Traffic

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Industrial Mathematics Project 2

Executive Summary

Characterised by slower speeds, longer trip times and traffic jams, congestion plays a major role in determining the efficiency of road networks. The Perthshire Road Agency asked us to investigate the optimal implementation of speed and traffic flow controls. We applied this to the real world to investigate the notorious Broxden Junction.

We initially investigated speed limits using a simple cellular automation model. We then modelled our system using the Intelligent Driver Model as per industry standard. We used a variety of measures to investigate minimising traffic congestion, including the speed performance index, the proportion of time spent at crawling speeds and a measure of traffic flow.

We found a number of interesting results, namely (1) when speed limits are too low, the likelihood of congestion increases, (2) congestion positively correlates with the number of vehicles on the road and (3) the implementation of traffic lights at the Broxden Junction could be beneficial.

Traffic congestion is clearly a very complex issue with numerous assumptions needing to be made in order to simplify models; these compromise the validity of applications to the real-world. Future research should build upon our current model. This could involve further investigation into the behaviour of drivers in approach to roundabouts. In addition, implementing different types of vehicles and multiple lanes could allow for interesting investigation into optimal paths for journeys.

1 Evaluating Traffic Flow

Before introducing our models to investigate congestion, we must first explicitly define congestion and the appropriate measures for minimising it. We define congestion as periods characterised by slower speeds and increased vehicular queuing. We use the Speed Performance Index (SPI) defined as $SPI = (v_{\text{avg}}/v_{\text{max}}) \cdot 100$ (Afrin & Yodo, 2020), where a value of < 50 represents congestion and < 25 heavy congestion. We also use the proportion of time spent at crawling speeds and a measure of traffic flow, i.e. cars entering versus cars exiting.

2 Simple Model

Initially, we use the Agent Based Model as described in (Nagel & Schreckenberg, 1992). In this simple model, our agents behaviour (each vehicle) is dictated by the following actions:

1. **Acceleration:** If the velocity v of a vehicle is lower than v_{max} and the distance to the vehicle ahead is larger than $v + 1$, we increase the speed $v \to v + 1$.
2. **Deceleration:** If a vehicle at site i sees the next vehicle at site $i + j$ with $j \leq v$, they reduce their speed $v \to j - 1$.
3. **Randomisation:** With probability p, the vehicle's velocity is decreased by one to a minimum of zero, $v \to \max(v - 1, 0)$.

2.1 Variable Speed-Limits

We now investigate the formation of traffic jams due to changes in speed-limit. Introduced in the early 1970s in Germany, the effects of variable speed limits on traffic are still not fully understood (Soriguera et al., 2017). Generally implemented to provide greater road safety, they can result in congestion - typically suggested to be due to over-breaking (Stoelhorst, 2008). The length of the road is taken to be 200 units, and cars at the end of the road are then reintroduced at the start where one car occupies precisely one unit. We take $p = 0.7$ as seen in (Liu, 2012). We hold the speed limit constant at 5 units per frame (upf). Each simulation was run for 100 frames. Figure 1 shows the results under a closed system.

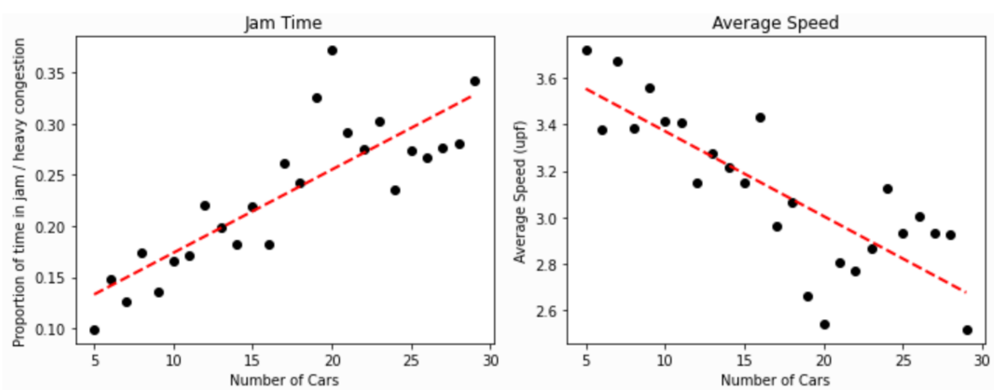

Figure 1: Cellular Automation: Variable Speed Limits.

We define time spent in a jam / heavy congestion as being time spent at speeds < 2 upf. We also find an increase in the number of cars in our closed system is highly correlated (Pearson correlation coefficient of 0.966) with an increase to the proportion of time spent in jams.

We now investigate the effects of a change in speed limits to the formation of traffic jams using a closed system. Cars reaching the end of the road will be re-introduced at the start, ensuring a constant number of cars remain in the system, which we take to be 20. We note that we have not altered the randomisation for each speed limit, it is still as described in (Nagel & Schreckenberg, 1992). Thus, the random behaviour of drivers will be comparatively less extreme as speed limits increase.

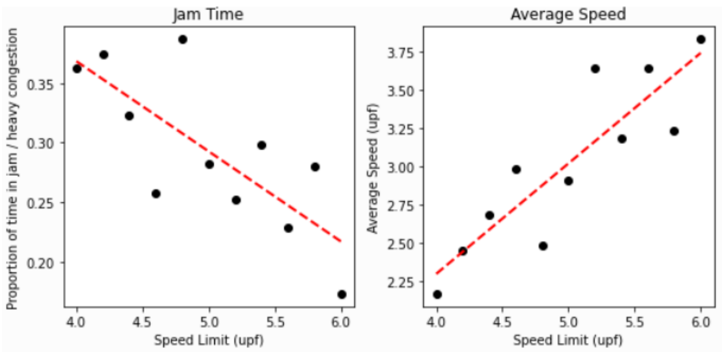

Figure 2: Variable Speed Limits.

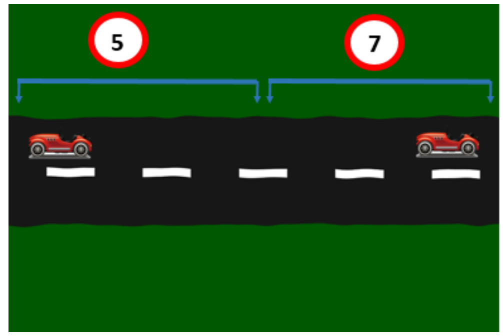

Figure 3: Road.

Figure 2 indicates the importance of setting proper speed limits. As our speed limit increases, we see that the proportion of time spent in jams decreases. We also see an increase to the average speed, and this is a clear indication that lower speed limits result in greater congestion.

We also investigate the congestion in regions after a change in speed limits. We introduce a change in speed limit at the midpoint of the road as seen in Figure 3. Figure 4 shows that in both regions the average speed decreases as we increase the number of cars, this difference is greater proportionally at lower speeds. We propose this difference to be the result of over breaking as cars enter the slower region. Investigation into the speed performance index is shown in Figure 5, we note that congestion (SPI < 50) appears to be avoidable for the chosen flow if speed limits are set appropriately in our closed system.

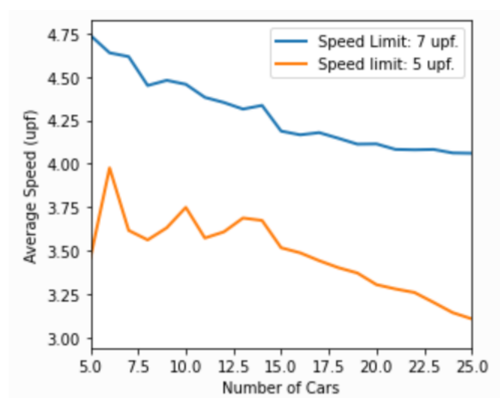

Figure 4: Change in Speed Limits.

We understand that we must be careful when implementing speed limits to our full model. We made over simplistic assumptions about the driver behaviour, most notably they wish to travel at the speed limit. This is now dealt with by implementing real world data to create a more advanced model.

Speed Limit (upf)	SPI Value
4	49.9
5	55.2
6	63.4

Figure 5: SPI Values.

3 Full Model

The Perthshire Road Agency asked us to investigate the implementation of our model into the real world. To do this, we investigate the renowned Broxden Junction. We model the four roads leading into the junction - working clockwise from the northern exit which is the A9, then to A93, M90 and the A9. This is seen in Figure 6.

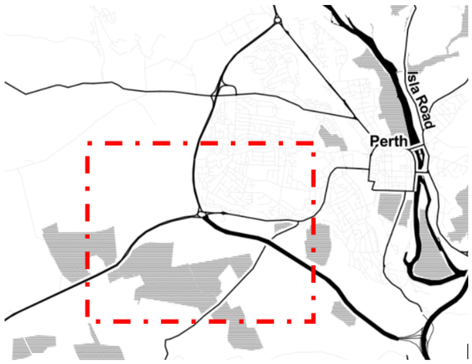

Figure 6: Our Region (Stamen Design, 2021).

Road	Speed Limit
A9	70 mph
M90	70mph
A93	60 mph

Figure 7: Limits.

Road	Average Speeds
A9	67mph
M90	67mph
A93	47 mph

Figure 8: Speeds.

The speed limits for the respective roads are seen in Figure 7 (Government Digital Service, 2015). Our junction speed limit is taken to be the minimum of the roads entering it (Transport Scotland, 2012), which we take to be 60 mph. Nationally, the average observed speed limits on dual carriageways has been found to be 67 mph, this decreases to 47 mph on single carriageways (Department for Transport, 2018), shown in Figure 8. We account for this to model more realistic driving. We assume drivers maintain a 2 second stopping distance (*The AA*, 2021). We also introduce speeding, incorporating data from (Department for Transport, 2020) to model our distribution of car speeds as:

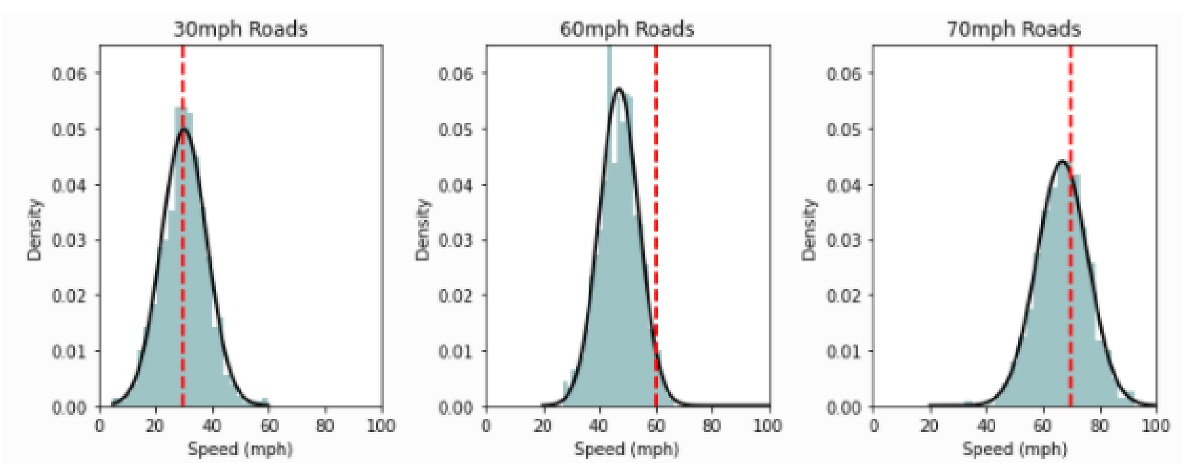

Figure 9: Speeding Distributions: the red dotted line represents the speed limit.

We assume faster drivers will likely have higher levels of maximum vehicle acceleration (Semin & Fiedler, 1986). We assume this is also true for the the comfortable breaking deceleration. We take the maximum vehicle acceleration as 0.73m/s^2 and comfortable deceleration as 1.67m/s^2 (Bokare & Maurya, 2017).

3.1 Intelligent Driver Model

Developed in 2000, the Intelligent Driver Model (IDM) simulates realistic traffic behaviour (Treiber et al., 2000). Unlike the simple cellular automation model previously investigated, car following models like the IDM model car behaviour as a reaction to the behaviour of the car in front, and more realistically simulate driver behaviour. The dynamics of our vehicle are dictated by the ODEs:

$$\dot{x}_\alpha = \frac{dx_\alpha}{dt} = v_\alpha \qquad \text{and} \qquad \dot{v}_\alpha = \frac{dv_\alpha}{dt} = a\Big(1 - \big(\frac{v_\alpha}{v_0}\big)^\delta - \big(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha}\big)^2\Big) \tag{1.1}$$

We define $s^*(v_\alpha, \Delta v_\alpha) = s_0 + v_\alpha T + v_\alpha \Delta v_\alpha / 2\sqrt{ab}$. s_0 represents the minimum desired distance to the car in front, taken to be 2 meters (Ahmad et al., 2015). T is the minimum desired interval to the car in front, taken to be 2 seconds (*The AA*, 2021). v_0 represents the car's desired velocity and is randomly sampled from the distribution in Figure 9. (Treiber & Kesting, 2013). We take δ is taken to be 4 (Treiber et al., 2000). s_α is the distance to the car in front, and is calculated dynamically based on driver positions (as, depending on driver interaction, the car order will not stay constant throughout a simulation).

We now investigate the Broxden Roundabout. Roundabouts are an efficient way to allow traffic to flow through a system (Transport Scotland, 2020). As traffic builds up, the delay from cars waiting at the entrance of a roundabout for a pathway to take means that there is a limit to the traffic that a roundabout can support (safely, that is — we recommend further research to investigate the ethics of prioritising roundabout throughput over human safety). Our initial implementation of a one lane Broxden Junction is seen in Figure 10 (the dashed white lane markers are purely poetic license — this is a one lane model).

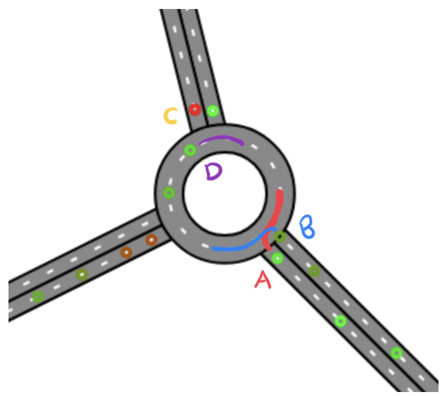

Figure 10: Broxden Junction.

The colour of a car represents its' individual congestion. This is seen in Figure 11. Looking at car C in Figure 10, we can see it pulling to a complete stop at the roundabout as it waits for D to continue. As D is not directly in front of C, our vehicle is looking back around the roundabout. We take this maximal "look-back" angle to be 90 degrees based on an average from a paper on the design of roundabouts (Mussone, 2013). This ensures the cars in our model follow the UK's 'right of way' system.

Colour	Interpretation
(light green)	Car is very close to desired speed, v_0.
(green)	Car is close to desired speed, v_0.
(red)	Car is experiencing heavy congestion.

Figure 11: Colour Table.

The Broxden junction is able to support traffic levels up to $0.4 - 0.5$ cars per second (spread across the three incoming roads), before insurmountable tailbacks begin to form. As incoming traffic is (in our model, as in reality) randomly distributed across the three lanes, it is common for some roads to become far more congested than others at the same roundabout (see the third figure below for an example of this situation). We compare the introduction of traffic lights via two models: one with 2 states (interior lights green and roundabout link roads green); and with 3 states (this is harder to describe, but can be seen in Figure 12, and cycles through turning each link road green in turn). A full light cycle is ≈ 2 minutes and we note the length of the light cycle does not change the results significantly (2021).

For both traffic light setups, the maximum traffic flow was on the order of 0.2 cars per second; far below the peaks achieved by the system with no traffic lights. However, both traffic light systems were better able to share the load of incoming link roads, and keep the traffic flowing more smoothly. Interestingly, although the second diagram hews closely to how real-world roundabout traffic lights are currently timed, the first diagram achieves slightly better throughput (a peak of $18 - 19$ cars per second compared to a peak of $17 - 18$), and should be considered by Perthshire Road Agency.

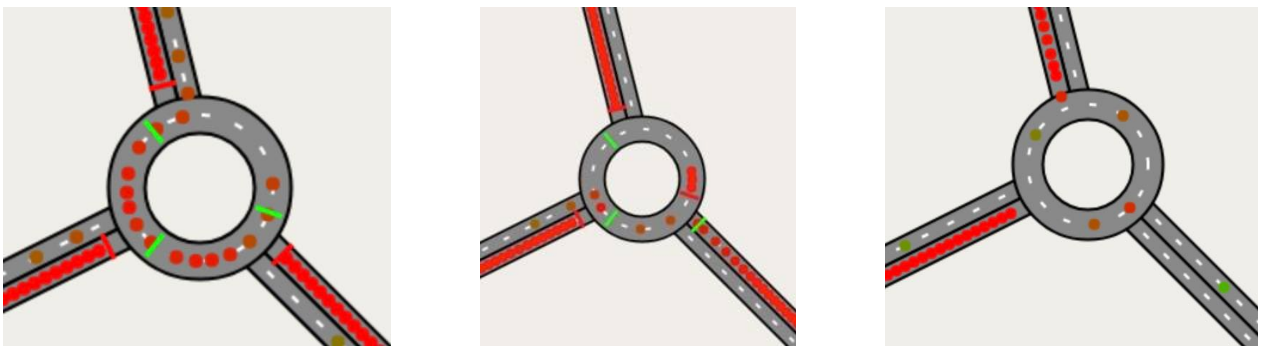

Figure 12: From left to right Broxden Roundabout with the 2-state traffic light setup, 3-state traffic light setup and no traffic lights.

4 Conclusions & Future Research

Traffic congestion is a complicated issue. With countless factors left unaccounted for, producing a realistic model is difficult. The introduction of traffic lights allows for better regularity of flow, at the cost of best-case journey time. Necessary caution should be taken when introducing speed limits, as these have a big impact on both average speeds and the formation of traffic jams.

Future research should look to build upon our model. We do not introduce pedestrians, thus a variety of areas remain simplistic. More research on driver behaviour would also be beneficial, especially concerning the decision factors in entering the roundabout. Although we made significant progress towards extending our model of Broxden Roundabout to the entirety of Perth's roads using OSM data (Stamen Design, 2021), this could not be finished within the time constraints. Further development will allow modelling the complex interplay between different road elements in a much more realistic way. This will allow for more holistic modelling across Perth, with many applications. As just one example, Perthshire Road Agency would be able to investigate optimal bus routes in order to reduce costs.

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