Piglet with value iteration

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$P_{000} = \frac{4}{7} =$ $P_{001} = \frac{5}{7}$ $P_{010} = \frac{2}{5}$

$P_{011} = \frac{3}{5}$ $P_{100} = \frac{4}{5}$ $P_{110} = \frac{2}{3}$

$S = (i, j, k)$

$A = \{roll, hold\}$

$i \rightarrow$ players score

$j \rightarrow$ opponent score

$k \rightarrow$ turn score

Transitions:

if roll:

if heads:

$i, j, k \rightarrow i, j, k+1 \;/\; W, L, 0$ ½

if tails:

$i, j, k \rightarrow j, i, 0$ ½

if hold:

$i, j, k \rightarrow j, i+k, 0$ 1

Reward

if $i, j, k \rightarrow W, L, 0$ +1

otherwise 0

$$P_{i,j,k} = \max \left\{ \underbrace{\frac{1}{\text{die size}} \left[(1 - P_{j,i,0}) + P_{i,j,k+1} \right]}_{\text{roll}}, \underbrace{1 - P_{j,i+k,0}}_{\text{hold}} \right\}$$

$P_{000} = 0.25$

$P_{001} = 0.625$

$P_{010} = 0.5$

$P_{011} = 0.875$

$P_{100} = 0.75$

$P_{011} = 0.875$

$P_{100} = 0.75$

$P_{110} = 0.75$

	iteration 1:	iteration 2	iteration 3	iteration 4	iteration 5
P_{000}	$\max\{\frac{1}{2}, 1\} = 1$	$\max\{\frac{1}{2}, 0\} = \frac{1}{2}$	$\max\{\frac{1}{2}, \frac{1}{2}\} = \frac{1}{2}$	$\max\{\frac{5}{8}, \frac{1}{2}\} = \frac{5}{8}$	$\max\{\frac{9}{16}, \frac{3}{8}\} = \frac{9}{16}$
P_{001}	$\max\{1, 1\} = 1$	$\max\{\frac{1}{2}, 0\} = \frac{1}{2}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{3}{4}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{3}{4}$	$\max\{\frac{11}{16}, \frac{1}{2}\} = \frac{11}{16}$
P_{010}	$\max\{\frac{1}{2}, 1\} = 1$	$\max\{\frac{1}{2}, 0\} = \frac{1}{2}$	$\max\{\frac{1}{2}, \frac{1}{2}\} = \frac{1}{2}$	$\max\{\frac{1}{2}, \frac{1}{2}\} = \frac{1}{2}$	$\max\{\frac{1}{2}, \frac{1}{4}\} = \frac{1}{2}$
P_{011}	$\max\{1, 1\} = 1$	$\max\{\frac{1}{2}, 0\} = \frac{1}{2}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{1}{2}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{3}{4}$	$\max\{\frac{5}{8}, \frac{1}{2}\} = \frac{5}{8}$
P_{100}	$\max\{1, 1\} = 1$	$\max\{\frac{1}{2}, 0\} = \frac{1}{2}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{1}{2}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{3}{4}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{3}{4}$
P_{110}	$\max\{1, 1\} = 1$	$\max\{\frac{1}{2}, 0\} = \frac{1}{2}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{1}{2}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{1}{2}$	$\max\{\frac{3}{4}, \frac{1}{2}\} = \frac{3}{4}$