Quantum Machine Learning

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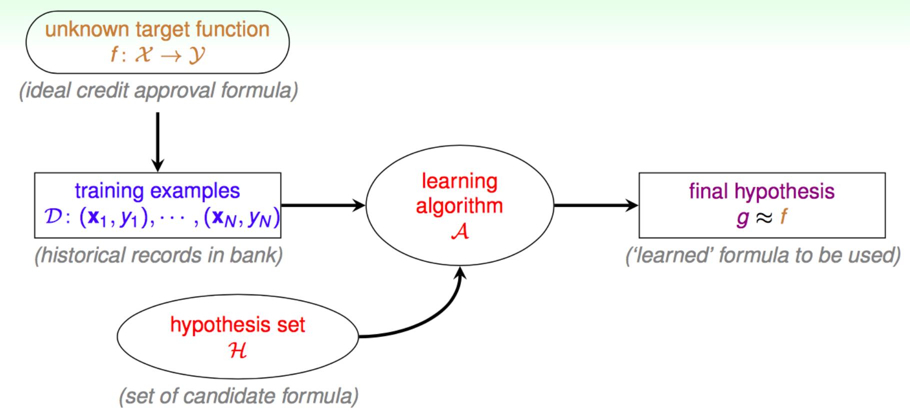
September 1, 2017

Outline

1. Machine Learning
   - ML model
   - complexity issues
2. Quantum Computation
   - Quantum Mechanics
   - Quantum Circuits
   - Quantum Power and Limitation
3. Quantum Machine Learning
   - Machine Learning with Quantum Resources
   - Quantum Mechanics with ML model

Goals of Machine Learning

learning model



Goals of Machine Learning

1. find target function with high accuracy and high probability
2. low time complexity
3. low sample complexity

High accuracy with high probability

Error

- Training error: $L_D(g) := \frac{\sum_{i=1}^{m} |g(x_i) - y_i|}{m}$
- True error: $L_U(g) := E_{(x,y)\sim U}[g(x) \neq y]$

ϵ-representative sample

$$\Pr_{D\sim U^m}[\forall h \in H, |L_D(h) - L_U(h)| \leq \epsilon] \geq 1 - \delta$$

PAC learnability

$$\Pr_{D\sim U^m}[L_U(g) - min_{h^* \in H} L_U(h^*) \leq \epsilon] \geq 1 - \delta$$

Low Time Complexity

- algorithm
- GPU[1]

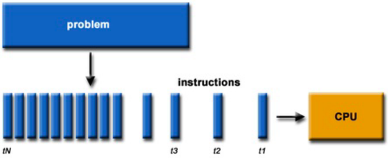


Figure: Serial processor

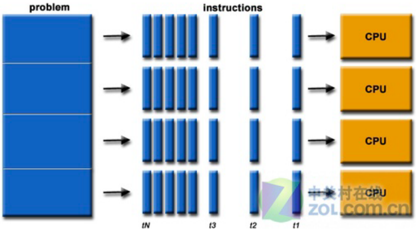


Figure: Parallel processor

- quantum computation

Sample Complexity

Error

- Training error: $L_D(g) := \frac{\sum_{i=1}^{m} |g(x_i) - y_i|}{m}$
- True error: $L_U(g) := E_{(x,y)\sim U}[g(x) \neq y]$

PAC learnability

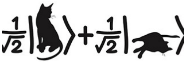
$$\Pr_{S\sim U^m}[L_U(g) - min_{h^*\in H} L_U(h^*) \leq \epsilon] \geq 1 - \delta$$

H is PAC learnable under ERM paradigm when

$$m \geq \lceil \frac{log(|H|/\delta)}{\epsilon^2} \rceil$$

Quantum Mechanics

- superposition state[2]



- measurement[3]

- result
  dead / alive

Quantum Circuit

Quantum bit[4]

- $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{bmatrix}1\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\end{bmatrix}$

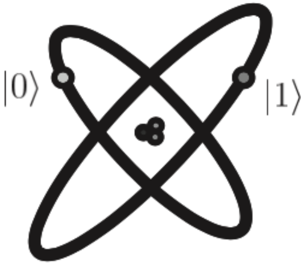


Figure: Qubit represented by two electronic levels in an atom.

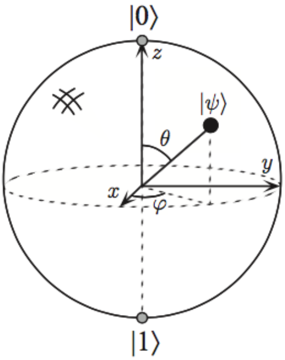
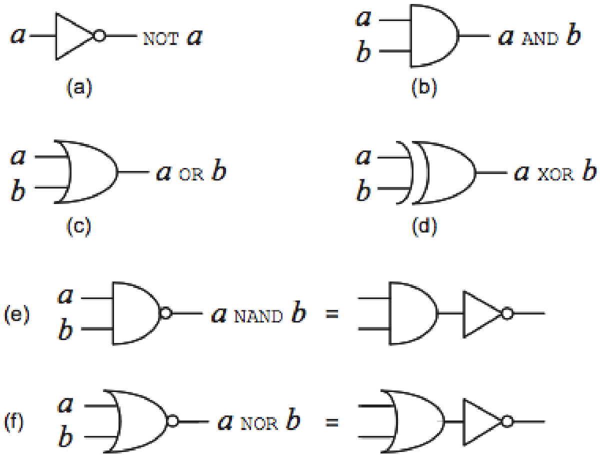


Figure: Bloch sphere representation of a qubit.

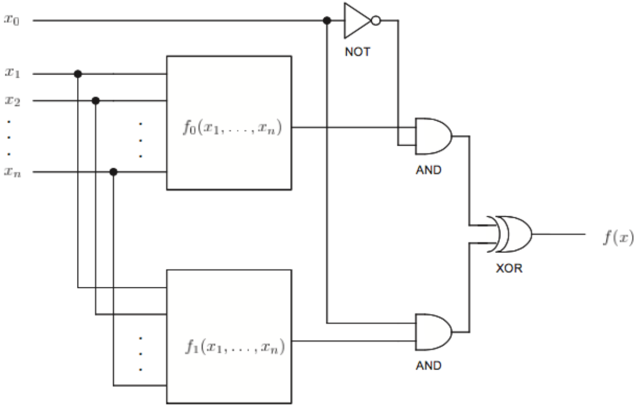
Quantum Circuit

- Classical gates[4]



Quantum Circuit

- Classical circuit[4]

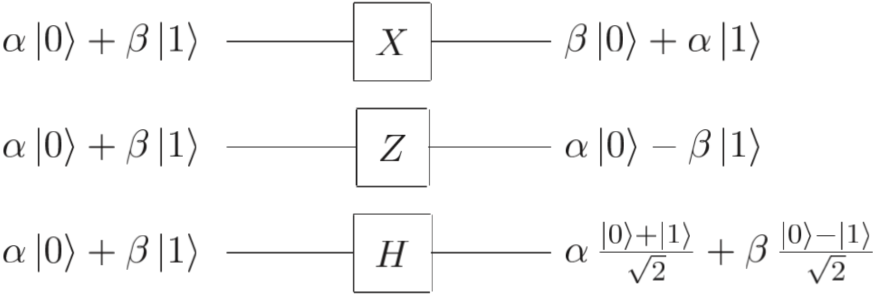


Quantum Circuit

- Quantum single qubit gates[4]

Hadamard	$-\boxed{H}-$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Pauli-X	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y	$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z	$-\boxed{Z}-$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Phase	$-\boxed{S}-$	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$	$-\boxed{T}-$	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$

- Quantum single qubit gates[4]



Quantum Circuit

- Quantum single qubit gates[4]

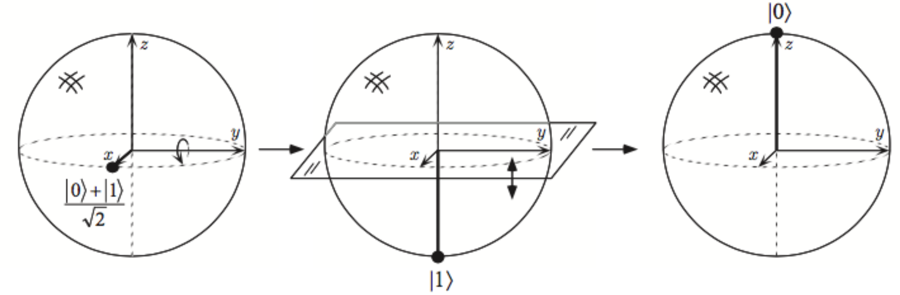


Figure: Hadamard gate on Block sphere

Quantum Circuit

- multi-qubit gates[4]
  - unitary
  - reversible

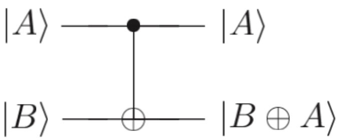


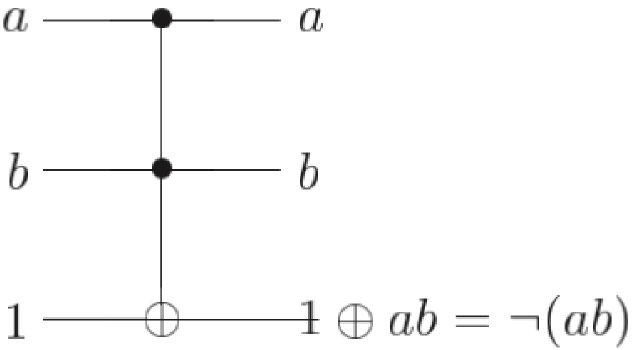
Figure: Controlled-not gate
$|c\rangle\,|t\rangle \rightarrow |c\rangle\,|c \oplus t\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure: matrix representation of a CNOT gate.

Quantum Circuit

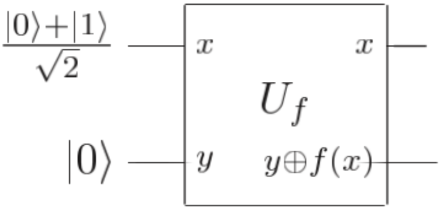
- Quantum NAND gates[4]



Quantum Parallelism

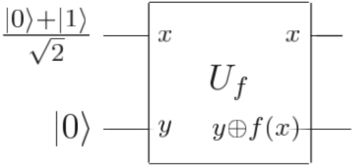
Function $f(x) : \{0, 1\} \rightarrow \{0, 1\}$

- Classical:
  $f(0)$, $f(1)$
- Quantum:[4]
  $|x\rangle |y\rangle \rightarrow |x\rangle |y \oplus f(x)\rangle$



Quantum Parallelism

$|x\rangle |y\rangle \rightarrow |x\rangle |y \oplus f(x)\rangle$



$$U_f\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle\right) = U_f\left(\frac{|0\rangle |0\rangle}{\sqrt{2}} + \frac{|1\rangle |0\rangle}{\sqrt{2}}\right) = \frac{|0\rangle |f(0)\rangle}{\sqrt{2}} + \frac{|1\rangle |f(1)\rangle}{\sqrt{2}}$$

Quantum Parallelism

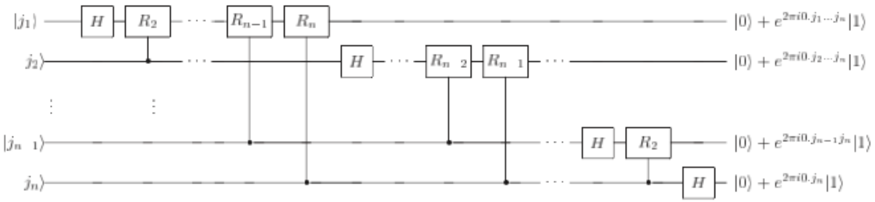
If we have 2 qubits

$$(\frac{|0\rangle + |1\rangle}{\sqrt{2}}) \otimes (\frac{|0\rangle + |1\rangle}{\sqrt{2}}) = \frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{\sqrt{2}^2}$$

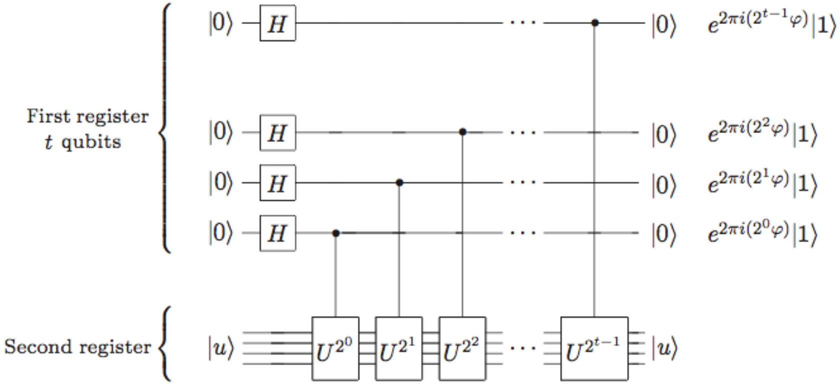
$$\Rightarrow U_f(\frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{\sqrt{2}^2} \otimes |0\rangle)$$

$$= \frac{1}{2}[|00\rangle |f(00)\rangle + |01\rangle |f(01)\rangle + |10\rangle |f(10)\rangle + |11\rangle |f(11)\rangle]$$

- Fourier transform
  $$|j\rangle \to \sum_{k=0}^{2^n-1} e^{2\pi ijk/(2^n)}\,|k\rangle$$



- Quantum Phase Estimation
  Goal: find ϕ of $U|\psi\rangle = e^{2\pi i\phi}|\psi\rangle$



Quantum Limitation

Measurement

Get 1 information per measurement (as classical)

State preparation

- classical data to quantum state: by QRAM
- directly prepare quantum state

Famous Quantum Algorithms

Shor's algorithm

Factoring $n = p \times q$, where p and q are prime numbers
time: exponential speed-up

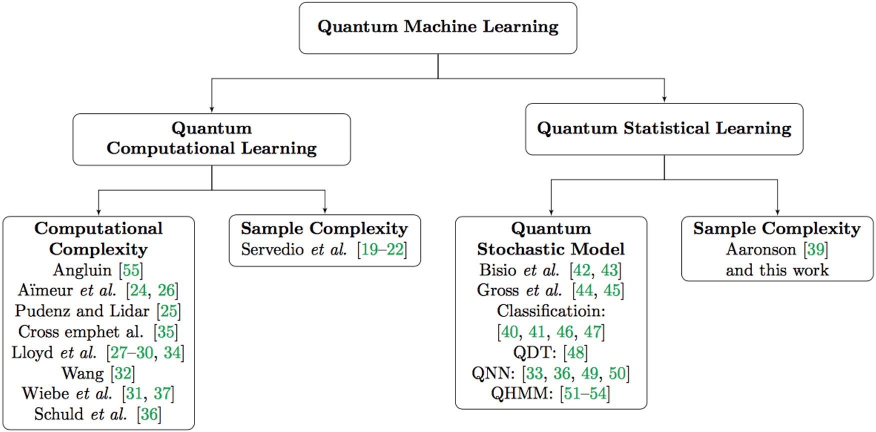
Grover search algorithm

Find a unique data
time: quadratic speed-up

HHL algorithm

Solving linear systems of equations $Ax = b$
time: $O(log(N)\kappa^2 s^2/\epsilon)$ for positive semidefinite matrices
(best classical algorithm: $O(N^{2.373})$)

Quantum Machine Learning



Grover's search for machine learning

Grover's search

1. clustering via minimum spanning tree
2. divisive clustering
3. k-medians

Grover's search + other Q. algorithm

1. k-means clustering
2. quantum recommendation systems
3. quantum neural networks

HHL for machine learning

solving linear system of equations

Scaling	Applications
C: $\tilde{\mathcal{O}}(s\kappa N \log(1/\epsilon))$ [She94][a]	Least-square-SVM [RML14]
Q: $\tilde{\mathcal{O}}(s^2\kappa^2 \log(N)/\epsilon)$ [HHL09][b]	GP Regression [ZFF15]
P: $\mathcal{O}(\log^2(N)\log(1/\epsilon))$ [Csa76][c]	Kernel Least Squares [SSP16]

Limitation

1. matrix is sparse
2. classical data be loaded in quantum state in $O(log(N))$
3. write down $x = (x_1, x_2, \cdots, x_n)$ requires n steps
4. condition number (κ) is at most sublinearly with N

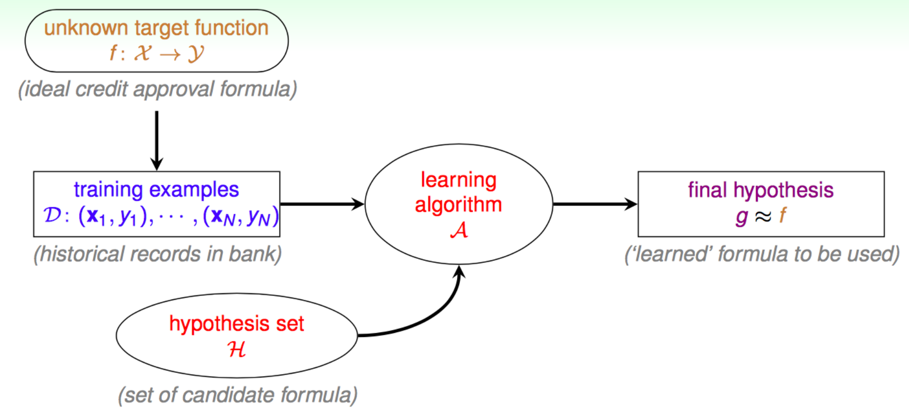
Other possible works and hardness

1. Possible Work:
   - Quantum Neural Network
   - optimization
2. Hardness:
   - Nonlinear functions in NN
   - QRAM (not clear now)
   - learn DNF efficiently from quantum examples under uniform distribution
   - no significant benefit to sample complexity
   - write down output

- Quantum State/Measurement Tomography
  Number of measurement to determine a n-particle system grows exponentially in n.
  Ex. requires 6561000 measurements to reconstruct an entangled state of 8 Ca

Quantum statistical learning

learning model



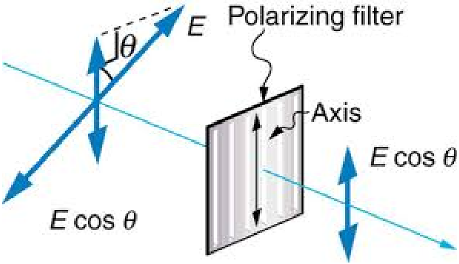
Quantum v.s. Classical

Quantum	v.s.	**Classical**
measurement $E = \{E_1, \cdots E_m\}$	sample	$D = \{x_1, \cdots x_m\}$
state ρ (n-qubit system)	target function	f
$\sigma \approx \rho$	hypothesis	$g \approx f$
$\{0,1\}^m$	label	$\{0,1\}^m$

Table: Quantum v.s. Classical model

Basis:

- $|0\rangle$, $|1\rangle$
- $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, $|-\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}$



- Unknown state ρ is a n-qubit state. (Can be represented by $2^n \times 2^n$ PSD matrix with $Tr(\rho) = 1$)
- A set of two-outcome ($\{0, 1\}$) measurements: $\{E_1, E_2, \cdots\}$

$$\begin{cases} \text{output 1 with probability } Tr(E_i\rho) \\ \text{output 0 with probability } 1 - Tr(E_i\rho) \end{cases}$$

Quantum state tomography

Theorem

Let ρ be a n-qubit mixed state, let $E = \{E_1, \cdots E_m\}$ be a "training set" of two outcome measurements drawn independently from a set D. Also, fix $\epsilon, \eta, \gamma > 0$ with $\gamma\epsilon \geq 7\eta$. E is a "good" training set if any hypothesis $\sigma \in H$ satisfies

$$|Tr(E_i\sigma) - Tr(E_i\rho)| \leq \eta, \forall E_i \in E$$

also satisfies

$$Pr_{E \in D}[Tr(E\sigma) - Tr(E\rho)| \geq \gamma] \leq \epsilon$$

Then there exists a constant $K > 0$ such that E is a good training set with probability at least $1 - \delta$, provided that

$$m \geq \frac{K}{\gamma^2\epsilon^2}(\frac{n}{\gamma^2\epsilon^2} log^2(\frac{1}{(\gamma\eta)}) + log\frac{1}{\delta})$$

References

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The End