Vladimir M. Vishnevskiy
Konstantin E. Samouylov
Dmitry V. Kozyrev (Eds.)

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Distributed Computer and Communication Networks

19th International Conference, DCCN 2016
Moscow, Russia, November 21–25, 2016
Revised Selected Papers

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Preface

This volume contains a collection of revised selected full-text papers presented at the 19th International Conference on Distributed Computer and Communication Networks (DCCN 2016), held in Moscow, Russia, November 21–25, 2016.

The conference is a continuation of traditional international conferences of the DCCN series, which took place in Bulgaria (Sofia, 1995, 2005, 2006, 2008, 2009, 2014), Israel (Tel Aviv, 1996, 1997, 1999, 2001), and Russia (Moscow, 1998, 2000, 2003, 2007, 2010, 2011, 2013, 2015) in the past 19 years. The main idea of the conference is to provide a platform and forum for researchers and developers from academia and industry from various countries working in the area of theory and applications of distributed computer and communication networks, mathematical modeling, methods of control and optimization of distributed systems, by offering them a unique opportunity to share their views, discuss prospective developments, and pursue collaborations in this area. The content of this volume is related to the following subjects:

1. Computer and communication networks architecture optimization
2. Control in computer and communication networks
3. Performance and QoS/QoE evaluation in wireless networks
4. Analytical modeling and simulation of next-generation communications systems
5. Queuing theory and reliability theory applications in computer networks
6. Wireless 4G/5G networks, cm- and mm-wave radio technologies
7. RFID technology and its application in intellectual transportation networks
8. Internet of Things, wearables, and applications of distributed information systems
9. Probabilistic and statistical models in information systems
10. Mathematical modeling of high-tech systems
11. Mathematical modeling and control problems
12. Distributed and cloud computing systems, big data analytics

The DCCN 2016 conference gathered 208 submissions from authors from 20 different countries. From these, 141 high-quality papers in English were accepted and presented during the conference, 56 of which were recommended by session chairs and selected by the Program Committee for the Springer proceedings.

All the papers selected for the proceedings are given in the form presented by the authors. These papers are of interest to everyone working in the field of computer and communication networks.

We thank all the authors for their interest in DCCN, the members of the Program Committee for their contributions, and the reviewers for their peer-reviewing efforts.

November 2016

Vladimir M. Vishnevskiy
Konstantin E. Samouylov

Organization

DCCN 2016 was jointly organized by the Russian Academy of Sciences (RAS), the V. A. Trapeznikov Institute of Control Sciences of RAS (ICS RAS), the Peoples' Friendship University of Russia (RUDN), the National Research Tomsk State University, and the Institute of Information and Communication Technologies of Bulgarian Academy of Sciences (IICT BAS).

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Contents

Computer and Communication Networks

Mathematical Modeling and Computation

Diagram Representation for the Stochastization of Single-Step Processes

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Abstract. Background. By the means of the method of stochastization of one-step processes we get the simplified mathematical model of the original stochastic system. We can explore these models by standard methods, as opposed to the original system. The process of stochastization depends on the type of the system under study. **Purpose.** We want to get a unified abstract formalism for stochastization of one-step processes. This formalism should be equivalent to the previously introduced. **Methods.** To unify the methods of construction of the master equation, we propose to use the diagram technique. **Results.** We get a diagram technique, which allows to unify getting master equation for the system under study. We demonstrate the equivalence of the occupation number representation and the state vectors representation by using a Verhulst model. **Conclusions.** We have suggested a convenient diagram formalism for unified construction of stochastic systems.

Keywords: Occupation numbers representation · Fock space · Dirac notation · One-step processes · Master equation · Diagram technique

1 Introduction

When modeling various physical and technical systems, we often can model them in the form of a one-step processes (see [1,3,4,24]). Then there is the problem

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of adequate representation and study of the resulting model. For the statistical systems in addition to representation of the state vectors (combinatorial approach) [3,4] the representation of the occupation numbers (operator approach) (see [12,13,17,19,20,23]) is also used. This representation is especially well suited for the system with a variable number of elements description.

However, technique for obtaining models for the combinatorial approach is quite different from the technique for the operator approach. In this paper, we want to propose a unified methodology for both approaches on the basis of the diagram technique.

The structure of the article is as follows. In the Sect. 2 basic notations and conventions are introduced. The ideology of the method of stochastization of one-step process and its components are described in the Sect. 3. Then the interaction schemes and master equation overview are presented in the next Sects. 5 and 4. The combinatorial method of modelling is discussed in the following Sect. 6. The operator model approach is presented in the Sect. 7. In fact diagram technique introduced in Sect. 8. Application of of this technique is described in Sect. 9 on the example of Verhulst model.

2 Notations and Conventions

1. The abstract indices notation (see [22]) is used in this work. Under this notation a tensor as a whole object is denoted just as an index (e.g., x^i), components are denoted by underlined index (e.g., $x^{\underline{i}}$).
2. We will adhere to the following agreements. Latin indices from the middle of the alphabet (i, j, k) will be applied to the space of the system state vectors. Latin indices from the beginning of the alphabet (a) will be related to the Wiener process space. Greek indices (α) will set a number of different interactions in kinetic equations.

3 General Review of the Methodology

Our methodology is completely formalized in such a way that it is sufficient when the original problem is formulated accordingly. It should be noted that the most of the models under our study can be formalized as a one-step process (see [10,18]). In fact, for this type of models we developed this methodology, but it may be expanded for other processes.

First we transform our model to the one-step process (see Fig. 1). Next, we need to formalize this process in the form of interaction schemes[1] (see [3,4,13]).

Each scheme has its own interaction semantics. Semantics leads directly to the master equation (see [10,18]). However, the master equation has usually rather complex structure that makes it difficult for direct study and solution. Our technique involves two possibilities (see Fig. 2):

[1] The analogs of the interaction schemes are the equations of chemical kinetics, reaction particles and etc.

– computational approach—the solution of the master equation with help of perturbation theory;
– modeling approach—the approximate models are obtained in the form of Fokker–Planck and Langevin equations.

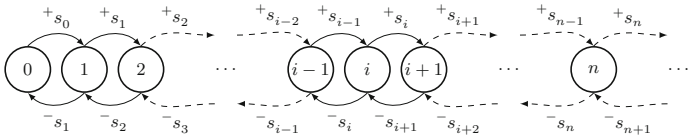

Fig. 1. One-step process

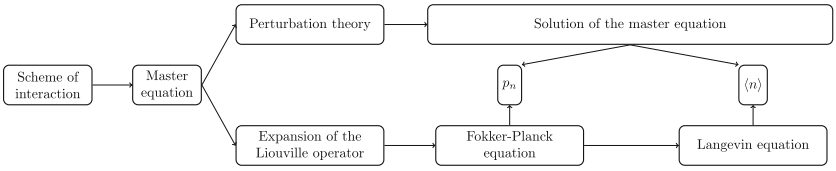

Fig. 2. The general structure of the methodology

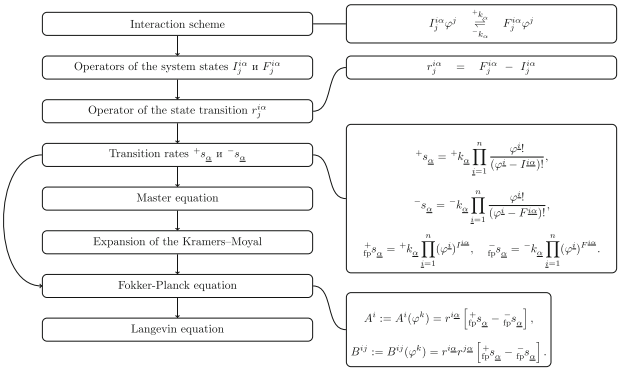

Fig. 3. Combinatorial modeling approach

The computational approach allows to obtain a concrete solution for the studied model. In our methodology, this approach is associated with perturbation theory (see [14–16]).

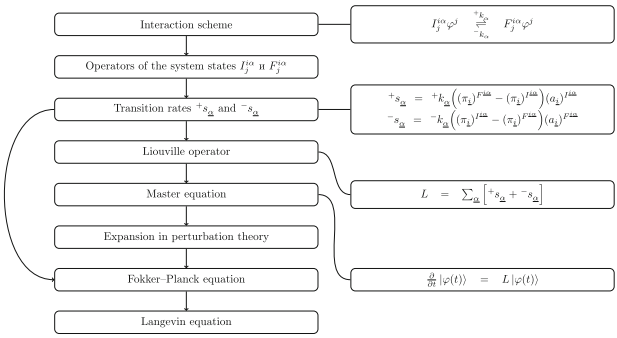

Fig. 4. Operator modeling approach

The model approach provides a model that is convenient to study numerically and qualitatively. In addition, this approach assumes the iterative process of research: the obtained approximate model can be specified and changed, which leads to the correction of initial interaction schemes.

There are two ways of building the master equation[2]

– combinatorial approach (see Fig. 3);
– operator approach (see Fig. 4).

In the combinatorial approach, all operations are performed in the space of states of the system, so we deal with a particular system throughout manipulations with the model.

For the operator approach we can abstract from the specific implementation of the system under study. We are working with abstract operators. We return to the state space only at the end of the calculations. In addition, we choose a particular operator algebra on the basis of symmetry of the problem.

4 Interaction Schemes

The system state is defined by the vector $\varphi^i \in \mathfrak{R}^n$, where n is system dimension[3]. The operator $I_j^i \in \mathfrak{N}_0^n \times \mathfrak{N}_0^n$ describes the state of the system before the interaction, the operator $F_j^i \in \mathfrak{N}_0^n \times \mathfrak{N}_0^n$ describes the state of the system after the interaction[4]. The result of interaction is the system transition from one state to another one.

There are s types of interaction in our system, so instead of I_j^i and F_j^i operators we will use operators $I_j^{i\alpha} \in \mathfrak{N}_0^n \times \mathfrak{N}_0^n \times \mathfrak{N}_+^s$ and $F_j^{i\alpha} \in \mathfrak{N}_0^n \times \mathfrak{N}_0^n \times \mathfrak{N}_+^s$ [5].

[2] In quantum field theory the path integrals approach can be considered as an analogue of the combinatorial approach and the method of second quantization as analog of the operator approach.

[3] We denote the module over the field $\mathbb{R}$ as $\mathfrak{R}$. Accordingly, $\mathfrak{N}$, $\mathfrak{N}_0$, $\mathfrak{N}_+$ are modules over rings $\mathbb{N}$, $\mathbb{N}_0$ (cardinal numbers with 0), $\mathbb{N}_+$ (cardinal numbers without 0).

[4] The component dimension indices take on values $\underline{i}, \underline{j} = \overline{1, n}$.

[5] The component indices of number of interactions take on values $\underline{\alpha} = \overline{1, s}$.

The interaction of the system elements will be described by interaction schemes, which are similar to schemes of chemical kinetics [11,26]:

$$I_j^{i\underline{\alpha}}\varphi^j \underset{{}^{-}k_{\underline{\alpha}}}{\overset{{}^{+}k_{\underline{\alpha}}}{\rightleftharpoons}} F_j^{i\underline{\alpha}}\varphi^j, \qquad \underline{\alpha} = \overline{1,s}, \tag{1}$$

the Greek indices specify the number of interactions and Latin are the system order. The coefficients ${}^{+}k_{\underline{\alpha}}$ and ${}^{-}k_{\underline{\alpha}}$ have meaning intensity (speed) of interaction.

We can also write (1) not in the form of vector equations but in the form of sums:

$$I_j^{i\underline{\alpha}}\varphi^j\delta_i \underset{{}^{-}k_{\underline{\alpha}}}{\overset{{}^{+}k_{\underline{\alpha}}}{\rightleftharpoons}} F_j^{i\underline{\alpha}}\varphi^j\delta_i, \tag{2}$$

where $\delta_{\underline{i}} = (1,\ldots,1)$.

Also the following notation will be used:

$$I^{i\underline{\alpha}} := I_j^{i\underline{\alpha}}\delta^j, \quad F^{i\underline{\alpha}} := F_j^{i\underline{\alpha}}\delta^j, \quad r^{i\underline{\alpha}} := r_j^{i\underline{\alpha}}\delta^j.$$

The state transition is given by the operator:

$$r_j^{i\underline{\alpha}} = F_j^{i\underline{\alpha}} - I_j^{i\underline{\alpha}}. \tag{3}$$

5 The Master Equation

For the system description we will use the master equation,[6] which describes the transition probabilities for Markov process (see [10,18]):

$$\frac{\partial p(\varphi_2,t_2|\varphi_1,t_1)}{\partial t} = \int \left[w(\varphi_2|\psi,t_2)p(\psi,t_2|\varphi_1,t_1) - w(\psi|\varphi_2,t_2)p(\varphi_2,t_2|\varphi_1,t_1)\right]\mathrm{d}\psi,$$

where $w(\varphi|\psi,t)$ is the probability of transition from the state ψ to the state φ for unit time.

By fixing the initial values of φ_1, t_1, we can write the equation for subensemble:

$$\frac{\partial p(\varphi,t)}{\partial t} = \int [w(\varphi|\psi,t)p(\psi,t) - w(\psi|\varphi,t)p(\varphi,t)]\mathrm{d}\psi. \tag{4}$$

For the discrete domain of φ, the (4) can be written as follows (the states are numbered by n and m):

$$\frac{\partial p_n(t)}{\partial t} = \sum_m \left[w_{nm}p_m(t) - w_{mn}p_n(t)\right], \tag{5}$$

where the p_n is the probability of the system to be in a state n at time t, w_{nm} is the probability of transition from the state m into the state n per unit time.

[6] Master equation can be considered as an implementation of the Kolmogorov equation. However, the master equation is more convenient and has an immediate physical interpretation (see [18]).

6 Combinatorial Approach

There are two types of system transition from one state to another (based on one–step processes) as a result of system elements interaction: in the forward direction $(\varphi^i + r^{i\underline{\alpha}})$ with the probability ${}^{+}s_{\underline{\alpha}}(\varphi^k)$ and in the opposite direction $(\varphi^i - r^{i\underline{\alpha}})$ with the probability ${}^{-}s_{\underline{\alpha}}(\varphi^k)$ (Fig. 1). The matrix of transition probabilities has the form:

$$w_{\underline{\alpha}}(\varphi^i|\psi^i,t) = {}^{+}s_{\underline{\alpha}}\delta_{\varphi^i,\psi^i+1} + {}^{-}s_{\underline{\alpha}}\delta_{\varphi^i,\psi^i-1}, \qquad \underline{\alpha} = \overline{1,s},$$

where $\delta_{i,j}$ is Kronecker delta.

Thus, the general form of the master equation for the state vector φ^i, changing by steps with length $r^{i\underline{\alpha}}$, is:

$$\frac{\partial p(\varphi^i,t)}{\partial t} = \sum_{\underline{\alpha}=1}^{s} \Big\{ {}^{-}s_{\underline{\alpha}}(\varphi^i + r^{i\underline{\alpha}},t)p(\varphi^i + r^{i\underline{\alpha}},t) + {}^{+}s_{\underline{\alpha}}(\varphi^i - r^{i\underline{\alpha}},t)p(\varphi^i - r^{i\underline{\alpha}},t) - \left[{}^{+}s_{\underline{\alpha}}(\varphi^i) + {}^{-}s_{\underline{\alpha}}(\varphi^i)\right] p(\varphi^i,t) \Big\}. \quad (6)$$

7 Operator Approach

7.1 Occupation Numbers Representation

Occupation number representation is the main language in the description of many-body physics. The main elements of the language are the wave functions of the system with information about how many particles are in each single-particle state. The creation and annihilation operators are used for system states change.

The method of application of the formalism of second quantization for the non-quantum systems (statistical, deterministic systems) was studied in a series of articles (see [6,7,12,21]).

The Dirac notation is commonly used for occupation numbers representation recording.

7.2 Dirac Notation

This notation is proposed by Dirac[7] (see [5]). Under this notation, state of the system is described by an element of the projective Hilbert space $\mathcal{H}$. The vector $\varphi^i \in \mathcal{H}$ is defined as $|i\rangle$, and covariant vector (covector) $\varphi_i \in \mathcal{H}^* := \mathcal{H}_\bullet$ is defined as $\langle i|$. Conjunction operation is used for raising and lowering of indices[8]:

$$\varphi_i^* := \varphi_i = (\varphi^i)^\dagger \equiv \langle i| = |i\rangle^\dagger.$$

[7] The notation is based on the notation, proposed by G. Grassmann in 1862 (see [2, p. 134]).

[8] In this case, we use Hermitian conjugation $\bullet^\dagger$. The sign of the complex conjugate $\bullet^*$ in this case is superfluous.

The scalar product is as follows:

$$\varphi_i \varphi^i \equiv \langle i|i\rangle.$$

The tensor product is:

$$\varphi_j \varphi^i \equiv |i\rangle\langle j|.$$

7.3 Creation and Annihilation Operators

The transition to the space of occupation numbers is not a unitary transformation. However, the algorithm of transition (specific to each task) can be constructed.

Let's write the master equation (5) in the occupation number representation. We will consider a system that does not depend on the spatial variables. For simplicity, we consider the one-dimensional version.

Let's denote in (5) the probability that there are n particles in our system as φ_n:

$$\varphi_n := p_n(\varphi, t).$$

The vector space $\mathcal{H}$ consists of states of φ.

Depending on the structure of the model, we can introduce the probability-based or the moment-based inner products [12]. We introduce a scalar product, exclusive ($\langle|\rangle_{\text{ex}}$) and inclusive ($\langle|\rangle_{\text{in}}$). Let $|n\rangle$ are basis vectors.

$$\langle\varphi|\psi\rangle_{\text{ex}} = \sum_n n! p_n^*(\varphi) p^n(\psi); \tag{7}$$

$$\langle\varphi|\psi\rangle_{\text{in}} = \sum_n \frac{1}{k!} n_k^*(\varphi) n^k(\psi).$$

There n_k are factorial moments:

$$n_k(\varphi) = \langle n(n-1)\cdots(n-k+1)\rangle = \frac{\partial k}{\partial z^k} G(z,\varphi)|_{z=1},$$

G is generating function:

$$G(z,\varphi) = \sum_n z^n p_n(\varphi).$$

Let's use creation and annihilation operators:

$$\pi|n\rangle = |n+1\rangle,$$
$$a|n\rangle = n|n-1\rangle$$

and commutation rule[9]:

$$[a,\pi] = 1. \tag{8}$$

If the form of scalar product is (7) then from (8) follows that our system is described by Bose–Einstein statistics.

[9] In fact, $a\pi|n\rangle - \pi a|n\rangle = (n+1)|n\rangle - n|n\rangle = |n\rangle$.

7.4 The Liouville Operator

In the occupation numbers formalism the master equation becomes the Liouville equation:

$$\frac{\partial}{\partial t}|\varphi(t)\rangle = L|\varphi(t)\rangle.$$

The Liouville operator L satisfies the relation:

$$\langle 0|L = 0. \tag{9}$$

8 Diagram Representation

We describe our proposed diagram technique for the stochastization of one-step processes.

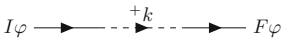

Fig. 5. Forward interaction

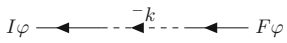

Fig. 6. Backward interaction

We will write the the scheme of interaction in the form of diagrams. Each scheme (1) or (2) corresponds to a pair of diagrams (see Figs. 5 and 6) for forward and backward interaction respectively.[10] The diagram consists of the following elements.

- Incoming lines (in the Fig. 5 are denoted by the solid line). These lines are directed to the line of interaction. These lines are marked with the number and type of interacting entities. You can write a single entity per a line or group them.
- Outgoing lines (in the Fig. 5 are denoted by the solid line). These lines are directed from the line of interaction. These lines are marked with the number and type of interacting entities. You can write a single entity per a line or group them.
- Line of interaction (in the Fig. 5 is denoted by the dotted line). The direction of time is denoted by the arrow. This line is marked by the coefficient of intensity of the interaction.

Each line is attributed to a certain factor (depending on the the approach chosen). The resulting expression is obtained by multiplying these factors.

[10] In order not to clutter the diagram, we have only one type of interacting entities left in these schemes.

8.1 Operator Approach

We obtain the Liouville operator when using interaction diagrams in the operator approach. Let us assign the corresponding factor for each line. The resulting term is obtained as the normal ordered product of factors.[11]

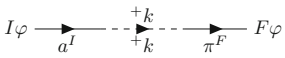

Fig. 7. Forward interaction (operator approach)

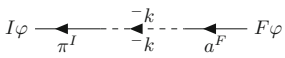

Fig. 8. Backward interaction (operator approach)

We use the following factors for each type of line (Fig. 7).

- Incoming line. This line corresponds to the disappearance of one entity from the system. Therefore, it corresponds to the annihilation operator a. It is clear that the line with combined capacity I corresponds to the operator a^I.
- Outgoing line. This line corresponds to the emergence of one entity in the system. Therefore, it corresponds to the creation operator π. It is clear that the line with combined capacity F corresponds to the operator π^F.
- Line of interaction. This line corresponds to the ratio of the interaction intensity.

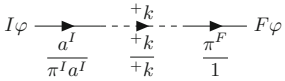

Fig. 9. Forward interaction (operator approach), extended notation

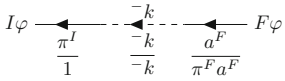

Fig. 10. Backward interaction (operator approach), extended notation

That is, for the Fig. 7 we obtain a factor ${}^+k\pi^F a^I$. However, this violates the Eq. (9). Redressing this, we have to subtract the number of entities that have entered into interaction, multiplied by the intensity of the interaction. Then we get a following term of the Liouville operator:

$$ {}^+k\pi^F a^I - {}^+k\pi^I a^I = {}^+k\left(\pi^F - \pi^I\right) a^I. \tag{10} $$

To backward interactions (Fig. 8), we use the same rules.

[11] In normal ordering product all creation operators are moved so as to be always to the left of all the annihilation operators.

To account for the additional factor of (10) we will use the extended diagrams (Figs. 9 and 10). Here, from the the normal ordered product of the numerators the normal product of the denominators is subtracted.

Thus, the following Liouville operator corresponds to the scheme (1):

$$L = \sum_{\underline{\alpha},\underline{i}} \left[{}^{+}k_{\underline{\alpha}} \left((\pi_{\underline{i}})^{F^{\underline{i\alpha}}} - (\pi_{\underline{i}})^{I^{\underline{i\alpha}}} \right) (a_{\underline{i}})^{I^{\underline{i\alpha}}} + {}^{-}k_{\underline{\alpha}} \left((\pi_{\underline{i}})^{I^{\underline{i\alpha}}} - (\pi_{\underline{i}})^{F^{\underline{i\alpha}}} \right) (a_{\underline{i}})^{F^{\underline{i\alpha}}} \right]. \tag{11}$$

8.2 Combinatorial Approach

For the combinatorial approach we get the master equation in the representation of the state vectors. In this approach, with the help of diagrams, we obtain the transition probability ${}^{+}s_{\underline{\alpha}}$ and ${}^{-}s_{\underline{\alpha}}$. They are, as in the case of operator approach, obtained by multiplying the diagrams factors.

However, the structure of the right-hand side of the Eq. (6) more complicated than Liouville operator. In the representation the state vectors the additive terms are presented in the functions arguments (dependency of the arguments from the operator r, see (3).). Therefore, we can not use only the factors multiplication.

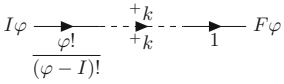

Fig. 11. Forward interaction (combinatorial approach)

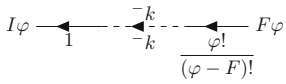

Fig. 12. Backward interaction (combinatorial approach)

We use the following factors for each type of line (Fig. 11).

– Incoming line. If all lines correspond to different state vectors, the factor of each line is the corresponding state vector. If there are several lines corresponding to the same state vector, the first line corresponds to the actual state vector (φ), the second line corresponds to the value of $\varphi - 1$ (as the first line has reduced the number of entities of this type in the system by one), and so further. That is, for a combined line factor can be written as follows:

$$\frac{\varphi!}{(\varphi - I)!}.$$

– Outgoing line do not give multiplicative contribution. It serves to obtain the step coefficient r:

$$r = F - I.$$

– Line of interaction. This line corresponds to the ratio of the interaction intensity.

In addition, we need the transition probabilities for the previous and the next steps:

$$\varphi^i + r^{i\underline{\alpha}},$$
$$\varphi^i - r^{i\underline{\alpha}}.$$

Thus, for the Fig. 11 transition probability will be as follows:

$${}^{+}s(\varphi) = {}^{+}k\frac{\varphi!}{(\varphi - I)!},$$
$${}^{-}s(\varphi) = {}^{-}k\frac{\varphi!}{(\varphi - F)!}.$$

For the general case:

$${}^{+}s_{\underline{\alpha}}(\varphi^i) = {}^{+}k_{\underline{\alpha}}\prod_{\underline{i}=1}^{n}\frac{\varphi^{\underline{i}}!}{(\varphi^{\underline{i}} - I^{i\underline{\alpha}})!},$$
$${}^{-}s_{\underline{\alpha}}(\varphi^i) = {}^{-}k_{\underline{\alpha}}\prod_{\underline{i}=1}^{n}\frac{\varphi^{\underline{i}}!}{(\varphi^{\underline{i}} - F^{i\underline{\alpha}})!}.$$

To backward interactions (Fig. 12), we use the same rules.

The general form of the master equation for the state vector φ^i we obtain on the basis of formula (6).

9 Verhulst Model

As a demonstration of the method, we consider the Verhulst model [8,9,25], which describes the limited growth[12]. Initially, this model was written down as the differential equation:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \lambda\varphi - \beta\varphi - \gamma\varphi^2,$$

where λ denotes the breeding intensity factor, β—the extinction intensity factor, γ—the factor of population reduction rate (usually the rivalry of individuals is considered)[13].

The interaction scheme for the stochastic version of the model is:

$$\begin{aligned} \varphi &\underset{\gamma}{\overset{\lambda}{\rightleftharpoons}} 2\varphi, \\ \varphi &\xrightarrow{\beta} 0. \end{aligned} \tag{12}$$

The interaction schemes (12) match Figs. 13, 14, and 15.

The first relation means that an individual who eats one unit of meal is immediately reproduced, and in the opposite direction is the rivalry between individuals. The second relation describes the death of an individual.

[12] The attractiveness of this model is that it is one-dimensional and non-linear.

[13] The same notation as in the original model [25] is used.

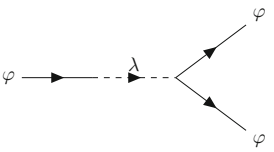

Fig. 13. First forward interaction

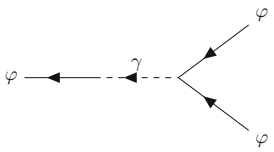

Fig. 14. First backward interaction

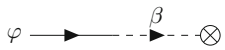

Fig. 15. Second forward interaction

9.1 Combinatorial Approach

The interaction schemes (12) in combinatorial approach match Figs. 16, 17, and 18.

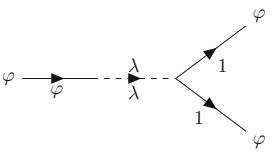

Fig. 16. First forward interaction (combinatorial approach)

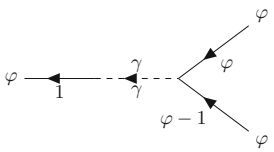

Fig. 17. First backward interaction (combinatorial approach)

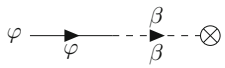

Fig. 18. Second forward interaction (combinatorial approach)

Let's define transition rates within the Verhults model as follows:

$$
\begin{array}{lll}
{}^{+}s_1(\varphi) = \lambda\varphi, & {}^{+}s_1(\varphi-1) = \lambda(\varphi-1), & {}^{+}s_1(\varphi+1) = \lambda(\varphi+1), \\
{}^{-}s_1(\varphi) = \gamma\varphi(\varphi-1), & {}^{-}s_1(\varphi-1) = \gamma(\varphi-1)(\varphi-2), & {}^{-}s_1(\varphi+1) = \gamma(\varphi+1)\varphi, \\
{}^{+}s_2(\varphi) = \beta\varphi. & {}^{+}s_2(\varphi-1) = \beta(\varphi-1). & {}^{+}s_2(\varphi+1) = \beta(\varphi+1).
\end{array}
$$

$$
r^1 = 1, \qquad r^2 = -1.
$$

Then, based on (6), the form of the master equation is:

$$
\begin{aligned}
\frac{\partial p(\varphi,t)}{\partial t} = &-\left[\lambda\varphi + \beta\varphi + \gamma\varphi(\varphi-1)\right] p(\varphi,t) \\
&+ \left[\beta(\varphi+1) + \gamma(\varphi+1)\varphi\right] p(\varphi+1,t) + \lambda(\varphi-1)p(\varphi-1,t).
\end{aligned}
$$

For particular values of φ (as in (5)):

$$
\begin{aligned}
\frac{\partial p_n(t)}{\partial t} := \left.\frac{\partial p(\varphi,t)}{\partial t}\right|_{\varphi=n} = &-\left[\lambda n + \beta n + \gamma n(n-1)\right] p_n(t) \\
&+ \left[\beta(n+1) + \gamma(n+1)n\right] p_{n+1}(t) + \lambda(n-1)p_{n-1}(t). \quad (13)
\end{aligned}
$$

9.2 Operator Approach

The interaction schemes (12) in operator approach match Figs. 19, 20, and 21.

From (12) and (11) the Liouville operator is:

$$\begin{aligned} L &= \lambda(\pi^2 - \pi)a + \gamma(\pi - \pi^2)a^2 + \beta(1-\pi)a \\ &= \lambda\left((a^\dagger)^2 - a^\dagger\right)a + \gamma\left(a^\dagger - (a^\dagger)^2\right)a^2 + \beta\left(1 - a^\dagger\right)a \\ &= \lambda\left(a^\dagger - 1\right)a^\dagger a + \beta\left(1 - a^\dagger\right)a + \gamma\left(1 - a^\dagger\right)a^\dagger a^2. \end{aligned}$$

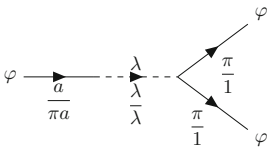

Fig. 19. First forward interaction (operator approach)

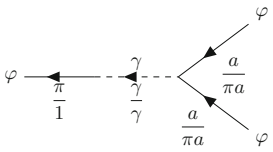

Fig. 20. First backward interaction (operator approach)

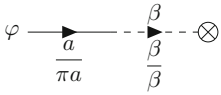

Fig. 21. Second forward interaction (operator approach)

The master equation by Liouville operator:

$$\begin{aligned} \frac{\partial p_n(t)}{\partial t} &= \frac{1}{n!}\langle n|L|\varphi\rangle \\ &= \frac{1}{n!}\langle n| - \left[\lambda a^\dagger a + \beta a^\dagger a + \gamma a^\dagger a^\dagger aa\right] + \left[\beta a + \gamma a^\dagger aa\right] + \lambda a^\dagger a^\dagger a|\varphi\rangle \\ &= -\left[\lambda n + \beta n + \gamma n(n-1)\right]\langle n|\varphi\rangle \\ &+ \left[\beta(n+1) + \gamma(n+1)n\right]\langle n+1|\varphi\rangle + \lambda(n-1)\langle n-1|\varphi\rangle \\ &= -\left[\lambda n + \beta n + \gamma n(n-1)\right]p_n(t) + \\ &+ \left[\beta(n+1) + \gamma(n+1)n\right]p_{n+1}(t) + \lambda(n-1)p_{n-1}(t). \quad (14) \end{aligned}$$

The result (14) coincides with the formula (13), which was obtained by combinatorial method.

10 Conclusions

The authors proposed a diagram technique for the stochastization of one-step processes. At the moment, this technique allows to get main master equation. Also, this technique makes it possible to unify different approaches to the stochastization of one-step processes.

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