Characteristics of Lost and Served Packets for Retrial Queueing System with General Renovation and Recurrent Input Flow

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Abstract. The retrial queuing system with general renovation is under investigation. The mechanism of general renovation with retrials means that the packet at the end of its service in accordance with a given probability distribution discards a certain number of other packets from the buffer and itself stays in the system for another round of service, or simply leaves the system without any effect on it. In order to obtain some probability and time related performance characteristics the embedded Markov chain technique is applied. Under the assumption of the existence of a stationary regime, the steady-state probability distribution (for the embedded Markov chain) of the number of packets in the system is obtained, as well as some other characteristics, such as the probability of the accepted task to be served or the probability of the accepted task to be dropped from the buffer, the probability distribution of number of repeated services. Also time characteristics are given.

Keywords: Retrial queueing system · General renovation
Recurrent input flow · Repeated service
Probability–time characteristics · Lost packet · Served packet

1 Introduction

Even though mathematical modelling of telecommunication systems with possible losses of information has been the subject of numerous research papers, this topic still attracts attention from the research community. The main research directions, to name a few, are:

V. M. Vishnevskiy and D. V. Kozyrev (Eds.): DCCN 2018, CCIS 919, pp. 327–340, 2018.
https://doi.org/10.1007/978-3-319-99447-5_28

– dropping mechanisms which regulate queue (buffer) lengths by discarding the incoming packets (see [1–8]);
– disaster arrivals, when the incoming signals cause the buffer to drop some or all the packets (see [9–20]);
– unreliable servers, which cause the packet dropping (see [21–25]);
– repairable and reliability systems (see [26–30])
– renovation, when the queue (partially or fully) empties out upon service completions (see [31–37]).

In [31] the authors have introduced the so-called renovation mechanism, when the packet at the end of its service empties the buffer with the probability q and leaves the system or with the complementary probability $p = 1 - q$ leaves the system, having no effect on it. The application of the renovation mechanism in finance and some other application fields was shown in [32]. In [33] Bocharov proposed the mathematical model of renovation mechanism with retrials (or repeated service): if the served packet empties the buffer it enters the server for another round of service. Later on the renovation mechanism was further generalized by A. V. Pechinkin, who proposed the mathematical model of general renovation: at the end of service the packet discards from the buffer of capacity $0 < r < \infty$ exactly i, $i \geq 1$, other packets with probability $q(i)$ and leaves the system, or just leaves the system without any effect on it with the complementary probability $p = 1 - \sum_{i=1}^{r} q(i)$.

In [34–36] the $GI|M|n|r$ queueing system with various types of service disciplines and renovation was studied. The $M|G|1|r$ queue was analysed in [37,38]. The first paper to analyse the queue with the general renovation and retrials is apparently [33], where the authors obtained the main steady-state characteristics. In [39] the $GI|M|1|\infty$ queueing system with renovation (when the buffer is fully emptied, in case of renovation) was thoroughly investigated: the expressions for the steady-state probabilities, the probabilities of incoming packet to be served (or dropped from the buffer) as well as main stationary waiting and sojourn time characteristics were derived in analytic form.

In [8] such performance characteristics as stationary loss rate, moments of the number in the system for $M/D/1/N$ queue were obtained in order to compare the renovation mechanism with well known active queue mechanisms like RED.

The first attempt to apply the general renovation to systems with repeated service (retrials) was done in [40] for the $M|M|1|\infty$ system. In [41] some possible approaches to the investigation of the $GI|M|1|\infty$ system were formulated. The main goal of this article is to present some new analytic results concerning the steady-state analysis of $GI|M|1|\infty$ queue with general renovation and retrials.

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The structure of the article is follows. In Sect. 2 the description of retrial queueing system with general renovation is presented Sect. 2.1, some auxiliary probabilities are formulated Sect. 2.2, in Sect. 2.3 the embedded Markov chain and transition probabilities matrix are defined and the formulas of the steady-state probability distribution of embedded Markov chain are obtained in Subsect. 2.4. The Sect. 3 is devoted to characteristics of served (lost) packets: in Sect. 3.1 the probability p^{serv} that the incoming into the system packet will be served is defined; the probability p^{loss} that the arriving into the system packet will be dropped from the buffer is obtained in Sect. 3.2; some other probabilities (such as the probability $\tilde{p}_1$ that none of the incoming packets will be lost, the probability $\tilde{p}_2$ that all the incoming packets will be served only once) are presented in Sect. 3.3; in Sect. 3.4 the probability distribution of number of repeated services is obtained; and in Sect. 3.5 the Laplace-Stieltjes transformation of waiting time steady-state distribution $\omega^{\text{serv}}(s)$ of accepted and served packet as well as the Laplace-Stieltjes transformation of waiting time steady-state distribution $\omega^{\text{loss}}(s)$ of accepted and lost packet are defined. In Sect. 4 the future goals are formulated.

2 The Description Of the Retrial Queueing System, the Steady-State Probability Distribution

2.1 The General Renovation with Retrials Mechanism

Consideration is given to the queueing system $GI|M|1|\infty$ with recurrent input flow, exponentially distributed service times, unlimited buffer capacity and general renovation mechanism with retrials.

The general renovation with retrial is defined as follows. The packet at the end of its service with probability $q(i)$, $i \geq 0$, drops exactly i packets from the buffer (if there are more the i packets present in it) or empties out the buffer (if there are i or less packets in it) and stays in the system for another round of service (retrial). There are two possible types of retrials: either the served packet occupies the first free place in the buffer, or remains in the server for repeated service. With probability $p = 1 - \sum_{i=0}^{\infty} q(i)$ the served packet leaves the system having no effect on it.

In order to analyse this system we use the embedded Markov chain technique. Before we can proceed some auxiliary probabilities are needed.

2.2 The Auxiliary Probabilities

In order to obtain time-probability characteristics of the system some auxiliary probabilities are needed. The first type of auxiliary probabilities—$\pi(\cdot)$—will be defined for the case, when the exact number of customers leaves the system (from the server or (and) from the buffer) and system is not empty. The second type auxiliary probabilities—$\pi^*(\cdot)$—when the buffer is emptied by one of the served packets (the server remains busy). The results of [40] are used for auxiliary probabilities deriving.

The probability of the first type: $\pi(k, n, m)$—is the probability that between successive arrival moments exactly k ($k \geq 0$) packets will be served, exactly m ($m \geq 0$) packets will be dropped from the buffer, and n ($0 \leq n \leq k$) served packets will leave the system if at the previous arrival moment there were $n+m$ packets. For $\pi(k,n,m)$ the following formulas are valid:

$$\pi(0,0,0)=1, \qquad \pi(0,n,m)=0, \quad n\geq 1, m\geq 1; \tag{1}$$

$$\pi(k,n,0)=C_k^n p^n q^{k-n}(0), \quad p+q(0)\neq 1, \quad k\geq 1, \quad n=\overline{0,k}; \tag{2}$$

$$\pi(1,0,m)=q(m), \quad m\geq 0; \tag{3}$$

$$\pi(k,0,m)=\sum_{i=0}^{m}\pi(1,0,i)\pi(k-1,0,m-i), \quad k>1, \quad m>1; \tag{4}$$

$$\pi(k,n,m)=C_k^n p^n \pi(k-n,0,m), \quad k>1, \quad n=\overline{0,k}, \quad m\geq 1. \tag{5}$$

For the second type probability $\pi^*(k,n,m)$—the probability that k $(k\geq 1)$ served packets will empty the buffer of the system and exactly n $(0\leq n\leq k-1)$ served packets will leave the system, the following relations are valid:

$$\pi^*(0,0,0)=1; \quad \pi^*(1,0,m)=\sum_{j=m}^{\infty}q(j)=Q(m), \quad m\geq 0; \tag{6}$$

$$\pi^*(k,0,0)=(1-p)^k, \quad k\geq 1; \tag{7}$$

$$\pi^*(k,0,m) = \sum_{i=0}^{m-1}\pi(1,0,i)\pi^*(k-1,0,m-i)+\pi^*(1,0,m)\pi^*(k-1,0,0), \quad k\geq 2, m\geq 1; \tag{8}$$

$$\pi^*(k,n,0)=C_k^n p^n(1-p)^{k-n}, \quad k\geq 1, \quad n=\overline{0,k}; \tag{9}$$

$$\pi^*(k,n,m)=C_k^n p^n \pi^*(k-n,0,m), \quad k>1, n=\overline{0,k}, m\geq 0. \tag{10}$$

Also we need to define the following transformations:

$$\pi(g)=\sum_{i=0}^{\infty}g^i\pi(1,0,i), \quad \pi^*(g)=\sum_{i=0}^{\infty}g^i\pi^*(1,0,i), \tag{11}$$

where g is some variable, $0<g<1$, which will be defined in the Subsect. 2.4.

Also we may define:

$$\pi_k(g)=\sum_{i=0}^{\infty}g^i\pi(k,0,i)=\pi^k(g), \quad \pi_k^*(g)=\sum_{i=0}^{\infty}g^i\pi^*(k,0,i)=\left(\pi^*(g)\right)^k, k>1. \tag{12}$$

For $\pi(g)$ and $\pi^*(g)$ the following relation is true:

$$\pi^*(g)=\frac{1-p-g\pi(g)}{1-g}. \tag{13}$$

Now we may define the embedded Markov chain and transition ptobabilities matrix.

2.3 The Embedded Markov Chain, Transition Probabilities Matrix

To investigate our system we will construct the embedded upon arrival times Markov chain $\nu_n = \nu(\tau_n - 0)$ (τ_n—the moment of the n-th task arrival) with enumerable number of states $\mathcal{X} = \{0, 1, 2, \ldots\}$ and the matrix $P = (p_{ij})_{i,j\geq 0}$ of transition probabilities.

Now the transition probability matrix $P = (p_{i,j})_{i,j=\overline{1,n+r}}$ of embedded Markov chain may be defined in the following form:

$$P = \begin{pmatrix} P_1^* & \tilde{P}_0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots \\ P_2^* & \tilde{P}_1 & P_0 & 0 & \ldots & 0 & 0 & 0 & \ldots \\ P_3^* & \tilde{P}_2 & P_1 & P_0 & \ldots & 0 & 0 & 0 & \ldots \\ P_4^* & \tilde{P}_3 & P_2 & P_1 & \ldots & 0 & 0 & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\ P_{k-1}^* & \tilde{P}_{k-2} & P_{k-3} & P_{k-4} & \ldots & P_1 & P_0 & 0 & \vdots \\ P_k^* & \tilde{P}_{k-1} & P_{k-2} & P_{k-3} & \ldots & P_2 & P_1 & P_0 & \vdots \\ P_{k+1}^* & \tilde{P}_k & P_{k-1} & P_{k-2} & \ldots & P_3 & P_2 & P_1 & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{14}$$

The probability $P_{i+1}^* = p_{i,0}$ $(i \geq 0)$ corresponds to the transition probability from the state i $(i \geq 0)$ to the state 0 of the system being empty.

$$P_1^* = 1 - \alpha(p\mu), \tag{15}$$

$$P_{i+1}^* = \int_0^\infty \left(\int_0^x \left(\sum_{n=0}^{i} \sum_{k=n+1}^{\infty} \pi^*(k, n, i-n) \frac{(\mu y)^k}{k!} e^{-\mu y} \right) p\mu e^{-\mu(x-y)} dy \right) dA(x), \quad i > 1. \tag{16}$$

The probability $\tilde{P}_i = p_{i,0}$ $(i \geq 0)$ is the transition probability from the state i $(i \geq 0)$ to the state 1 when the buffer of the system is empty and the server is occupied.

$$\tilde{P}_0 = \alpha(p\mu), \tag{17}$$

$$\tilde{P}_i = \int_0^\infty \left(\sum_{n=0}^{i} \sum_{k=n+1}^{\infty} \pi^*(k, n, i-n) \frac{(\mu x)^k}{k!} e^{-\mu x} \right) dA(x), \quad i \geq 1. \tag{18}$$

The probability $P_k = p_{i+k,i+1}$ $(k \geq 0,\ i \geq 0)$—the transition probability from the state $i + k$ to the state $i + 1$ (the exactly k packets leave the system and the buffer is nor empty):

$$P_0 = \alpha\left(\mu(1 - q(0))\right), \tag{19}$$

$$P_k = \int_0^\infty \left(\sum_{j=k}^{\infty} \pi(j,k,0) \frac{(\mu x)^j}{j!} e^{-\mu x} \right) dA(x)$$
$$+ \int_0^\infty \left(\sum_{n=0}^{k-1} \sum_{j=n+1}^{\infty} \pi(j,n,k-n) \frac{(\mu x)^j}{j!} e^{-\mu x} \right) dA(x), \quad k \geq 1. \tag{20}$$

Here, $\alpha(s)$ is the Laplace-Stieltjes transformation of an interarrival time probability distribution function $A(x)$.

Now we may derive the formulas for steady-state probability distribution of embedded Markov chain.

2.4 The Steady-State Probability Distribution

Let's define the steady-state probability distribution of the embedded Markov chain (in assumption that the steady-state regime exists) as p_k^- $(k \geq 0)$. Here, the probability p_k^- means that there were k $(k \geq 0)$ packets in the system at the moment of arrival.

The steady-state probabilities p_k^-, $k \geq 0$, satisfy the following system of equations

$$p_0^- = \sum_{i=0}^{\infty} P_{i+1}^* p_i^-, \quad p_1^- = \sum_{i=0}^{\infty} \tilde{P}_i p_i^-, \tag{21}$$

$$p_k^- = \sum_{i=0}^{\infty} P_i p_{k-1+i}^-, \quad k \geq 2, \tag{22}$$

with the normalization requirement:

$$\sum_{k=0}^{\infty} p_k^- = 1. \tag{23}$$

The steady-state probabilities p_i^- for $i \geq 2$ may be written down in the geometric form as in [34,35,39]:

$$p_i^- = p_2^- \cdot g^{i-2}, \quad i \geq 2, \tag{24}$$

where the constant g is the unique solution $(0 < g < 1)$ of the following equation (obtained by substituting (24) in (22)):

$$g = \sum_{i=0}^{\infty} P_i g^i = \alpha\left(\mu(1 - pg - \pi(g))\right). \tag{25}$$

From the (21) and (22) (by using (24)) we may derive the probabilities p_1^- and p_2^- via p_0^-:

$$p_1^- = p_0^- \tilde{P}_0 \frac{\tilde{P}(g) + P^*(g)}{P_2^* \tilde{P}(g) + \left(1 - \tilde{P}_1\right) P^*(g)}, \tag{26}$$

$$p_2^- = p_0^- \frac{\tilde{P}_0}{\tilde{P}(g)} \left(\frac{\left(1-\tilde{P}_1\right)\left(\tilde{P}(g)+P^*(g)\right)}{P_2^*\tilde{P}(g)+\left(1-\tilde{P}_1\right)P^*(g)} - 1 \right), \qquad (27)$$

where $P^*(g) = \sum_{i=2}^{\infty} P_{i+1}^* g^{i-2}$, $\tilde{P}(g) = \sum_{i=1}^{\infty} \tilde{P}_i g^{i-2}$.

Finally, by using the equations (23), (25), (26) and (27), the probability p_0^- of the system being empty is derived:

$$p_0^- = \left(1 + \tilde{P}_0 \frac{\tilde{P}(g)+P^*(g)}{P_2^*\tilde{P}(g)+\left(1-\tilde{P}_1\right)P^*(g)} \right.$$
$$\left. + \frac{\tilde{P}_0}{(1-g)\tilde{P}(g)} \left(\frac{\left(1-\tilde{P}_1\right)\left(\tilde{P}(g)+P^*(g)\right)}{P_2^*\tilde{P}(g)+\left(1-\tilde{P}_1\right)P^*(g)} - 1 \right) \right)^{-1}. \qquad (28)$$

In the next section the main probabilistic and time characteristics of lost packets and served packets will be presented.

3 The Characteristics of Lost and Served Packets

In this section some additional probability and time characteristics will be presented. But first let's make some assumptions about the service discipline and the discipline of the reset of packets from the buffer, as well as the behavior of the remaining packet for possible repeated service.

- packets are served from the queue in the FCFS (First-Come-First-Served) order;
- packets to be dropped from the buffer are chosen successively starting from the head of the queue;
- the served packet occupies the first free space in the buffer.

These assumptions determine the characteristics presented below.

3.1 The Probability that the Incoming Packet Will Be Served

The probability p^{serv} that the incoming into the system packet will be served is defined as follows:

$$p^{\text{serv}} = p_0^- + \sum_{i=1}^{\infty} p_i^- \sum_{n=0}^{i} \pi(i,n,0) + \sum_{i=2}^{\infty} p_i^- \sum_{k=1}^{i-1} \sum_{n=0}^{k-1} \pi(k,n,i-k),$$

and with the help of (1)–(10), (11) and (12), (25) takes the form:

$$\begin{aligned} p^{\text{serv}} &= p_0^- + p_1^- (p+q(0)) + p_2^- \frac{(p+q(0))^2}{1-g(p+q(0)} \\ &\quad + p_2^- \frac{1}{g}\left(\frac{p+\pi(g)}{1-g(p+\pi(g))} - \frac{p+q(0)}{1-g(p+q(0))}\right) \\ &= p_0^- + p_1^- (p+q(0)) + p_2^- \frac{\pi(g) - q(0) + g\,(p+\pi(g))\,(p+q(0))}{g\,(1-g\,(p+\pi(g)))} \end{aligned} \tag{29}$$

3.2 The Probability that the Incoming Packet Will Be Dropped from the Buffer

The probability p^{loss} that the arriving into the system packet will be dropped from the buffer by one of the served packets is defined as:

$$\begin{aligned} p^{\text{loss}} &= \sum_{i=1}^{\infty} p_i^- \pi^*(1,0,i) + \sum_{i=2}^{\infty} p_i^- \sum_{k=1}^{i-1}\sum_{n=0}^{k} \pi(k,n,0)\pi^*(1,0,i-k) \\ &\quad + \sum_{i=3}^{\infty} p_i^- \sum_{k=1}^{i-2}\sum_{n=0}^{k-1}\sum_{j=1}^{i-k-1} \pi(k,n,j)\pi^*(1,0,i-k-j), \end{aligned}$$

and with the help of (1)–(10), (11) and (12), (25) takes the form:

$$\begin{aligned} p^{\text{loss}} &= p_1^- Q(1) + p_2^- \frac{\pi^*(g) - Q(0) - gQ(1)}{g^2} + p_2^- \frac{\pi^*(g) - Q(0)}{g} \frac{p+\pi(g)}{1-g(p+\pi(g))} \\ &= p_1^- Q(1) + p_2^- \frac{\pi^*(g) - Q(0) - gQ(1) + g^2 Q(1)\,(p+\pi(g))}{g^2\,(1-g\,(p+\pi(g)))}. \end{aligned} \tag{30}$$

3.3 Some Other Probability Characteristics

The probability $\tilde{p}_1$ that none of the incoming packets will be lost is

$$\tilde{p}_1 = p_0^- + p_1^- (p+q(0)) + p_2^- \frac{(p+q(0))^2}{1-g(p+q(0)}.$$

The probability $\tilde{p}_2$ that all the incoming packets will be served (and only once) is

$$\tilde{p}_2 = \sum_{i=0}^{\infty} p_i^- \pi(i+1,i+1,0) = pp_0^- + p^2 p_1^- + p_2^- \frac{p^3}{1-pg}.$$

The probability $\tilde{p}_3$ that the incoming packet will be dropped by the first served packet is

$$\tilde{p}_3 = \sum_{i=1}^{\infty} p_i^- \pi^*(1,0,i) = p_1^- Q(1) + p_2^- \frac{\pi^*(g) - Q(0) - Q(1)}{g^2}.$$

3.4 The Probability Distribution of Number of Repeated Services

Let's define the probability $\tilde{q}_k$, $k \geq 0$, that the incoming packet will be served exactly k times. Then the following expressions are valid:

$$\tilde{q}_0 = p^{\text{loss}}, \tag{31}$$

$$\tilde{q}_k = \left(p^{\text{serv}}\right)^k (1-p)^{k-1} p + \left(p^{\text{serv}}\right)^k (1-p)^k p^{\text{loss}}, \quad k \geq 1. \tag{32}$$

It's easy to see that $\sum\limits_{k=0}^{\infty} \tilde{q}_k = 1$.

The mean number of services of accepted packets $\tilde{N}$ is

$$\tilde{N} = \sum_{k=0}^{\infty} k\tilde{q}_k = \frac{p^{\text{serv}}}{p^{\text{loss}} + pp^{\text{serv}}}.$$

3.5 The Time Characteristics

The Laplace-Stieltjes transformation of waiting time steady-state distribution $\omega^{\text{serv}}(s)$ of accepted and served packet is defined by the formula:

$$\omega^{\text{serv}}(s) = \frac{1}{p^{\text{serv}}} \left(p_0^- + \sum_{i=1}^{\infty} p_i^- \omega^i(s) \sum_{n=0}^{i} \pi(i,n,0) + \sum_{i=2}^{\infty} p_i^- \sum_{k=1}^{i-1} \omega^k(s) \sum_{n=0}^{k-1} \pi(k,n,i-k) \right),$$

where $\omega(s)$—Laplace-Stieltjes transformation of service time distribution function.

By using relations (1)–(10), (11) and (12), $\omega^{\text{serv}}(s)$ takes form:

$$\begin{aligned}
\omega^{\text{serv}}(s) &= \frac{1}{p^{\text{serv}}} \left(p_0^- + p_1^- \omega(s) (p + q(0)) + p_2^- \frac{\omega^2(s) (p+q(0))^2}{1 - g\omega(s)(p+q(0))} \right. \\
&\left. + p_2^- \frac{\omega(s)}{g} \left(\frac{p + \pi(g)}{1 - g\omega(s)(p+\pi(g))} - \frac{p+q(0)}{1 - g\omega(s)(p+q(0))} \right) \right) \\
&= \frac{1}{p^{\text{serv}}} \left(p_0^- + p_1^- \omega(s)(p+q(0)) \right. \\
&\left. + p_2^- \omega(s) \frac{\pi(g) - q(0) + g\omega(s)(p+q(0))(p+\pi(g))}{g\left(1 - g\omega(s)(p+\pi(g))\right)} \right).
\end{aligned} \tag{33}$$

The Laplace-Stieltjes transformation of waiting time steady-state distribution $\omega^{\text{loss}}(s)$ of accepted and lost packet is defined as

$$\begin{aligned}
\omega^{\text{loss}}(s) &= \frac{1}{p^{\text{loss}}} \left(\omega(s) \sum_{i=1}^{\infty} p_i^- \pi^*(1,0,i) \right. \\
&+ \sum_{i=2}^{\infty} p_i^- \sum_{k=1}^{i-1} \omega^{k+1}(s) \sum_{n=0}^{k} \pi(k,n,0) \pi^*(1,0,i-k) \\
&\left. + \sum_{i=3}^{\infty} p_i^- \sum_{k=1}^{i-2} \omega^{k+1}(s) \sum_{n=0}^{k-1} \sum_{j=1}^{i-k-1} \pi(k,n,j) \pi^*(1,0,i-k-j) \right),
\end{aligned}$$

and due to relations (1)–(10), (11) and (12), $\omega^{\text{loss}}(s)$ takes form:

$$\omega^{\text{loss}}(s) = \frac{\omega(s)}{p^{\text{loss}}}\Big(p_1^- Q(1) + p_2^- \frac{\pi^*(g) - Q(0) - gQ(1) + g^2 Q(1)\omega(s)\left(p+\pi(g)\right)}{g^2\left(1 - g\omega(s)\left(p+\pi(g)\right)\right)}\Big), \quad (34)$$

If w^{serv} and w^{loss} are mean waiting times for a served packet and a lost packet (they may be easily found from (33) and (34)), then the mean dwell time (time in system) w for an arbitrary packet is

$$w = w^{\text{loss}} p^{\text{loss}} + \sum_{k=1}^{\infty} \left(p^{\text{serv}}\right)^k (1-p)^{k-1} pk\left(w^{\text{serv}} + \mu^{-1}\right) + \sum_{k=1}^{\infty} \left(p^{\text{serv}}\right)^k (1-p)^k p^{\text{loss}}\left(k\left(w^{\text{serv}} + \mu^{-1}\right) + w^{\text{loss}}\right) = w^{\text{loss}} p^{\text{loss}} + \frac{p^{\text{serv}}\left(w^{\text{serv}} + \mu^{-1}\right) + w^{\text{loss}} p^{\text{loss}}}{1 - p^{\text{serv}}(1-p)}. \quad (35)$$

4 Conclusion

The conception of the retrial queueing system with general renovation was introduced in this article.

The main probability-time characteristics of retrial queueing system with general renovation such as the probability distribution (24), (26), (27) and (28), as well as the probability of arrival packet to be served p^{serv} (29) or to be dropped from the queue p^{loss} (30), as well as Laplace-Stieltjes transformation of waiting time steady-state distribution $\omega^{\text{serv}}(s)$ of accepted and served packet (33) and Laplace-Stieltjes transformation of waiting time steady-state distribution $\omega^{\text{loss}}(s)$ of accepted and lost packet (34) are presented in analytical form. Also the mean dwell time (time in system) w (35) for an arbitrary packet via mean waiting times for a served packet w^{serv} and a lost packet w^{loss} is presented in analytical form.

The future goals are to obtain the same probability-time characteristics for different combination of initial assumptions:

- packets are served from the queue in theFCFS or LCFS (Last-Come-First-Served) order;
- packets to be dropped from the buffer are chosen successively starting from the head or from the end of the queue;
- the served packet (if it remains in the system) occupies the first free space in the buffer or immediately goes to the server for repeated service.

It is also of interest to combine renovation mechanism and hysteretic overload control policies [42–51] in order to construct more adequate mathematical models of real telecommunication systems (for example, SIP server [42,45,46,49,52,53] or RED-like AQM algorithms).

Acknowledgments. The publication has been prepared with the support of the "RUDN University Program 5-100" and has been funded by Russian Foundation for Basic Research (RFBR) according to the research project No. 18-07-00692 and No. 16-07-00556.

References

1. Floyd, S., Jacobson, V.: Random early detection gateways for congestion avoidance. IEEE/ACM Trans. Netw. **4**(1), 397–413 (1993)
2. Ramakrishnan, K., Floyd, S., Black, D.: The Addition of Explicit Congestion Notification (ECN) to IP. RFC 3168. Internet Engineering Task Force (2001). https://tools.ietf.org/html/rfc3168
3. Korolkova, A.V., Zaryadov, I.S.: The mathematical model of the traffic transfer process with a rate adjustable by RED. In: International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), Moscow, Russia, pp. 1046–1050. IEEE (2010)
4. Sharma, V., Purkayastha, P.: Performance analysis of TCP connections with RED control and exogenous traffic. Queueing Syst. **48**(3), 193–235 (2004)
5. Velieva, T.R., Korolkova, A.V., Kulyabov, D.S.: Designing installations for verification of the model of active queue management discipline RED in the GNS3. In: 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), pp. 570–577. IEEE Computer Society (2015)
6. Korolkova, A.V., Kulyabov, D.S., Sevastianov, L.A.: Combinatorial and operator approaches to RED modeling. Math. Model. Geom. **3**, 1–18 (2015)
7. Zaryadov, I., Korolkova, A., Kulyabov, D., Milovanova, T., Tsurlukov, V.: The survey on Markov-modulated arrival processes and their application to the analysis of active queue management algorithms. In: Vishnevskiy, V.M., Samouylov, K.E., Kozyrev, D.V. (eds.) DCCN 2017. CCIS, vol. 700, pp. 417–430. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-66836-9_35
8. Konovalov, M., Razumchik, R.: Queueing Systems with Renovation vs. Queues with RED. Supplementary Material (2017). https://arxiv.org/abs/1709.01477
9. Gelenbe, E.: Product-form queueing networks with negative and positive customers. J. Appl. Probab. **28**(3), 656–663 (1991)
10. Pechinkin, A., Razumchik, R.: Waiting characteristics of queueing system $Geo/Geo/1/\infty$ with negative claims and a bunker for superseded claims in discrete time. In: International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, ICUMT 2010, pp. 1051–1055 (2010). https://doi.org/10.1109/ICUMT.2010.5676508
11. Pechinkin, A.V., Razumchik, R.V.: A method for calculating stationary queue distribution in a queuing system with flows of ordinary and negative claims and a bunker for superseded claims. J. Commun. Technol. Electr. **57**(8), 882–891 (2012)
12. Pechinkin, A.V., Razumchik, R.V.: The stationary distribution of the waiting time in a queueing system with negative customers and a bunker for superseded customers in the case of the LAST-LIFO-LIFO discipline. J. Commun. Technol. Electr. **57**(12), 1331–1339 (2012)
13. Razumchik, R.V.: Analysis of finite capacity queue with negative customers and bunker for ousted customers using chebyshev and gegenbauer polynomials. Asia-Pacific J. Oper. Res. **31**(04), 1450029 (2014). https://doi.org/10.1142/S0217595914500298

14. Semenova, O.V.: Multithreshold control of the $BMAP/G/1$ queuing system with MAP flow of Markovian disasters. Autom. Remote Control **68**(1), 95–108 (2007)
15. Li, J., Zhang, L.: $M^X|M|c$ queue with catastrophes and state-dependent control at idle time. Front. Math. China **12**(6), 1427–1439 (2017)
16. Gudkova, I., et al.: Modeling and analyzing licensed shared access operation for 5G network as an inhomogeneous queue with catastrophes. In: International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, December 2016, 7765372, pp. 282–287 (2016)
17. Sudhesh, R., Savitha, P., Dharmaraja, S.: Transient analysis of a two heterogeneous servers queue with system disaster, server repair and customers impatience. TOP **25**(1), 179–205 (2017)
18. Li, J., Zhang, L.: $M^X/M/c$ queue with catastrophes and state-dependent control at idle time. Front. Math. China **12**(6), 1427–1439 (2017)
19. Suranga Sampath, M.I.G., Liu, J.: Transient analysis of an $M/M/1$ queue with reneging, catastrophes, server failures and repairs. Bull. Iran. Math. Soc. (2018). https://doi.org/10.1007/s41980-018-0037-6
20. Azadeh, A., Naghavi Lhoseiny, M.S., Salehi, V.: Optimum alternatives of tandem $G/G/K$ queues with disaster customers and retrial phenomenon: interactive voice response systems. Telecommun. Syst. **68**(3), 535–562 (2018)
21. Dudin, A., Klimenok, V., Vishnevsky, V.: Analysis of unreliable single server queueing system with hot back-up server. Commun. Comput. Inf. Sci. **499**, 149–161 (2015)
22. Krishnamoorthy, A., Pramod, P.K., Chakravarthy, S.R.: Queues with interruptions: a survey. TOP **22**(1), 290–320 (2014)
23. Vishnevsky, V.M., Kozyrev, D.V., Semenova, O.V.: Redundant queuing system with unreliable servers. In: International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, pp. 283–286. IEEE Xplore (2014)
24. Xu, B., Xu, X.: Equilibrium strategic behavior of customers in the $M/M/1$ queue with partial failures and repairs. Oper. Res. **18**(2), 273–292 (2018)
25. Nazarov, A., Sztrik, J., Kvach, A., Berczes, T.: Asymptotic analysis of finite-source $M/M/$ retrial queueing system with collisions and server subject to breakdowns and repairs. Ann. Oper. Res. 1–17 (2018). https://doi.org/10.1007/s10479-018-2894-z
26. Ometov, A., Kozyrev, D., Rykov, V., Andreev, S., Gaidamaka, Y., Koucheryavy, Y.: Reliability-centric analysis of offloaded computation in cooperative wearable applications. Wirel. Commun. Mob. Comput. **2017**, 9625687 (2017)
27. Rykov, V., Kozyrev, D., Zaripova, E.: Modeling and simulation of reliability function of a homogeneous hot double redundant repairable system. In: Proceedings - 31st European Conference on Modelling and Simulation, ECMS 2017, pp. 701–705 (2017)
28. Houankpo, H.G.K., Kozyrev, D.V.: Sensitivity analysis of steady state reliability characteristics of a repairable cold standby data transmission system to the shapes of lifetime and repair time distributions of its elements. In: CEUR Workshop Proceedings, vol. 1995, pp. 107–113 (2017)
29. Rykov, V.V., Kozyrev, D.V.: Analysis of renewable reliability systems by Markovization method. In: Rykov, V.V., Singpurwalla, N.D., Zubkov, A.M. (eds.) ACMPT 2017. LNCS, vol. 10684, pp. 210–220. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-71504-9_19
30. Rykov, V., Kozyrev, D.: On sensitivity of steady-state probabilities of a cold redundant system to the shapes of life and repair time distributions of its elements. In:

Pilz, J., Rasch, D., Melas, V., Moder, K. (eds.) IWS 2015. Springer Proceedings in Mathematics & Statistics, vol. 231, pp. 391–402. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-76035-3_28
31. Kreinin, A.: Queueing systems with renovation. J. Appl. Math. Stochast. Anal. **10**(4), 431–443 (1997)
32. Kreinin, A.: Inhomogeneous random walks: applications in queueing and finance. In: CanQueue 2003. Fields Institute, Toronto (2003)
33. Bocharov, P.P., Zaryadov, I.S.: Probability distribution in queueing systems with renovation. Bull. Peoples' Friendsh. Univ. Russia. Ser. Math. Inf. Sci. Phys. **1–2**, 15–25 (2007)
34. Zaryadov, I.S., Pechinkin, A.V.: Stationary time characteristics of the $GI/M/n/\infty$ system with some variants of the generalized renovation discipline. Autom. Remote Control **70**(12), 2085–2097 (2009)
35. Zaryadov, I.S.: Queueing systems with general renovation. In: International Conference on Ultra Modern Telecommunications, ICUMT 2009, St.-Petersburg. IEEE (2009). https://doi.org/10.1109/ICUMT.2009.5345382
36. Zaryadov, I., Razumchik, R., Milovanova, T.: Stationary waiting time distribution in $G|M|n|r$ with random renovation policy. In: Vishnevskiy, V.M., Samouylov, K.E., Kozyrev, D.V. (eds.) DCCN 2016. CCIS, vol. 678, pp. 349–360. Springer, Cham (2016). https://doi.org/10.1007/978-3-319-51917-3_31
37. Bogdanova, E.V., Milovanova, T.A., Zaryadov, I.S.: The analysis of queueing system with general service distribution and renovation. Bull. Peoples' Friendsh. Univ. Russia. Ser. Math. Inf. Sci. Phys. **25**(1), 3–8 (2017)
38. Zaryadov, I.S., Bogdanova, E.V., Milovanova, T.A.: Probability-time characteristics of $M|G|1|1$ queueing system with renovation. In: CEUR Workshop Proceedings, vol. 1995, pp. 125–131 (2017)
39. Zaryadov, I.S., Scherbanskaya, A.A.: Time characteristics of queuing system with renovation and reservice. Bull. Peoples' Friendsh. Univ. Russia. Ser. Math. Inf. Sci. Phys. **2**, 61–66 (2014)
40. Matskevich, I.A.: Time-probabilistic characteristics of queueing system with general renovation and repeated service. In: Proceedings of the VI Conference on Information and Telecommunication Technologies and Mathematical Modeling of High-Tech Systems, Moscow, Russia, pp. 37–39 (2016)
41. Zaryadov, I.S., Matskevich, I.A., Scherbanskaya, A.A.: The queueing system with general renovation and repeated service – time-probability characteristics. In: Proceedings of the Nineteenth International Scientific Conference on Distributed Computer and Communication Networks: Control, Computation, Communications (DCCN 2016), vol. 3, pp. 458–462 (2016)
42. Abaev, P., Pechinkin, A., Razumchik, R.: On analytical model for optimal SIP server hop-by-hop overload control. In: International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, 6459680, pp. 286–291 (2012)
43. Abaev, P., Gaidamaka, Y., Samouylov, K.E.: Modeling of hysteretic signaling load control in next generation networks. In: Andreev, S., Balandin, S., Koucheryavy, Y. (eds.) NEW2AN/ruSMART -2012. LNCS, vol. 7469, pp. 440–452. Springer, Heidelberg (2012). https://doi.org/10.1007/978-3-642-32686-8_41
44. Abaev, P., Pechinkin, A., Razumchik, R.: On mean return time in queueing system with constant service time and bi-level hysteric policy. In: Dudin, A., Klimenok, V., Tsarenkov, G., Dudin, S. (eds.) BWWQT 2013. CCIS, vol. 356, pp. 11–19. Springer, Heidelberg (2013). https://doi.org/10.1007/978-3-642-35980-4_2

45. Pechinkin, A.V., Razumchik, R.V.: Approach for analysis of finite $M_2|M_2|1|R$ with hysteric policy for SIP server hop-by-hop overload control. In: Proceedings - 27th European Conference on Modelling and Simulation, ECMS 2013, pp. 573–579 (2013)
46. Abaev, P., Razumchik, R.V.: Queuing model for SIP server hysteretic overload control with bursty traffic. In: Balandin, S., Andreev, S., Koucheryavy, Y. (eds.) NEW2AN/ruSMART -2013. LNCS, vol. 8121, pp. 383–396. Springer, Heidelberg (2013). https://doi.org/10.1007/978-3-642-40316-3_34
47. Pechinkin, A.V., Razumchik, R.V.: Stationary characteristics of $M_2|G|1|r$ system with hysteretic policy for arrival rate control. J. Commun. Technol. Electron. **58**(12), 1282–1291 (2013)
48. Gaidamaka, Y., Pechinkin, A., Razumchik, R., Samouylov, K., Sopin, E.: Analysis of an $M|G|1|R$ queue with batch arrivals and two hysteretic overload control policies. Int. J. Appl. Math. Comput. Sci. **24**(3), 519–534 (2014)
49. Samouylov, K.E., Abaev, P.O., Gaidamaka, Y.V., Pechinkin, A.V., Razumchik, R.V.: Analytical modelling and simulation for performance evaluation of SIP server with hysteretic overload control. In: Proceedings - 28th European Conference on Modelling and Simulation, ECMS 2014, pp. 603–609 (2014)
50. Gaidamaka, Y., Pechinkin, A., Razumchik, R.: Time-related stationary characteristics in queueing system with constant service time under hysteretic policy. In: International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, January 2015, 7002158, pp. 534–540 (2015)
51. Abaev, P., Khachko, A., Beschastny, V.: Queuing model for SIP server hysteretic overload control with K-state MMPP bursty traffic. In: International Congress on Ultra Modern Telecommunications and Control Systems and Workshops 2015-January, 7002151, pp. 495–500 (2015)
52. Hilt, V., Widjaja, I., Labs, B.: Controlling overload in networks of SIP servers. In: Proceedings - International Conference on Network Protocols, ICNP, art. no. 4697027, pp. 83–93 (2008)
53. Abdelal, A., Matragi, W.: Signal-based overload control for SIP servers. In: 7th IEEE Consumer Communications and Networking Conference, CCNC 2010, art. no. 5421642 (2010). https://doi.org/10.1109/CCNC.2010.5421642