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Clustering of Cars in Cellular Automaton Model of Freeway Traffic

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A cellular automaton (CA) model is presented to simulate the clustering (or aggregation) process of cars in traffic flow on a highway. The CA model is an extended version of the one-dimensional asymmetric simple-exclusion model taking into account the variation of car velocity which depends on the distance between a car and the car ahead. Using computer simulation, it is found that clustering (or aggregation) of cars occurs in our model. The mean distance $\langle l \rangle$ between cars scales as $\langle l \rangle \approx t^{0.37\pm0.02}$ where t is time. Similarly, the mean cluster size $\langle s \rangle$ of cars scales as $\langle s \rangle \approx t^{0.37\pm0.02}$. It is shown that the cumulative cluster-size distribution N_s is not a power-law type but an exponential function $N_s \approx \langle s \rangle^{-1}\, e^{-1.72s/\langle s \rangle}$.

[traffic flow, cellular automaton, aggregation, scaling, asymmetric exclusion model]

Recently, traffic problems have attracted considerable attention.[1,2] Traffic simulations based on various hydrodynamic models have provided much insight. However, the simulation of traffic flow is a formidable task since it involves many degrees of freedom. Cellular automaton (CA) models are being applied successfully to simulations of complex physical systems.[3,4]

The one-dimensional (1D) asymmetric simple-exclusion model can be formulated into traffic flow problems. The 1D exclusion model is one of the simplest examples of a driven system.[5,6] The model has been extensively studied to understand systems of interacting particles.[7,8] The 1D exclusion model is used to study the microscopic structure of shocks[9] and is closely linked to growth processes.[10] Very recently, Biham *et al.*[11] applied the two-dimensional asymmetric exclusion model to the traffic-jam problem.

Nagel and Schreckenberg[12] extended the 1D asymmetric exclusion model to take into account car velocity in order to simulate freeway traffic. They showed that a transition from laminar traffic flow to start-stop waves occurs with increasing car density, as is observed in actual freeway traffic. Musha[13] has found that traffic flow on a highway shows a $1/f$ power spectrum, by directly measuring the traffic flow on an actual highway. He observed that cars flowing on a highway cluster more and more as they progress. He suggested that the traffic flow is described by the Burgers equation. Lighthill and Whithgam[14] were the first to propose that a traffic jam is described by the Burgers equation. The Burgers equation is derived from the 1D asymmetric exclusion model using appropriate coarse-graining.[15] The 1D asymmetric exclusion model is a microscopic model of the Burgers equation. The velocity field of the Burgers one-dimensional model of turbulence at extremely large Reynolds numbers is expressed as a train of random triangular shock waves. It has been shown that the number of shock fronts decreases with time as $t^{-\alpha}$ $(0<\alpha<1)$, and consequently, the mean interval increases as t^{α}.[16] However, a clustering of cars never occurs in the 1D asymmetric exclusion model. Aggregation of cars has not appeared in the CA models of traffic problems proposed until now. Recently, there has been increasing interest in a variety of aggregation phenomena.[17] The aggregation of cars is an attractive problem.

In this letter, we present a CA model of freeway traffic showing a clustering of cars. We extend the 1D asymmetric simple-exclusion model to take into account variation of car velocity. Using computer simulation, we calculate the traffic flow for various car densities. We study the scaling behavior of the aggregation (clustering) process of cars.

Our CA model is defined on a one-dimensional lattice of L sites with periodic boundary conditions. Each site is occupied by one car or it is empty. We extend the one-dimensional asymmetric exclusion model to take into account the dependence of car velocity on the distance between the car and the car ahead. In our model, the car velocity v depends only on the distance between the car and the car ahead. For an arbitrary configuration, one update of the system consists of the following rule which is performed in parallel for all cars. When the distance l between a car and the car ahead is longer than the critical distance $r_{\max}$, the car moves ahead by one step with probability p_{a1} and the car does not move ahead with probability $1-p_{a1}$. If the distance l between a car and the car ahead is equal to or less than the critical distance $r_{\max}$, the car moves ahead by one step with probability p_{a2} and does not move ahead with probability $1-p_{a2}$. Then, the car velocity is given by

$$\begin{aligned} v&=1 \quad \text{with probability } p_{a1} \\ v&=0 \quad \text{with probability } 1-p_{a1} \\ &\quad\ \text{for} \quad l>r_{\max}, \end{aligned}$$

and

$$\begin{aligned} v&=1 \quad \text{with probability } p_{a2} \\ v&=0 \quad \text{with probability } 1-p_{a2} \\ &\quad\ \text{for} \quad l\leq r_{\max}. \end{aligned} \tag{1}$$

In the case of $p_{a1}<p_{a2}$, a car with $l>r_{\max}$ moves slower than the car with $l\leq r_{\max}$. In this case, a clustering of cars occurs. If $p_{a1}\geq p_{a2}$, clustering does not occur. In this model, natural velocity fluctuations due to human behavior are taken into account in the parameters $r_{\max}$, p_{a1} and p_{a2}. At coarse-grained time scales, the velocity of cars with $l>r_{\max}$ is proportional to the probablity p_{a1} and the velocity of cars with $l\leq r_{\max}$ is proportional to the probability p_{a2}. In the limit of $p_{a1}=p_{a2}=1$, our model reproduces the 1D asymmetric simple-exclusion model. In actual traffic flow, even if the above condition for the occurrence of clustering is satisfied, the traffic jam may not occur because a car may pass the car ahead.

We have performed simulations with the CA model starting with an ensemble of random initial conditions where the system size is $L=6000$, the initial density of cars is $p=0.0$–0.5, and the critical distance is $r_{\max}=2$–10. Bach run is calculated up to 10^5 time steps. The data are averaged over 50 runs. For illustration, Fig. 1 shows a typical pattern of cars for the car density $p=0.1$, the critical distance $r_{\max}=2$, and the probabilities $p_{a1}=0.5$ and $p_{a2}=1.0$ up to 500 time steps where the system size is $L=300$. The vertical direction indicates that in which cars move ahead. The horizontal direction is that of time. A car is indicated by a dot. The trajectory of a car is indicated by a curve. Clustering of cars occurs more and more with increasing time. The cluster of cars becomes larger. We find that clustering of cars occurs if $p_{a1}<p_{a2}$. We study the scaling behavior of the mean cluster size of cars. A cluster of cars is defined as a group of cars for which the distance between cars is within the critical distance $r_{\max}$. We also define the mean cluster size $\langle s\rangle$ of cars as

$$\langle s\rangle\equiv\frac{\sum_{s=1}^{\infty} s^2 n_s}{\sum_{s=1}^{\infty} s n_s}, \tag{2}$$

where s is the number of cars within a cluster and n_s is the cluster-size distribution. Figure 2 shows the log-log plot of the mean cluster size $\langle s\rangle$ against time t for the car densities $p=0.1$, 0.2 and 0.3 where $r_{\max}=2$, $p_{a1}=0.5$ and

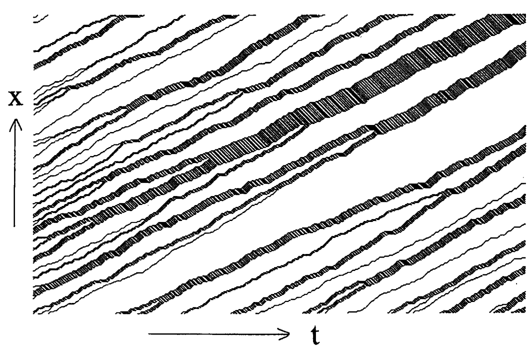

Fig. 1. Typical pattern of car configuration for the car density $p=0.1$ up to 500 time steps, where the system size is $L=300$, the critical distance $r_{\max}=2$, and the probabilities $p_{a1}=0.5$ and $p_{a2}=1.0$. The vertical and horizontal directions indicate respectively space and time. The trajectory of a car is indicated by a curve.

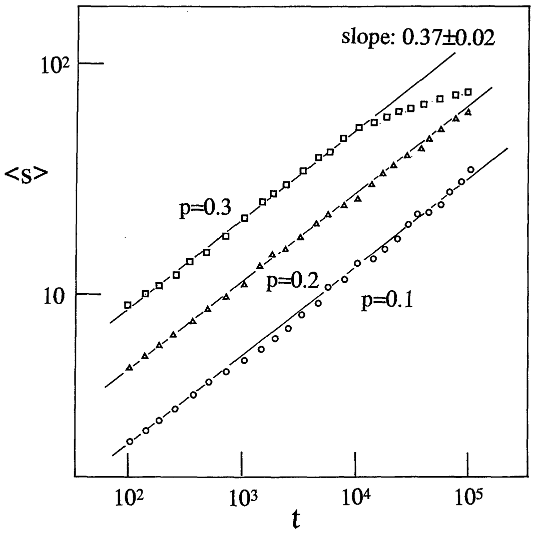

Fig. 2. The log-log plot of the mean cluster size $\langle s\rangle$ against time t for the car densities $p=0.1$, 0.2 and 0.3 where $r_{\max}=2$, $p_{a1}=0.5$ and $p_{a2}=1.0$.

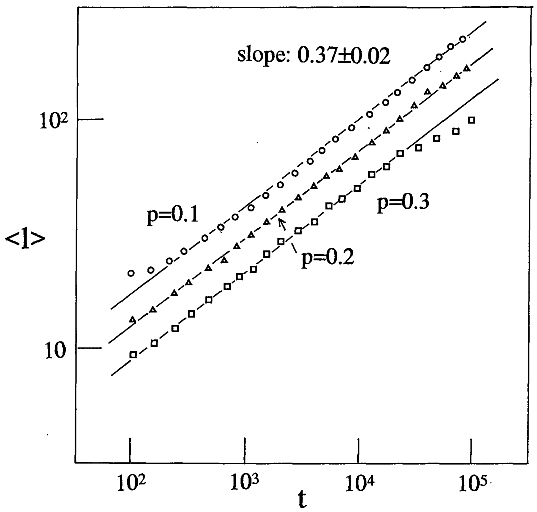

Fig. 3. The log-log plot of the mean distance $\langle l\rangle$ between a car and the car ahead against time t for the car densities $p=0.1$, 0.2 and 0.3 where $r_{\max}=2$, $p_{a1}=0.5$ and $p_{a2}=1.0$.

$p_{a2}=1.0$. For density lower than $p=0.3$, the mean cluster size $\langle s\rangle$ scales as

$$\langle s\rangle \approx t^{0.37\pm0.02}. \tag{3}$$

Similarly, we define the mean distance $\langle l\rangle$ between the car and the car ahead as

$$\langle l\rangle \equiv \frac{\sum_{l=1}^{\infty} l^2 n_l}{\sum_{l=1}^{\infty} l n_l}, \tag{4}$$

where l is the distance between a car and the car ahead and n_l is the distribution of the distance. Figure 3 shows the log-log plot of the mean distance $\langle l\rangle$ against time t for the car densities $p=0.1$, 0.2 and 0.3 where $r_{\max}=2$, $p_{a1}=0.5$ and $p_{a2}=1.0$. For density lower than $p=0.3$, the mean distance $\langle l\rangle$ scales as

$$\langle l\rangle \approx t^{0.37\pm0.02}. \tag{5}$$

The exponent of the mean distance $\langle l\rangle$ agrees with that of the mean cluster size $\langle s\rangle$. We study the dependence of the mean cluster size $\langle s\rangle$ upon the probability p_{a1}. Figure 4 shows the log-log plot of the mean cluster size $\langle s\rangle$ against time t for $p_{a1}=0.7$, 0.5 and 0.3 where $r_{\max}=2$, $p_{a2}=1.0$ and $p=0.1$. It is found that the scaling behavior of the mean cluster size $\langle s\rangle$ does not depend on the probability p_{a1} if $p_{a1}<p_{a2}$. We study the dependence of the mean cluster size $\langle s\rangle$ upon the critical distance $r_{\max}$. Figure 5 shows the log-log plot of the mean cluster size $\langle s\rangle$ against time t for $r_{\max}=2$, 5 and 10 where $p_{a1}=0.5$, $p_{a2}=1.0$ and $p=0.1$. For values of $r_{\max}$ larger than 10, the mean cluster size $\langle s\rangle$ increases slowly with time t at early stages. At late stages, the mean cluster size $\langle s\rangle$ scales as the scaling eq. (3). The scaling behavior of the mean cluster size $\langle s\rangle$ is little dependent upon the critical distance $r_{\max}$ after many time steps. Figure 6 shows the semi-log plot of the rescaled cumulative cluster-size distribution $t^{0.37}N_s$ against the res-

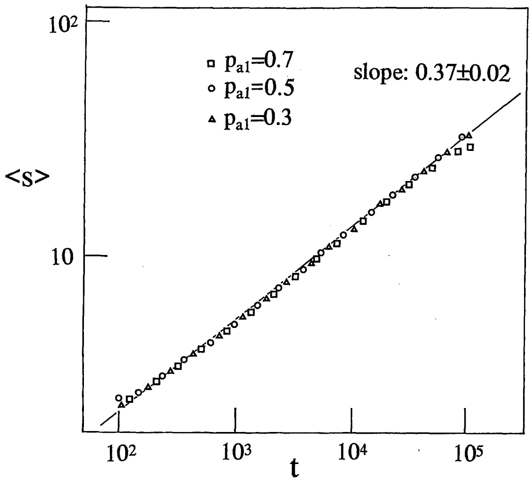

Fig. 4. The log-log plot of the mean cluster size $\langle s\rangle$ against time t for $p_{a1}=0.7$, 0.5 and 0.3 where $r_{\max}=2$; $p_{a2}=1.0$ and $p=0.1$.

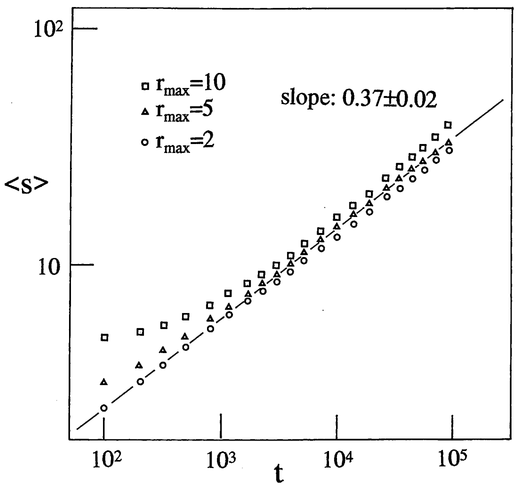

Fig. 5. The log-log plot of the mean cluster size $\langle s \rangle$ against time t for $r_{\max}=2$, 5 and 10 where $p_{a1}=0.5$, $p_{a2}=1.0$ and $p=0.1$.

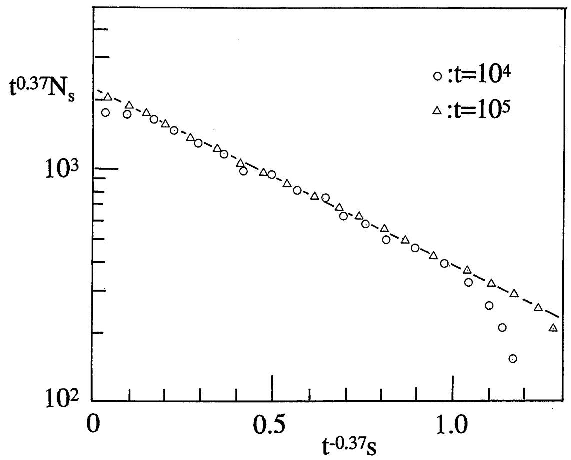

Fig. 6. The semi-log plot of the rescaled cumulative cluster-size distribution $t^{0.37}N_s$ against the rescaled size $t^{-0.37}s$ for $t=10^4$ and 10^5 where $r_{\max}=2$, $p=0.2$, $p_{a1}=0.5$ and $p_{a2}=1.0$.

caled size $t^{-0.37}s$ for $t=10^4$ and 10^5 where $r_{\max}=2$, $p=0.2$, $p_{a1}=0.5$, $p_{a2}=1.0$ and the cumulative cluster-size distribution is defined as $N_s=\sum_{s'=s}^{\infty} n_{s'}$. For many time steps, the cumulative cluster-size distribution N_s becomes an exponential function. We find that the cumulative size distribution is described by

$$N_s \approx \langle s \rangle^{-1} \mathrm{e}^{-1.72s/\langle s \rangle}, \tag{6}$$

where the mean cluster size $\langle s \rangle$ is given by the scaling eq. (3). The cumulative distance distribution is also described by the same exponential function (6). The scaling form (6) is consistent with the analytical result derived from the Burgers equation.[16] However, the scaling exponent of the mean distance $\langle l \rangle$ (eq. (5)) does not agree with the analytical result 0.5.[16] Our model is a different universality class from the Burgers model. The power spectrum of the waiting-time distribution does not show $1/f$ noise as was observed by Musha.[13] However, in our model, the power spectrum of the waiting time distribution may show $1/f^2$ noise for an appropriate frequency since the waiting time has an exponential distribution.

In summary, we present a cellular automaton model of freeway traffic showing a clustering of cars. We study the scaling behavior of the aggregation process by computer simulation.

References

1) N. H. Gatner and N. H. M. Wilson: *Transportation and Traffic Theory* (Elsevier, New York, 1987).
2) W. Leutzbach: *Introduction to the Theory of Traffic Flow* (Springer-Verlag, Berlin, 1988).
3) S. Wolfram: *Theory and Applications of Cellular Automata* (World Scientific, Singapore, 1986).
4) K. Kaneko: in *Formation, Dynamics and Statistics of Patterns*, ed. K. Kawasaki, M. Suzuki and A. Onuki (World Scientific, Singapore, 1990) Vol. 1, p. 1.
5) B. Derrida, M. R. Evans, V. Hakim and V. Pasquier: J. Phys. A **26** (1993) 1493.
6) L. H. Gwa and H. Spohn: Phys. Rev. Lett. **68** (1992) 725.
7) T. M. Liggett: *Interacting Particle Systems* (Springer-Verlag, New York, 1985).
8) H. Spohn: *Large Scale Dynamics of Interacting Particles* (Springer-Verlag, Berlin, 1991).
9) S. A. Janowsky and J. L. Lebowitz: Phys. Rev. A **45** (1992) 618.
10) J. Krug and H. Spohn: Phys. Rev. A **38** (1988) 4271.
11) O. Biham, A. A. Middleton and D. Levine: Phys. Rev. A **46** (1992) R6124.
12) K. Nagel and M. Schreckenberg: J. Phys. I France **2** (1992) 2221.
13) T. Musha: *World of Fluctuations* (Kodansha, Tokyo, 1980) p. 112 [in Japanese].
14) M. J. Lighthill and G. B. Whitham: Proc. Roy. Soc. London **A229** (1955) 317.
15) J. L. Lebowitz, E. Presutti and H. Spohn: J. Stat. Phys. **51** (1988) 841.
16) T. Tatsumi and S. Kida: J. Fluid Mech. **55** (1972) 659.
17) T. Vicsek: *Fractal Growth Phenomena* (World Scientific, Singapore, 1989).