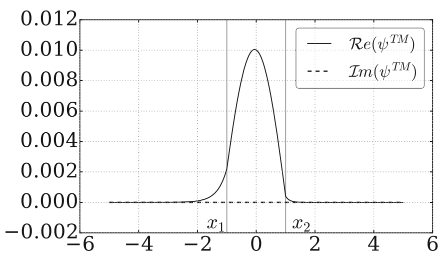

Fig. 4. Waveguide mode TM_0, $n_c = 1.0$, $n_f = 2.15$, $n_s = 1.515$, $\beta^{TE} = 1.5955$

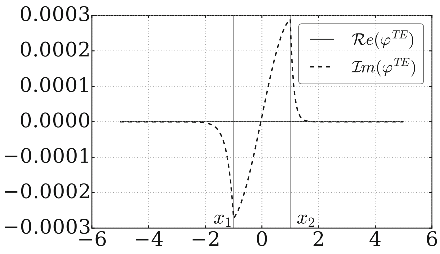

Fig. 5. Waveguide mode TE_0, $n_c = 1.0$, $n_f = 2.15$, $n_s = 1.515$, $\beta^{TE} = 1.6752$

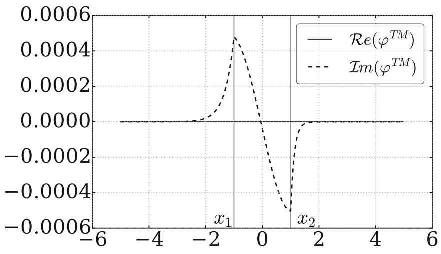

Fig. 6. Waveguide mode TM_0, $n_c = 1.0$, $n_f = 2.15$, $n_s = 1.515$, $\beta^{TE} = 1.5955$

4 Calculation of Cover Radiation Modes

Similarly to what was done in [15,26] for piecewise-constant potentials, let's move from solutions of the problem (9)–(14) satisfying the asymptotic Jost conditions, to the solutions satisfying the "scattering problem" conditions. A one-to-one correspondence between them is set in [15,26] for the potentials of a more general kind.

In particular, the asymptotics of the cover radiation modes $\psi_c(k,x)$ correspond to the problem of scattering of a plane Jost wave incident on the potential $V(x)$ from the right, from the region $x \sim +\infty$, which is reflected to the right with reflection coefficient $R_-(k)$, and is transmitted (through the potential $V(x)$) to the left with transmittance coefficient $T_-(k)$ in the form of a plane Jost wave propagating from right to left, in the region $x \sim -\infty$. All solutions $\psi_c(k,x)$ satisfy these asymptotics, when $k^2 \in (V_c, \infty)$. A sought solution, as in the case of guided modes, is constructed by matching at the boundaries of the general solutions of Eq. (9) in the regions of the argument $(-\infty, x_1)$, (x_1, x_2) and (x_2, ∞).

So, in the region $(-\infty, x_1)$ the general solutions of Eq. (9) with constant coefficient V_s are of the form (for TE modes):

$$\psi_c^{\mathrm{TE}}(k,x) = T_-^{\mathrm{TE}}(k)\exp\{-ip_s(x-x_1)\}. \tag{27}$$

In the region (x_2, ∞) the general solutions of Eq. (9) have the form

$$\psi_c^{\mathrm{TE}}(k,x) = \exp\{-ip_c(x-x_2)\} + R_-^{\mathrm{TE}}(k)\exp\{ip_c(x-x_2)\}. \tag{28}$$

In the region (a,b) the general solutions of Eq. (9) have the form (for TE and TM modes, respectively):

$$\psi_f(k,x) = C_f^1 Ai\left(\frac{a(x-x_2)+b}{(-a)^{2/3}}\right) + C_f^2 Bi\left(\frac{a(x-x_2)+b}{(-a)^{2/3}}\right). \tag{29}$$

Thus, the solutions (for TE modes) are given by sets of amplitude coefficients $(T_-^{\mathrm{TE}}, C_f^1, C_f^2, R_-^{\mathrm{TE}})^T$, satisfying the system of linear algebraic equations:

$$T_-^{\mathrm{TE}}(k) = C_f^1 Ai\left(\frac{-ad+b}{(-a)^{2/3}}\right) + C_f^2 Bi\left(\frac{-ad+b}{(-a)^{2/3}}\right), \tag{30}$$

$$-\frac{p_s}{k_0\mu_s}T_-^{\mathrm{TE}}(k) = -C_f^1(-a)^{1/3}\frac{dAi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right) - C_f^2(-a)^{1/3}\frac{dAi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right), \tag{31}$$

$$C_f^1 Ai(0) + C_f^2 Bi(0) = 1 + R_-^{\mathrm{TE}}(k), \tag{32}$$

$$-C_f^1(-a)^{1/3}\frac{dAi}{dx}(0) - C_f^2(-a)^{1/3}\frac{dBi}{dx}(0) = -\frac{p_c}{k_0\mu_c}\left[1 - R_-^{\mathrm{TE}}(k)\right]. \tag{33}$$

The resulting SLAE can be rewritten as:

$$\hat{M}^{\mathrm{TE}}(k)(T_-^{\mathrm{TE}}, C_f^1, C_f^2, R_-^{\mathrm{TE}})^T = \left(0, 0, 1, -\frac{p_c}{k_0\mu_c}\right)^T, \tag{34}$$

so that the solution exists for any $k^2 \in (V_c, \infty)$ and is unique up to a complex factor (Figs. 7 and 8).

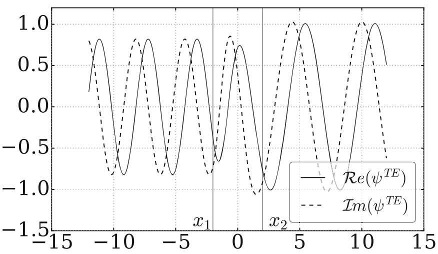

Fig. 7. Cover radiation TE mode $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = 0.250$

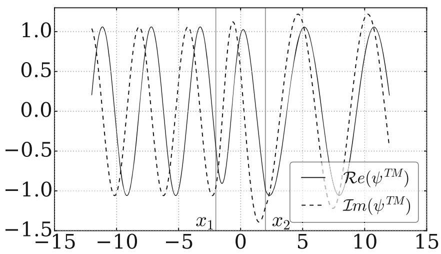

Fig. 8. Cover radiation TM mode $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = 0.250$

5 Calculation of Substrate Radiation Modes

The asymptotics of substrate radiation modes $\psi_s(k,x)$ correspond to the scattering problem of plane Jost wave, the potential $V(x)$ from the left, from the region $x \sim -\infty$, which is reflected to the left with reflection coefficient $R_+(k)$. At the same time, the Jost wave coming from the left, passing through the potential $V(x)$, propagates to the right as the plane Jost wave with the transmittance coefficient $T_+(k)$ when $k^2 \in (V_c, \infty)$, and as an evanescent wave decaying to the right with a weighting factor $A_c(k)$ when $k^2 \in (V_s, V_c)$.

Solutions have different view for different values of spectral parameter k from the two spectral subregions $k^2 \in (V_s, V_c)$ and $k^2 \in (V_c, \infty)$. But for both regions the solution, as in the case of guided modes, is constructed by matching at the boundaries of the general solutions of Eq. (9) in the regions of the argument $(-\infty, x_1)$, (x_1, x_2) and (x_2, ∞).

In the region $(-\infty, x_1)$ the general solutions of Eq. (9) with a spectral parameter $k^2 \in (V_s, V_c)$ have the form:

$$\psi_s^{\mathrm{TE}}(k,x) = \exp\{ip_s(k)(x - x_1)\} + R_+^{\mathrm{TE}}(k)\exp\{ip_s(k)(x - x_1)\}. \tag{35}$$

In the region (x_1, x_2) the general solutions of Eq. (9) with a spectral parameter $k^2 \in (V_s, V_c)$ have the form:

$$\psi_f(k,x) = C_f^1 Ai\left(\frac{a(x - x_2) + b}{(-a)^{2/3}}\right) + C_f^2 Bi\left(\frac{a(x - x_2) + b}{(-a)^{2/3}}\right). \tag{36}$$

In the region (x_2, ∞) the general solutions of Eq. (9) with a spectral parameter $k^2 \in (V_s, V_c)$ have the form (by virtue of the asymptotic decay at infinity):

$$\psi_s^{\mathrm{TE}}(k,x) = A_c \exp\{-\gamma_c(x - x_2)\}. \tag{37}$$

Thus, the solutions (for TE modes) are given by sets of amplitude coefficients $(R_+^{\mathrm{TE}}, C_f^1, C_f^2, A_c)^T$ satisfying the system of linear algebraic equations:

$$1 + R_+^{\mathrm{TE}}(k) = C_f^1 Ai\left(\frac{-ad + b}{(-a)^{2/3}}\right) + C_f^2 Bi\left(\frac{-ad + b}{(-a)^{2/3}}\right), \tag{38}$$

$$\frac{p_s}{k_0\mu_s}\left[1-R_+^{\mathrm{TE}}(k)\right] = -C_f^1(-a)^{1/3}\frac{dAi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right)-C_f^2(-a)^{1/3}\frac{dAi}{dx}\left(\frac{-ad+b}{(-a)^{2/3}}\right), \tag{39}$$

$$C_f^1 Ai(0) + C_f^2 Bi(0) = A_c, \tag{40}$$

$$-C_f^1(-a)^{1/3}\frac{dAi}{dx}(0) - C_f^2(-a)^{1/3}\frac{dBi}{dx}(0) = -\frac{\gamma_c}{ik_0\mu_c}A_c. \tag{41}$$

The resulting SLAE can be rewritten as:

$$\hat{M}^{\mathrm{TE}}(k)(R_+^{\mathrm{TE}}, C_f^1, C_f^2, A_c)^T = \left(-1, -\frac{p_c}{k_0\mu_s}, 0, 0\right)^T, \tag{42}$$

so that there exists a solution for any $k^2 \in (V_s, V_c)$ and it is unique up to a complex multiplier (Figs. 9 and 10).

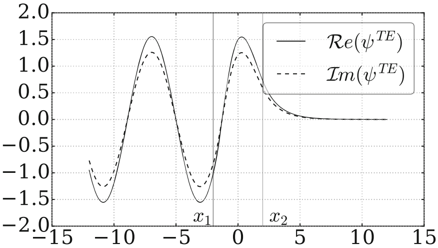

Fig. 9. Substrate radiation TE mode $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = -1.648 \in (V_s, V_c)$

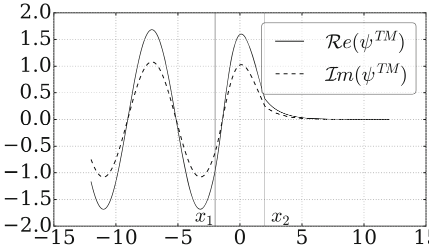

Fig. 10. Substrate radiation TM mode $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = -1.648 \in (V_s, V_c)$

For the spectral parameter k from the region $k^2 \in (V_c, \infty)$, in the coordinate regions $(-\infty, x_1)$ and (x_1, x_2) common solutions have the same form as in the case $k^2 \in (V_s, V_c)$, and in the region (x_2, ∞), they take the form:

$$\psi_s^{\mathrm{TE}}(k,x) = T_+^{\mathrm{TE}}(k)\exp\{ip_c(k)(x-x_2)\}. \tag{43}$$

Consequently, the second pair of boundary equations at the point $x = x_2$ for TE modes take the form:

$$C_f^1 Ai(0) + C_f^2 Bi(0) = T_+^{\mathrm{TE}}(k), \tag{44}$$

$$-C_f^1(-a)^{1/3}\frac{dAi}{dx}(0) - C_f^2(-a)^{1/3}\frac{dBi}{dx}(0) = \frac{p_c(k)}{k_0\mu_c}T_+^{\mathrm{TE}}(k). \tag{45}$$

The resulting SLAE can be rewritten as:

$$\hat{M}^{\mathrm{TE}}(k)(R_+^{\mathrm{TE}}, C_f^1, C_f^2, T_+^{\mathrm{TE}})^T = \left(-1, -\frac{p_s}{k_0\mu_s}, 0, 0\right)^T, \tag{46}$$

so that there exists a solution for any $k^2 \in (V_c, \infty)$ and it is unique up to a complex multiplier (Figs. 11 and 12).

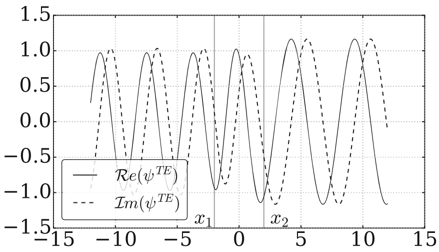

Fig. 11. Substrate radiation TE mode $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = 0.500 \in (V_c, \infty)$

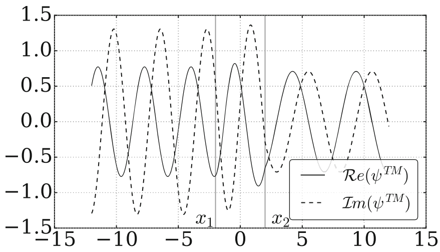

Fig. 12. Substrate radiation TM mode $n_c = 1.0$, $n_f = 1.59$, $n_s = 1.515$, $k^2 = 0.500 \in (V_c, \infty)$

6 Conclusion

The solution of many problems of integrated optics is realized by the Galerkin and by the Kantorovich methods, including the expansion of the desired solution in a complete set of waveguide modes of a regular comparison waveguide [23, 24]. Computer numerical and analytical implementations of all three types of waveguide modes are known for the class of multilayer waveguides with constant values of the refractive indices of the layers (see., e.g., [14]).

This paper presents the numerical implementations on a computer of square-integrable eigenfunctions corresponding to discrete spectrum $k_j = i\kappa_j$ for a piecewise-linear potential $V(x)$ (for the gradient waveguide). The present study also shows the numerical implementations on a computer of the cover radiation modes and substrate radiation modes. For modeling these modes, the problems of scattering on the potential $V(x)$ of Jost functions equivalent to the original problem in the case of the continuous spectrum were used: the problems of scattering on the left for the substrate radiation modes and the problems of scattering on the right for the cover radiation modes.

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