Executive Summary

This report highlights the critical role of traffic flow analysis in optimising road networks, essential for both road users and the economy, and aims to offer practical insights and recommendations for more efficient road systems in Leeds. The Nagel-Schreckenberg model served as the basis for investigating traffic management measures on single-lane roads. After validating the parameters used within this model and finding that results were consistent across a range of parameter values, the model was extended to incorporate variable speed limits as well as traffic lights located at road junctions and pedestrian crossings.

A comprehensive study was carried out to examine the factors that influence the relationship between the density of cars on the road and different metrics that measure traffic flow/congestion. Increasing the duration of a green traffic light signal and reducing the number of traffic lights on the road were both found to help reduce traffic congestion and improve traffic flow in some cases. Using these findings to introduce changes to a real road network in Leeds demonstrated the model's ability to determine optimal strategies for implementing traffic management measures such as enforcing speed limits or installing traffic lights in line with government regulations.

More realistic traffic simulations may be achieved through refining assumptions about vehicle acceleration and deceleration, and incorporating other types of vehicles, such as buses. Additionally, looking closely at how the studied factors interact with each other can help inform the design of more effective road networks.

1 Introduction

In 2022, the average driver in Leeds experienced a loss of 60 hours, amounting to a loss of £530 per driver, as a result of being stuck in traffic [1]. This is not only an issue for individuals, but also highlights a major problem for Leeds Traffic Management.

Recognising the gravity of the situation, this report aims to investigate how the introduction of speed and traffic flow controls may influence the occurrence and resolution of traffic jams. To do so, we extend an existing traffic model to incorporate dynamic elements such as variable speed limits, traffic lights, and other road users. Through the refinement of our model, we ultimately aim to demonstrate that our model can be used to determine an "optimal" strategy for installing traffic management measures to improve traffic flow.

2 The Nagel-Schreckenberg Model

We begin with the Nagel-Shreckenberg model as our basic model. This consists of a road of length L, or, equivalently, a one-dimensional array with L sites. Each site may either be empty or occupied by a car. Cars may only travel forwards, and cannot overtake. Each car has an individual integer velocity, v, describing how many sites it advances per iteration. We assume all cars follow the speed limit, v_M, so $0 < v < v_M$. These discrete speed values are not fully realistic, but over long times the model represents driver behaviour accurately [2]. Each iteration, the system is updated according to the acceleration, deceleration, randomisation and car motion rules as described in [3]. The probability of random deceleration due to driver error is denoted by p.

2.1 Simulation Procedure

Using object-oriented programming, we designed our model to have a road with periodic boundary conditions. This created a closed system, allowing traffic jams to accumulate in realistic ways [4]. The initial conditions (initial position and velocity of each vehicle) were randomly generated. Each site in our model represented 5 m of a road, based on the average size of a car parking space being 4.8 m long [5]. The velocities in the model were in metres per second, so each iteration represented a timestep of 5 seconds. The number of cars on the road, N, was fixed for the entirety of a simulation and determined by the density, $\rho = N/L$.

For our basic model, we set $L = 240$ sites, $v_M = 14$ sites/iteration $= 14$ m/s $\approx$ 30mph, $p = 0.127$ and $\rho = 1/12 \approx 0.08$, and we let the number of iterations be $nt = 100$. This resulted in the system shown in figure 1a. Here, we witness the backward motion of traffic jams [4], whilst observing the four rules of motion in [3], thus validating our implementation.

2.2 Model and Parameter Validation

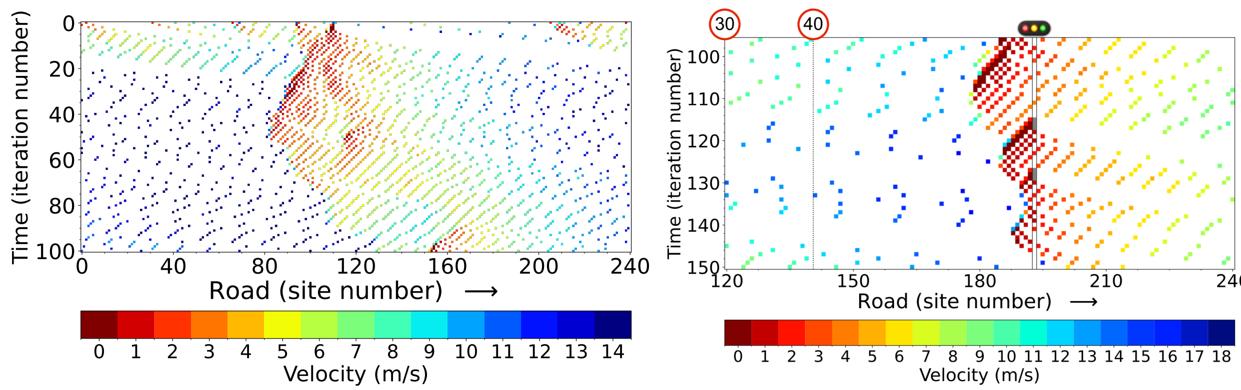
In order to validate our model and determine reasonable parameter values for further testing, we individually varied L, nt, p, and v_M within our basic model. These validation simulations were run under the assumption of "free-flow" traffic, where the entire road had a constant speed limit and contained no traffic management measures, such as traffic lights. Each test was averaged over multiple simulations to ensure that the variability of the randomised initial conditions in our model was accounted for.

We used the flow rate (FR), given as the average number of cars passing the boundary per iteration, to measure traffic flow in our model. This was analogous to counting the number of traffic jams, as a high number of jams corresponded to a lower FR, and vice versa. We also considered the Speed Performance Index (SPI), given by SPI $= (v_{\text{avg}}/v_M) \times 100$ in [6] as a metric since it allowed us to quantify congestion. A higher SPI value corresponded to lower levels of congestion. However, we primarily used FR in our analysis as it provided more clarity on the formation of traffic jams. FR was also found to be a popular metric in the surrounding literature [3] [4] [7]. In our validation analysis, we focused on the effect that varying a parameter had on the relationship between FR or SPI and ρ, accounting for the variability in results through error bars, as seen in figures 2 and 4.

First, we varied L. As shown in figure 2a, when $L \geq 100$ sites $= 500$ m (0.31 mi), it had no significant impact on the relationship between FR and ρ. Hence, we set $L = 240$ sites $=$ 0.75 mi (1.2 km) for all following simulations, unless stated otherwise.

In varying nt we found that for $nt \geq 100$ iterations (8 mins 20 secs), nt had a similar effect to L on the model. Therefore, each of the following simulations were run for 100 iterations.

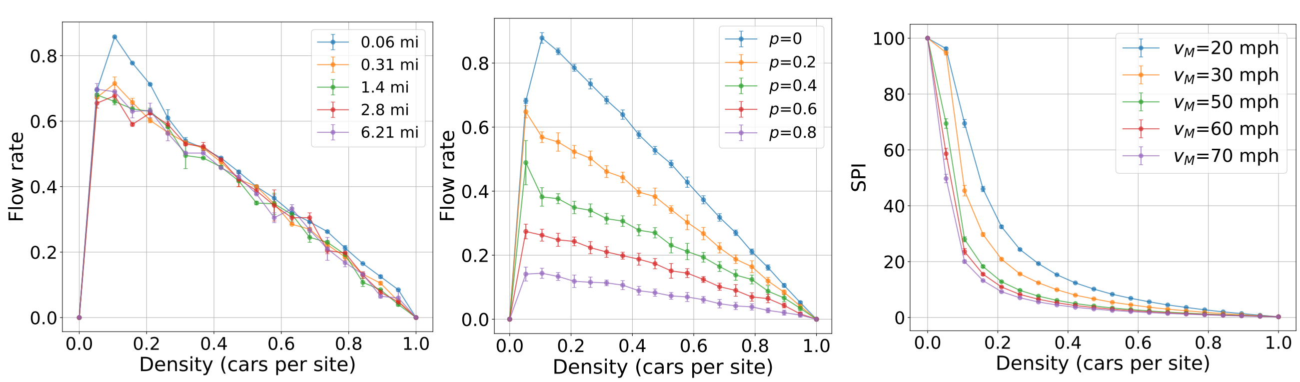
From figure 2b, we see that increasing p decreased FR overall. This was as expected, since a higher p caused more cases of random deceleration, leading to more traffic congestion. For all future simulations, we let $p = 0.127$, as suggested in [8].



(a) A visualisation of our basic (free-flow) model.

(b) A visualisation of a model involving speed limits (in mph) and a pedestrian crossing.

Figure 1: Visualisations of traffic flow in two of our models.



(a) Varying L, measuring FR.

(b) Varying p, measuring FR.

(c) Varying v_M, measuring SPI.

Figure 2: Graphs to show the effects of varying different parameters within our basic model.

Varying v_M had no effect on the relationship between FR and ρ. This was due to the structure of the NaSch model: as ρ increased, cars had less space ahead to accelerate, so they tended not to reach v_M, especially for higher v_M values. For $\rho < 0.1$, lower v_M led to lower FR. For $\rho \geq 0.1$, the FR converged for all 20 mph $< v_M <$ 70 mph. However, varying v_M had a clear effect on the relationship between SPI and ρ, as shown in figure 2c. For higher values of v_M, cars failed to reach v_M as ρ increased. With SPI being the ratio v_{avg}/v_M, we got lower SPI values for higher values of v_M overall.

In figures 2a and 2b, we see a clear trend between ρ and FR. Most importantly, they suggest the existence of a critical density at ~ 0.1, before which the traffic is in free-flow (FR increases with ρ), and after which the traffic is congested (FR decreases with ρ), as in [4].

We considered the impact of each parameter on the relationship between SPI and ρ, but generally found there to be minimal effects, unless specified otherwise.

3 Variable Speed Limits

Our first model extension incorporated roads with variable speed limits, shown in figure 3.

If the motion of the car took it from section A to section B in the next iteration and $v_{M_a} > v_{M_b}$, then we applied some new rules of motion. The car accelerated as before if $v < v_{M_b}$ and it had sufficient space ahead to do so. If $v_{M_b} < v < v_{M_a}$, then it decelerated to v_{M_b}. Otherwise, rules were unchanged.

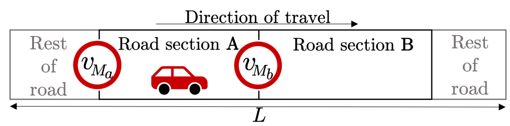


Figure 3: A road with variable speed limits.

We implemented this with various combinations of v_{M_a} and v_{M_b}, ensuring that each individual speed limit was implemented for a road segment at least 600 m long, adhering to government advice [9]. We expected a decrease in speed limit to cause congestion, particularly for larger decreases, to suggest that these changes should only be implemented where necessary. However, due to model limitations (road length, periodic boundary conditions, rules of acceleration and deceleration) we were unable to obtain conclusive results when testing variable speed limits in isolation.

4 Traffic Lights

We then extended the model to include two types of traffic lights. The first were “controlled” traffic lights, like those found at junctions. These had pre-determined red and green cycle times. The period of the traffic light cycle in which cars were allowed (not allowed) to pass was the green (red) cycle time, respectively. The total cycle time was the sum of one red and one green cycle. The second type were pedestrian (specifically pelican) crossings, which had pre-determined red time but variable green time, reflecting the random frequency with which pedestrians and other road users may cross the road. The average time a pedestrian takes to cross a single carriageway A-road in the UK is ≈ 10 s, calculated using the formulae in [10] and dimensions from [11]. This was our red time for all simulations, unless stated otherwise. Given that total cycle time should be $<$ 120 s [12], and allowing at least 10 s between red cycles, we let green time be a randomly generated multiple of 5 between 10 and 100 s.

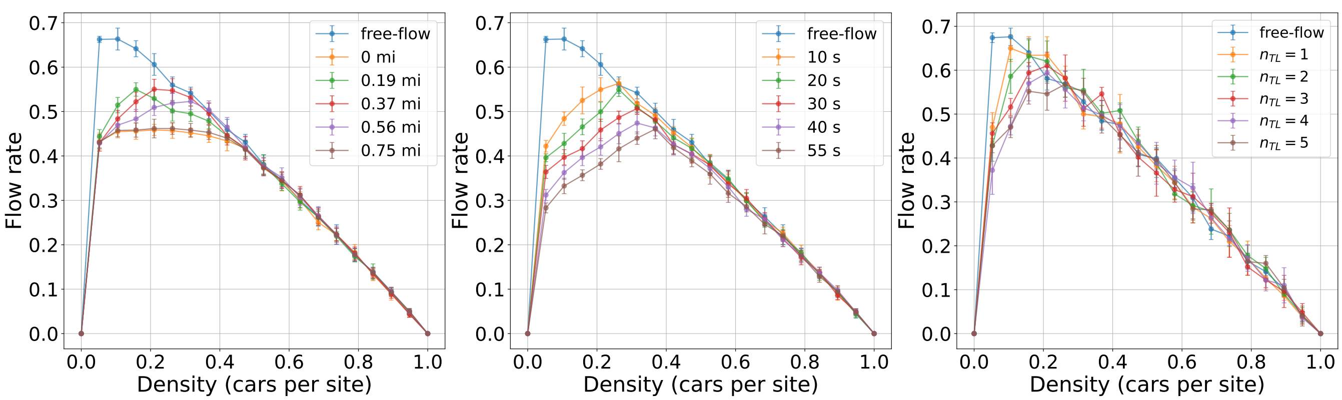
To implement this, we defined additional rules for our model. During green time, cars behaved according to the original model rules. When the light was red, cars decelerated to a halt on the site directly before the light (or the next closest empty site), i.e. the site occupied by the traffic light acted as a car with $v = 0$. Figure 1b illustrates this (grey squares show red time).

4.1 Effects of Varying Parameters

We wished to understand the impact of several parameters on the road's overall traffic flow.

First, we varied the position of a singular traffic light along the road. Using a controlled traffic light allowed us to fix the red and green times. Since it was not possible to determine these cycle times for a junction or intersection without consideration of other unknown factors [12], we fixed red time = 10 s and green time = 60 s. As seen in figure 4a, this variation had no impact on the relationship between ρ and FR in general. However, when the traffic light was placed at our road boundaries (0 or 0.75 mi), it lowered FR overall for $0.05 < \rho < 0.45$. This was due to our FR count point being located on the boundary. Placing a light there resulted in no cars passing the boundary during a red light, causing FR to decrease. For $\rho \geq 0.45$, the impact of traffic light position on FR was dominated by that of ρ. From these results, to reduce any effect of our model's boundary conditions, we placed the traffic light at site $L/2$ for all following simulations, unless stated otherwise.

As seen in figure 4b, as red time increased, FR decreased. Longer red time caused cars to stop for longer periods, allowing fewer cars to pass the boundary each iteration. This trend was most prominent for $0.05 < \rho < 0.32$. Outside of this range, the effect of ρ was, once again, dominant. Conversely, we found that a longer green time increased FR, most noticeably for $0.05 < \rho < 0.27$, with a similar pattern of convergence for $\rho \geq 0.27$.



(a) Varying the position of a single traffic light.

(b) Varying the red cycle time.

(c) Varying the number of traffic lights n_{TL}.

Figure 4: Graphs of flow rate vs. density for varying traffic light parameters.

We then tested the effects of reduced speed limits around traffic lights on traffic flow. We placed a controlled traffic light at site $L/2$ on a road of $L = 856$ sites (≈ 2.66 mi) and centred a "slow zone" around it. The base speed limit for the road was 18 m/s (40mph), but the slow zone had $v_M = 14$ m/s ($\sim$ 30 mph). The size of the slow zone varied from 0 to 1 km. We ultimately found the size of the slow zone to have no discernible impact on the traffic flow.

We also considered a road with two traffic lights. We varied the distance between them and investigated the effect on the relationship between FR and ρ. We found that the lights placed closer together caused FR to increase more quickly with increasing ρ for $\rho \leq 0.25$. For $\rho > 0.25$, the density was once again the dominating variable.

We then tested whether the total number of traffic lights on a road, n_{TL}, impacted the relationship between the FR and ρ. For this, we used $L = 856$ sites ($\approx$2.66 mi), allowing us to place more traffic lights but still keep them reasonably spaced. As shown in figure 4c, we found that the maximum FR decreased as n_{TL} increased for $\rho < 0.1$.

We found that, naturally, introducing traffic lights lowered the FR relative to free-flow conditions, especially for lower ρ. This is illustrated in both figures 4a and 4c.

Like in Section 2.2, the SPI was used as a metric in our investigations but not included here, as the FR results were more prominent. Moreover, these results show that these parameters may be varied to determine an optimal strategy for installing traffic lights.

5 Case Study: York Road, Leeds

We applied our model to a real road system in Leeds: York Road, eastbound, from 53°47′55.1″N 1°31′38.4″W to 53°48′30.2′N 1°27′59.7″W, shown in Figure 5.Figure 1b shows the simulation of a section of the road over a subset of iterations, modelled as a single carriageway despite it being a dual carriageway in reality.

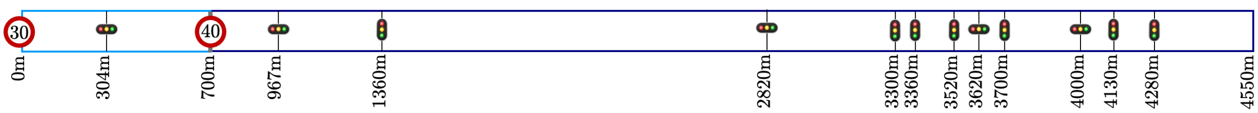


Figure 5: A diagram to illustrate the layout of York Road.[1] represents a controlled traffic light and represents a pedestrian crossing. Speed limits are given in mph.

Using our previous results, we conjectured that removing the controlled traffic light at 3300 m would increase FR, due to it being neither a junction nor a crossing (thus having no clear purpose). We also removed the pedestrian crossing at 3620 m, as the neighbouring controlled lights could be used to cross instead. For $\rho = 0.1$, these changes caused FR to increase by ≈ 0.05. We considered increasing v_M, but since the road passed through residential areas and over large bridges, this required further consideration. Introducing a 20 mph speed limit at 2700m improved the SPI by almost 10.8 units (but did not improve FR), suggesting that while fewer cars travelled along the road, there was also less congestion. However, implementing this in reality may not be entirely practical.

While these improvements cannot be generalised for any ρ, our extended model can be used to implement an optimal strategy for designing the road.

6 Conclusion and Future Work

In this report, we extended the Nagel-Schreckenberg model to incorporate variable speed limits, controlled traffic lights, and crossings for other road users. We investigated the parameters within this model (e.g. red/green cycle times or number of traffic lights) and their effects on the flow rate and speed performance index, demonstrating that our model could ultimately be used to determine an "optimal" strategy for implementing traffic management measures (with this being highly dependent on the metric being optimised). However, we note that our model is idealised and consideration should be given to aspects such as more realistic acceleration and deceleration rules for smoother flow, alternate boundary conditions for more effective modelling of variable speed limits, or other types of vehicles (e.g. lorries). To provide more meaningful and accurate results, parameter values should be chosen that are relevant to specific roads. Since the density of cars on the road was found to be a very influential factor in our investigations, extra care should be taken to obtain accurate values of this before designing a strategy for traffic control measures.

[1]The data used to construct this diagram was obtained from Google Maps on 29/11/2023.

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