Parametric Portfolio Policy Adaptive Elastic Net

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Outline

Outline

1. Parametric Portfolio Policy
   - Utility Maximization
   - Transaction cost
2. Adaptive Elastic Net
3. Estimation
   - Including transaction cost
   - Cross Validation
   - Overview of PPPAENET
4. Illustration: PPPAENET Trend following

- Find portfolio weights ($w_{i,t}$) that maximize a given utility function.
- Solve for the optimal portfolio weights:

$$
\begin{aligned}
w_{i,t} &= f\left(x_{i,t};\theta\right) \\
&= \bar{w}_{i,t} + \frac{\theta^T x_{i,t}}{N_t} \\
&= \bar{w}_{i,t} + \left(\theta_1 x_{i,t}^1 + \theta_2 x_{i,t}^2 + \cdots + \theta_p x_{i,t}^p\right)\frac{1}{N_t}
\end{aligned} \tag{1}
$$

where:

- $\bar{w}_{i,t}$ is weight of asset i in the benchmark portfolio at time t
- N_t is number of asset at time t
- $x_{i,t}$ is a vector of characteristic
- θ is a vector of coefficients to be estimated

PPP: Examples

Trend following strategy

$\bar{w}_{i,t} = 0$, and $x^j_{i,t}$= trend signals

Cross sectional strategy

$\bar{w}_{i,t}$= benchmark weghts, and $x^j_{i,t}$= cross sectional signals (carry, momentum, value, and economic)

Time series strategy

$\bar{w}_{i,t}$= benchmark weghts, and $x^j_{i,t}$= time series signals (carry, momentum, value, and economic)

PPP: portfolio returns

- Given the portfolio weight function, we compute portfolio returns:

$$
\begin{aligned}
r_{p,t+1} &= \sum_{i=1}^{N_t} w_{i,t} r_{i,t+1} \\
&= \sum_{i=1}^{N_t} \left(\bar{w}_{i,t} + \frac{\theta^T x_{i,t}}{N_t} \right) r_{i,t+1} \\
&= r_{ben,t+1} + r_{\theta,t+1} \qquad (2)
\end{aligned}
$$

where:

- $r_{ben,t+1}$ is the benchmark portfolio return
- $r_{\theta,t+1}$ is parametric portfolio return

Utility Maximization

- Find θ that maximizes expected utility:

$$\max_{\theta} E\left[u\left(r_{p,t+1}\right)\right] \tag{3}$$

- Estimated b y maximizing average realized utility in sample:

$$\max_{\theta} \frac{1}{T} \sum_{t=1}^{T} u\left(r_{p,t+1}\right) \tag{4}$$

- Power Utility:

$$u\left(r_{p,t+1}\right) = \frac{\left(1 + r_{p,t+1}\right)^{1-\gamma}}{1-\gamma} \tag{5}$$

- Mean Variance (quadratic) utility:

$$u\left(r_{p,t+1}\right) = E_t\left(r_{p,t+1}\right) - \frac{\gamma}{2} Var_t\left(r_{p,t+1}\right) \tag{6}$$

Utility Maximization

- In the case of power utility:

$$\min_{\theta} -\frac{1}{T}\sum_{t=1}^{T}\frac{(1+r_{p,t+1})^{1-\gamma}}{1-\gamma} \tag{7}$$

- In the case of mean variance utility:

$$\min_{\theta} \frac{\gamma}{2}\theta^T \hat{\Sigma}_c\theta + \gamma\theta^T\hat{\sigma}_{ben} - \theta^T\hat{\mu}_c \tag{8}$$

  - $\hat{\Sigma}_c$: sample covariance matrix of the characteristic return vector
  - $\hat{\sigma}_{ben}$: sample covariance (vector) between the characteristic return vector and benchmark portfolio return
  - $\hat{\mu}_c$: sample mean of the characteristic return vector

Transaction cost

- Turnover of the portfolio:

$$T_t = \sum_{i=1}^{N_t} |w_{i,t} - w_{i,t-1}| \tag{9}$$

- Re-balancing costs:

$$Cost_t = \sum_{i=1}^{N_t} c_{i,t} |w_{i,t} - w_{i,t-1}| \tag{10}$$

Utility Maximization with transaction cost

- In the case of power utility:

$$\min_{\theta} -\frac{1}{T}\sum_{t=1}^{T}\frac{(1+r_{p,t+1})^{1-\gamma}}{1-\gamma} \tag{11}$$

where:

$$r_{p,t+1} = \sum_{i=1}^{N_t} w_{i,t} r_{i,t+1} - \sum_{i=1}^{N_t} c_{i,t}\left|w_{i,t} - w_{i,t-1}\right| \tag{12}$$

- In the case of mean variance utility:

$$\min_{\theta} \frac{\gamma}{2}\theta^T \hat{\Sigma}_c \theta + \gamma\theta^T \hat{\sigma}_{ben} - \theta^T \hat{\mu}_c + \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N_t} c_{i,t}\left|w_{i,t} - w_{i,t-1}\right| \tag{13}$$

Outline

Adaptive Elastic Net

- Estimator:

$$\hat{\theta}_{Aenet} = \arg\min_{\theta} f(\theta) + \underbrace{\lambda_1 \sum_{j=1}^{p} \hat{\varpi}_j |\theta_j|}_{Lasso} + \underbrace{\lambda_2 \sum_{j=1}^{p} |\theta_j|^2}_{Ridge} \qquad (14)$$

- Lasso term: shrinkage (variables selection).
- Ridge term: stabilize solution path.
- Adaptive Elastic Net takes care of over-fitting issue and has the oracle property when there are large number of variables that are correlated.

Adaptive Elastic Net

- Let $\lambda_2 = 0$ and $\hat{\varpi}_j = 1$, we have the Lasso estimator:

$$\hat{\theta}_{Lasso} = \arg\min_{\theta} f(\theta) + \lambda_1 \sum_{j=1}^{p} |\theta_j| \tag{15}$$

- Let $\lambda_1 = 0$, we have the Ridge estimator:

$$\hat{\theta}_{Ridge} = \arg\min_{\theta} f(\theta) + \lambda_2 \sum_{j=1}^{p} |\theta_j|^2 \tag{16}$$

- Let $\hat{\varpi}_j = 1$, we have the Elastic Net estimator:

$$\hat{\theta}_{Enet} = \arg\min_{\theta} f(\theta) + \lambda_1 \sum_{j=1}^{p} |\theta_j| + \lambda_2 \sum_{j=1}^{p} |\theta_j|^2 \tag{17}$$

- In Adaptive Elastic Net estimator

$$\hat{\varpi}_j = \frac{1}{\left|\hat{\theta}_{Enet}\right|^{\alpha}} \simeq \frac{1}{\left|\hat{\theta}_{Enet} + 1/n\right|^{\alpha}}$$

Outline

Estimation

- PPPAENET with No transaction cost:
  - Quadratic approximation (optional)
  - Coordinate descent algorithm.
- PPPAENET with transaction cost:
  - L1 norm approximation.
  - Gradient based algorithm (e.g. BFGS, Newton Conjugate Gradient)

Power Utility with no transaction cost

Coordinate descent algorithm

$$\arg\min_{\theta} -\frac{1}{T}\sum_{t=1}^{T}\frac{(1+r_{p,t+1})^{1-\gamma}}{1-\gamma}+\lambda_1\sum_{j=1}^{p}\hat{\varpi}_j\left|\theta_j\right|+\lambda_2\sum_{j=1}^{p}\left|\theta_j\right|^2 \tag{18}$$

- Initialize all the $\theta_j = 0$.
  Cycle over $j = 1, 2, \ldots, p, 1, 2, \ldots$ till convergence:
- Compute: r^{*}_{t+1}, δ_j, ρ_j
- Update θ_j by *soft-thresholding*:

$$\begin{aligned}\theta_j &\leftarrow \frac{S\left(\rho_j, \lambda_1\hat{\varpi}_j\right)}{\delta_j+\lambda_2} \\ &= \frac{sign\left(\rho_j\right)\left(\left|\rho_j\right|-\lambda_1\hat{\varpi}_j\right)}{\delta_j+2\lambda_2}\end{aligned} \tag{19}$$

Power Utility with no transaction cost

Coordinate descent algorithm

where:

- $r^*_{t+1} = \sum_{i=1}^{N_t} \left(\bar{w}_{i,t} + \hat{\theta}^T x_{i,t} / N_t \right) r_{i,t+1}$
- $\delta_j = -\frac{1}{T} \sum_{t=1}^{T} u'' \left(r^*_{t+1} \right) \left[\sum_{i=1}^{N_t} \left(x^j_{i,t} r_{i,t+1} \frac{1}{N_t} \right) \right]^2$
- $\rho_j = \frac{1}{T} \sum_{t=1}^{T} u' \left(r^*_{t+1} \right) \sum_{i=1}^{N_t} \left(x^j_{i,t} r_{i,t+1} \frac{1}{N_t} \right)$

Mean Variance Utility with no transaction cost

Coordinate descent algorithm

$$\arg\min_{\theta} \frac{\gamma}{2}\theta^T \hat{\Sigma}_c \theta + \gamma\theta^T \hat{\sigma}_{ben} - \theta^T \hat{\mu}_c + \lambda_1 \sum_{j=1}^{p} \hat{\varpi}_j \left|\theta_j\right| + \lambda_2 \sum_{j=1}^{p} \left|\theta_j\right|^2 \quad (20)$$

- Initialize all the $\theta_j = 0$.
  Cycle over $j = 1, 2, \ldots, p, 1, 2, \ldots$ till convergence:
- Compute: $\rho_j = \gamma \sum_{k=1}^{p} \theta_k \sigma_{j,k} + \gamma \hat{\sigma}_{ben}^{j} - \hat{\mu_c}$
- Update θ_j by *soft-thresholding*:

$$\theta_j \leftarrow \frac{S\left(-\rho_j, \lambda_1 \hat{\varpi}_j\right)}{\gamma\sigma_j^2 + 2\lambda_2} \quad (21)$$

Including transaction cost

Problem:

Transaction cost term: $\sum_{i=1}^{N_t} c_{i,t} \left| w_{i,t} - w_{i,t-1} \right|$

- The computation depends on θ.
- $L1$ norm is not differential-able.

Including transaction cost

1. *No cost*
   - No transaction cost in optimization process.
   - Not optimally re-balancing the portfolio.
   - Coordinate descent works as before.
2. *Extra cost*
   - Approximation: cost associated with trading each characteristic independently.
   - Ignore the reduction in re-balancing the portfolio.
   - Coordinate descent works well.
3. *Exact cost*
   - Approximation of $L1$ norm by a smooth function.
   - Reduction in transaction cost via optimally re-balancing the portfolio.
   - Gradient based algorithm works.

Extra cost

Transaction costs approximation:

$$\sum_{i=1}^{N_t} c_{i,t} \left| w_{i,t} - w_{i,t-1} \right|$$

$$= \sum_{i=1}^{N_t} c_{i,t} \left| \bar{w}_{i,t} + \left(\theta_1 x_{i,t}^1 + \theta_2 x_{i,t}^2 + \cdots + \theta_p x_{i,t}^p\right) \frac{1}{N_t} \right.$$

$$\left. - \bar{w}_{i,t-1} - \left(\theta_1 x_{i,t-1}^1 + \theta_2 x_{i,t-1}^2 + \cdots + \theta_p x_{i,t-1}^p\right) \frac{1}{N_{t-1}} \right|$$

$$= \sum_{i=1}^{N_t} c_{i,t} \left| \Delta\bar{w}_{i,t} + \left(\theta_1 \Delta x_{i,t}^1 + \theta_2 \Delta x_{i,t}^2 + \cdots + \theta_p \Delta x_{i,t}^p\right) \right|$$

$$\approx \sum_{i=1}^{N_t} c_{i,t} \left[\left|\Delta\bar{w}_{i,t}\right| + |\theta_1| \left|\Delta x_{i,t}^1\right| + |\theta_2| \left|\Delta x_{i,t}^2\right| + \cdots + |\theta_p| \left|\Delta x_{i,t}^p\right| \right]$$

Extra cost

- Approximate the transaction cost with the sum of the cost associated with trading each characteristic independently.
- Works when combining various trading strategies (each strategy trades individual characteristic).
- Over estimate the transaction cost when combining characteristic.

Extra cost: Mean Variance Utility

Coordinate descent algorithm

$$\arg\min_{\theta} \frac{\gamma}{2}\theta^T \hat{\Sigma}_c \theta + \gamma\theta^T \hat{\sigma}_{ben} - \theta^T \hat{\mu}_c$$
$$+\frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N_t} c_{i,t}\left[\left|\Delta\bar{w}_{i,t}\right| + |\theta|^T \left|\Delta x_{i,t}\right|\right] + \lambda_1 \sum_{j=1}^{p} \hat{\varpi}_j \left|\theta_j\right| + \lambda_2 \sum_{j=1}^{p} \left|\theta_j\right|^2 \quad (22)$$

- Initialize all the $\theta_j = 0$.
  Cycle over $j = 1, 2, \ldots, p, 1, 2, \ldots$ till convergence:
- Compute: $\rho_j = \gamma \sum_{k=1}^{p} \theta_k \sigma_{j,k} + \gamma \hat{\sigma}_{ben}^{j} - \hat{\mu_c}$,
  $\delta_j = \gamma / T \sum_{t=1}^{T} \sum_{k=1}^{p} \left|\theta_k\right| \left|\Delta x_{i,t}^{k}\right|$
- Update θ_j by *soft-thresholding*:

$$\theta_j \leftarrow \frac{S\left(-\rho_j, \delta_j + \lambda_1 \hat{\varpi}_j\right)}{\gamma \sigma_j^2 + 2\lambda_2} \quad (23)$$

Exact cost: approximation of $L1$ norm

Definition

The pseudo-Huber function $\phi_\varepsilon : \Re \to \Re$

$$\phi_\varepsilon(x) = \sqrt{\varepsilon^2 + x^2} - \varepsilon \tag{24}$$

with first and second derivatives:

$$\phi'_\varepsilon(x) = \frac{x}{\sqrt{\varepsilon^2 + x^2}} \tag{25}$$

$$\phi''_\varepsilon(x) = \frac{\varepsilon^2}{(\varepsilon^2 + x^2)^{3/2}} \tag{26}$$

Exact cost: approximation of $L1$ norm

Problem: Mean Variance Utility with Transaction Cost

$$\begin{aligned}\arg\min_{\theta} \frac{\gamma}{2}\theta^T\hat{\Sigma}_c\theta + \gamma\theta^T\hat{\sigma}_{ben} - \theta^T\hat{\mu}_c \\ + \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N_t} c_{i,t}\left|w_{i,t} - w_{i,t-1}\right| + \lambda_1\sum_{j=1}^{p}\hat{\varpi}_j\left|\theta_j\right| + \lambda_2\sum_{j=1}^{p}\left|\theta_j\right|^2\end{aligned} \tag{27}$$

$$f(\theta) = \frac{\gamma}{2}\theta^T\hat{\Sigma}_c\theta + \gamma\theta^T\hat{\sigma}_{ben} - \theta^T\hat{\mu}_c + \lambda_2\sum_{j=1}^{p}\left|\theta_j\right|^2 \tag{28}$$

$$c(\theta) = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N_t} c_{i,t}\left|w_{i,t} - w_{i,t-1}\right| \tag{29}$$

$$\simeq \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N_t} c_{i,t}\phi_{\varepsilon}\left(w_{i,t} - w_{i,t-1}\right) \tag{30}$$

Exact cost: approximation of $L1$ norm

$$f'(\theta) = \gamma \hat{\Sigma}_c \theta + \gamma \hat{\sigma}_{ben} - \hat{\mu}_c + 2\lambda_2 \theta \tag{31}$$

$$\frac{\partial c}{\partial \theta_j} = \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N_t} c_{i,t} \phi'_{\varepsilon} \left(w_{i,t} - w_{i,t-1} \right) \tag{32}$$

$$= \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N_t} c_{i,t} \frac{\Delta \bar{w}_{i,t} + \theta^T \Delta x_{i,t}}{\sqrt{\varepsilon^2 + \left(\Delta \bar{w}_{i,t} + \theta^T \Delta x_{i,t} \right)^2}} \Delta x^j_{i,t} \tag{33}$$

The problem can be solved by the Orthant-Wise Limited- memory Quasi-Newton (*OWL-QN*) optimization algorithm.

OWL-QN is a modified Limited-memory Broyden-Fletcher-Goldfarb-Shanno (*L-BFGS*) algorithm that allows for allow for $L1$ penalties.

Cross Validation

- Cross validation
  - k-folds cross validation is performed to determine the tuning parameters.
  - Model is firstly estimated on the training set. The coefficients are then re-estimated for validation set.
  - Average sample criterion is calculated across folds and tuning parameters are chosen based on minimizing the criterion.
    - CEG: Certainty Equivalent Gain
    - SR: Sharpe Ratio
    - SOR: Sortino Ratio
    - OMG: Omega Ratio
  - Inference
    - Bootstrapping Method for Standard Errors and Confidence Intervals
    - Significance and Marginal contributions of characteristic
- Backtesting: Access the performance of the model in test set.

Overview of PPPAENET

PPPAENET

- Characteristics/signals selection.
- Utility functions.
- Transaction cost functions.
- Statistical inference.
- Cross Validation.
- Backtesting.

Overview of PPPAENET

Inputs

- Benchmark weights $\bar{w}_{i,t}$
- Asset prices/returns
- Transcation costs $c_{i,t}$

Notes

Benchmark weights:

- zero weighted
- equally weighted
- long only volatility parity

Utility Function

- Utility function to be optimised $u(.)$
- Parameters γ

Notes

- Power Utility
- Mean Variance Utility

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Overview of PPPAENET

Transaction Cost

- Method to include Transaction Cost in the optimisation

Notes

- No Cost
- Extra Cost
- Exact Cost

Cross validation

- Training set
- Criterion
- Determine the optimal tuning parameters
- Statistical inference

Notes

- 80% data set for Cross Validation
- Criterion: SR
- Bootstrapping

Backtest

- Strategy/Portfolio performance in Test set

Notes

- 20% data set for Testing

Overview of PPPAENET

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Outline

Trend following strategy

- Trend following strategy:

$$r_{p,t+1}^{TF} = \sum_{i=1}^{N_t} \frac{s_{i,t}}{N_t} r_{i,t+1} \tag{34}$$

- where:
- $s_{i,t}$ is trend signal
  - binary signal: sign
  - time series signal
  - cross sectional signal

Trend following strategy

Constant volatility Trend following strategy

$$r_{p,t+1}^{CV,TF} = \frac{\sigma_{TGT}}{\sigma_t^{TF}} \sum_{i=1}^{N_t} s_{i,t} r_{i,t+1} \tag{35}$$

Volatility Parity Trend following strategy

$$r_{p,t+1}^{VP,TF} = \frac{\sigma_{TGT}}{\sigma_t^{TF}} \sum_{i=1}^{N_t} s_{i,t} \frac{(\sigma_i)^{-1}}{\sum_{j=1}^{N} (\sigma_J)^{-1}} r_{i,t+1} \tag{36}$$

Data Description

- Futures Data
  - Source: Quandl.
  - Daily futures prices for 42 assets over the period 01/01/1995 to 11/05/2017.
  - Contract details:
    - Equity contracts:
    - Currency contracts:
    - Government Bond contracts:
    - Commodity contracts:
    - Energy contracts:

Trend signals

- Momentum:
    - $s =$past N months returns
    - signal $x = s/\sigma_{250}(s)$
    - $N =$5, 10 days, 1, 3, 6, 9, 12, 18, 24, 36, 48, 54, 60 months
- EMA1(n) Exponential Moving Average:
    - $s = Closes - EMA(n)$
    - signal $x = s/\sigma_{250}(s)$
    - $n =$10, 20, 50, 100, 200, 300, 400, 600, 800 days
- EMA2(n_1, n_2) Exponential Moving Average Crossover:
    - $s = EMA(n_1) - EMA(n_2)$
    - signal $x = s/\sigma_{250}(s)$
    - $(n_1, n_2) =$(10,20), (20,50), (40,80), (50,200), (100,300), (200,400), (300,500), (400,800) days

Trend signals

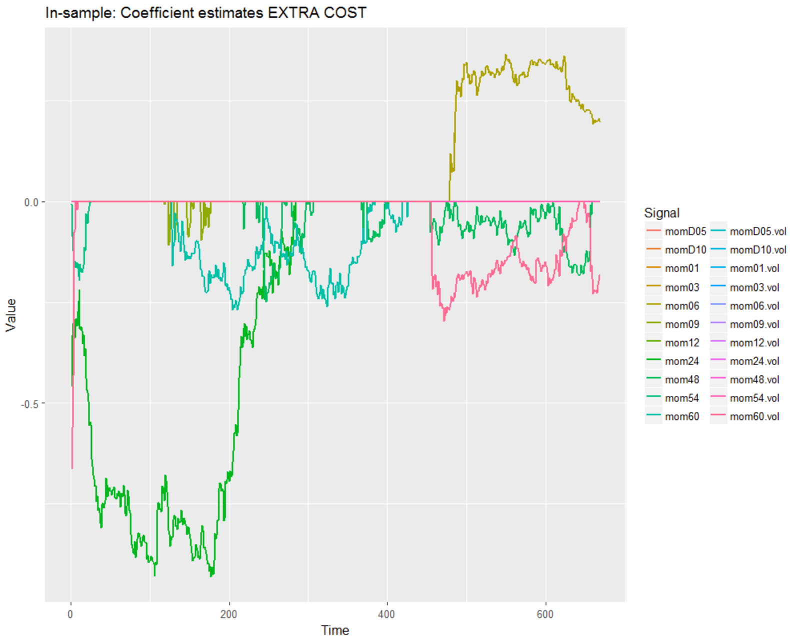
- MACD($nFAST, nSlow, nSig$)MACD Oscillator:
    - $s = MACDline - MACDsignal$
    - signal $x = s/\sigma_{250}(s)$
    - $(nFAST, nSlow, nSig)$ =(10,20,9), (20,50,9), (40,80,9), (500,200,9), (100,300,9), (200,400,9), (300,500,9), (400,800,9) days
- TRIX($n, nSig$)Triple Smoothed Exponential Oscillator:
    - $s = TRIX.value - TRIX.signal$
    - signal $x = s/\sigma_{250}(s)$
    - $(n, nSig) =$ (10,10), (20,10), (50,10), (100,10), (200,10), (300,10), (400,10), (600,10), (800,10) days
- RSI(n)Relative Strength Index:
    - $s = RSI - 1/2$
    - n =10, 20, 50, 100, 200, 300, 400, 600, 800 days

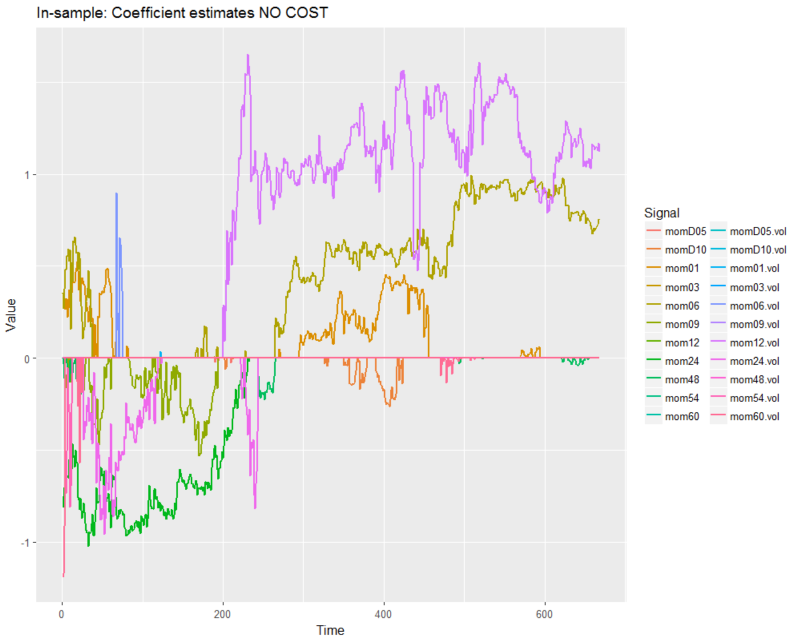
PPPAENET Trend following: set up

- PPPAENET:
  - Zero Benchmark weights.
  - $\gamma = 50$, $\alpha = 1$
  - Transaction cost: 10bps.
  - Training set (cross validation):
    - 01/01/1995 to 22/11/2012
    - Criterion: Omega Ratio
  - Test set: 23/11/2012 to 11/05/2017
  - In-sample:
    - initial window to estimate: 265
    - 04/02/2000 to 22/11/2012
  - Out-of-sample:
    - 23/11/2012 to 11/05/2017

PPPAENET Trend following: signals

- Trend signals (54):
  - $x^j_{i,t}$: Momentum, EMA1, EMA2, MACD, TRIX, RSI
- Volatility adjusted signals (54):
  - $\frac{x^j_{i,t}}{\sigma_{60}(r_t)}$: Volatility Adjusted Trend signals


In-sample: Coefficient estimates EXTRA COST
Value
0.0
-0.5
0
200
400
600
Time
Signal
momD05
momD10
mom01
mom03
mom06
mom09
mom12
mom24
mom48
mom54
mom60
momD05.vol
momD10.vol
mom01.vol
mom03.vol
mom06.vol
mom09.vol
mom12.vol
mom24.vol
mom48.vol
mom54.vol
mom60.vol



In-sample: Coefficient estimates NO COST
Value
Time
1
0
-1
0
200
400
600
Signal
momD05
momD10
mom01
mom03
mom06
mom09
mom12
mom24
mom48
mom54
mom60
momD05.vol
momD10.vol
mom01.vol
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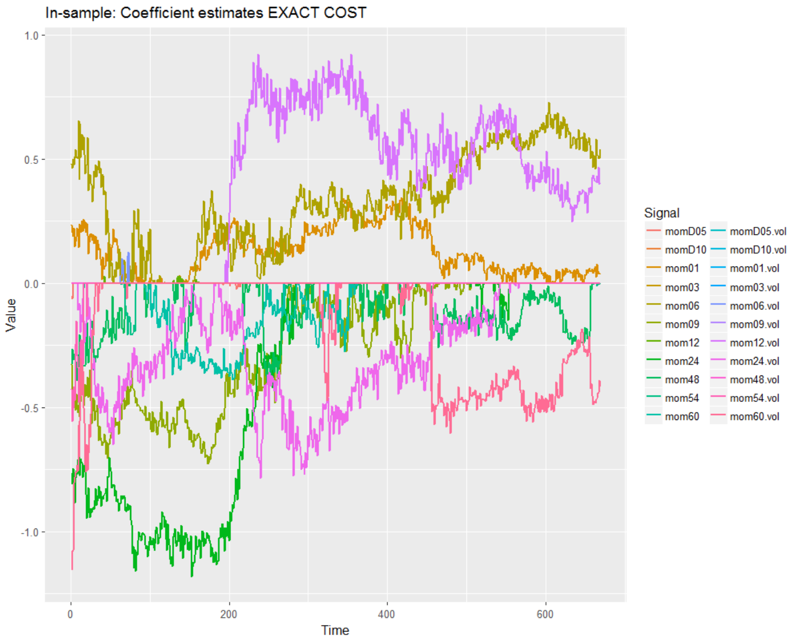
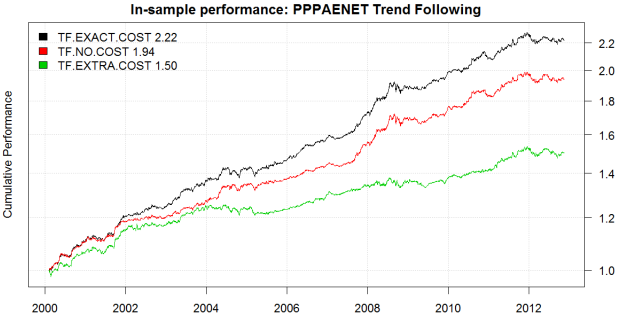
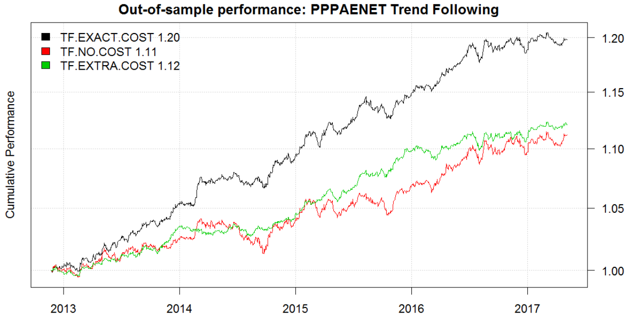
mom12.vol
mom24.vol
mom48.vol
mom54.vol
mom60.vol


In-sample: Coefficient estimates EXACT COST
1.0
0.5
0.0
-0.5
-1.0
Value
0
200
400
600
Time
Signal
momD05
momD10
mom01
mom03
mom06
mom09
mom12
mom24
mom48
mom54
mom60
momD05.vol
momD10.vol
mom01.vol
mom03.vol
mom06.vol
mom09.vol
mom12.vol
mom24.vol
mom48.vol
mom54.vol
mom60.vol



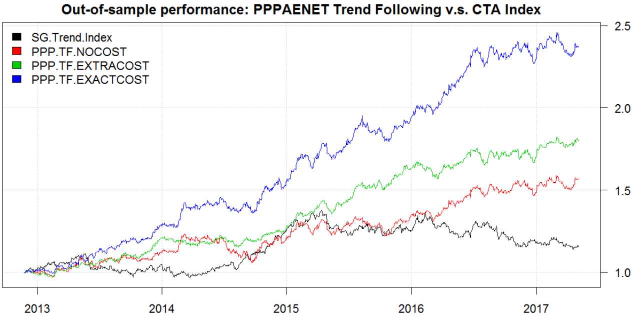
System	TF.EXACT.COST	TF.NO.COST	TF.EXTRA.COST
Period	Feb2000 - Nov2012	Feb2000 - Nov2012	Feb2000 - Nov2012
Cagr	6.44	5.32	3.23
Sharpe	1.94	1.71	1.1
DVR	1.9	1.67	1.06
Volatility	3.13	2.95	2.83
MaxDD	-4.06	-4.3	-4
AvgDD	-0.4	-0.4	-0.49
VaR	-0.28	-0.26	-0.28
CVaR	-0.4	-0.38	-0.38
Exposure	99.97	99.97	99.97
AveNetLeverage	11.73	18.69	11.04
AveGrossLeverage	99.94	99.95	99.97
MaxNetLeverage	96.65	96.87	90.99
MaxGrossLeverage	102.52	102.16	103.2



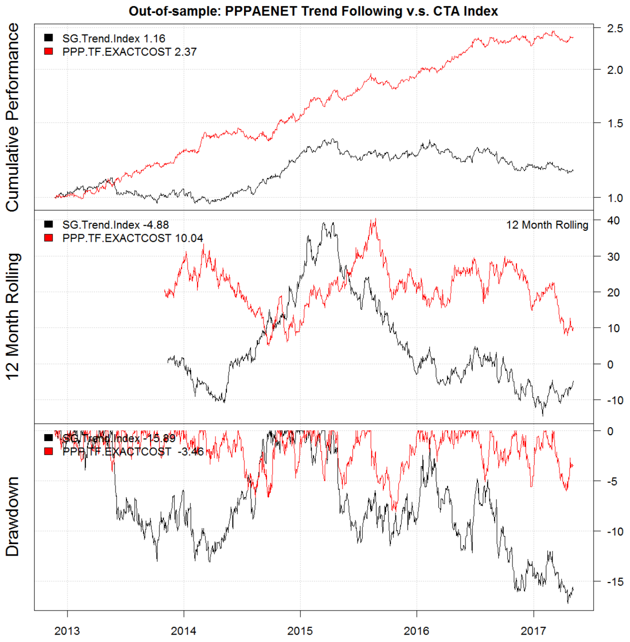
System	TF.EXACT.COST	TF.NO.COST	TF.EXTRA.COST
Period	Nov2012 - May2017	Nov2012 - May2017	Nov2012 - May2017
Cagr	4.15	2.42	2.6
Sharpe	1.78	1	1.58
DVR	1.75	0.93	1.53
Volatility	2.05	2.16	1.46
MaxDD	-1.73	-2.68	-1.43
AvgDD	-0.29	-0.41	-0.21
VaR	-0.19	-0.21	-0.13
CVaR	-0.26	-0.29	-0.2
Exposure	99.92	99.92	99.92
AveNetLeverage	-13.58	19.99	-22.06
AveGrossLeverage	99.94	99.95	99.97
MaxNetLeverage	36.35	92.52	20.16
MaxGrossLeverage	100.99	101.06	100.93

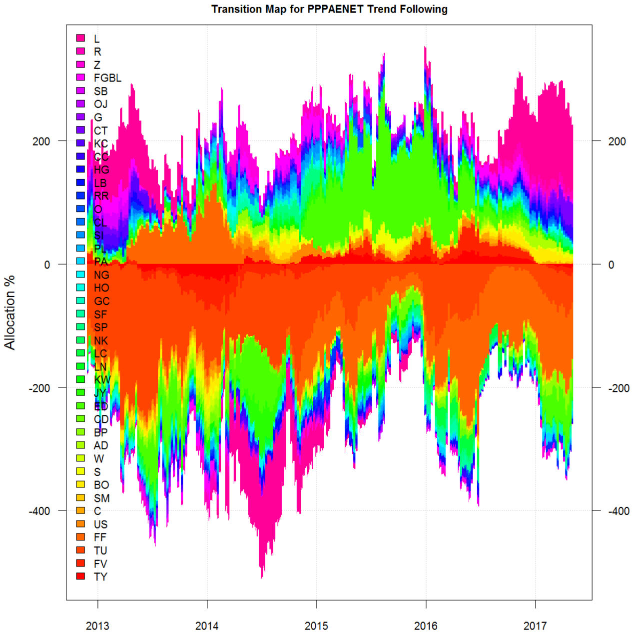
Backtesting Performance

- SG CTA Trend Index: calculates the net daily rate of return for a pool of trend following based hedge fund managers.
- Target volatility: 10%
- Max gross leverage: 600%

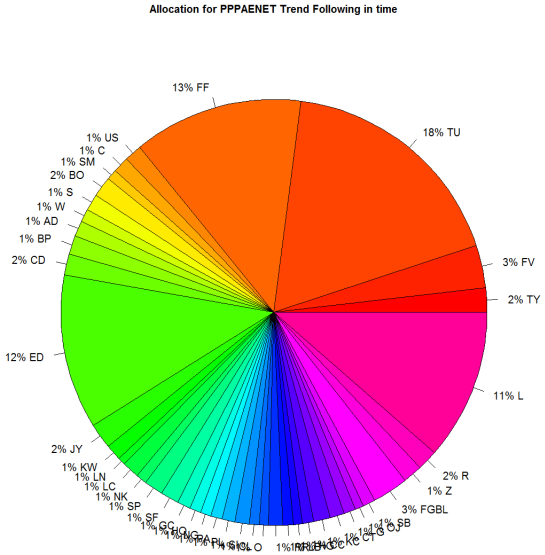


legendnames	SG.Trend.Index	PPP.TF.NOCOST	PPP.TF.EXTRACOST	PPP.TF.EXACTCOST
Period	Nov2012 - May2017	Nov2012 - May2017	Nov2012 - May2017	Nov2012 - May2017
AveReturns	3.71	9.55	12.05	17.8
Skewness	-33.02	31.74	0.93	22.3
Volatility	10.24	10.45	7.94	10.11
Sharpe	0.36	0.91	1.52	1.76
MaxDD	-17.21	-13.87	-6.2	-8.05
AveDD	-1.84	-2.06	-1.15	-1.44
Win.Percent.Month	50.91	54.55	65.45	74.55
Best.Month	7.29	9.96	6.41	11.18
Worst.Month	-4.86	-4.86	-2.83	-3.47


Out-of-sample: PPPAENET Trend Following v.s. CTA Index
Cumulative Performance
SG.Trend.Index 1.16
PPP.TF.EXACTCOST 2.37
2.5
2.0
1.5
1.0
12 Month Rolling
SG.Trend.Index -4.88
PPP.TF.EXACTCOST 10.04
12 Month Rolling
40
30
20
10
0
-10
Drawdown
SG.Trend.Index -15.89
PPP.TF.EXACTCOST -3.46
0
-5
-10
-15
2013
2014
2015
2016
2017


Transition Map for PPPAENET Trend Following
Allocation %
200
0
-200
-400
2013
2014
2015
2016
2017
L
R
Z
FGBL
SB
OJ
G
CT
KC
CC
HG
LB
RR
O
CL
SI
PL
PA
NG
HO
GC
SF
SP
NK
LC
LN
KW
JY
ED
CD
BP
AD
W
S
BO
SM
C
US
FF
TU
FV
TY

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	MaxDD
2012												0.2	0.7	-1.8
2013	0.2	3.1	-1.6	3.5	3.3	2.5	2.4	1.7	2.3	-0.1	0.8	6.2	27.1	-4.1
2014	0.7	7.5	0.3	1.2	-0.3	1.8	-2.8	1.9	-3.4	11.2	0.7	1.2	21.0	-6.6
2015	7.2	3.1	-0.4	1.3	-0.1	5.0	4.5	-1.5	0.7	-3.5	4.3	2.5	25.1	-8.0
2016	3.7	-0.3	2.7	4.5	1.7	7.2	-1.5	0.8	0.2	1.6	0.1	-3.0	18.5	-5.6
2017	3.5	1.9	-2.9	0.6	0.3								3.4	-6.0
Avg	3.1	3.1	-0.4	2.2	1.0	4.1	0.7	0.7	-0.1	2.3	1.5	1.4	16.0	-5.4


Allocation for PPPAENET Trend Following in time
13% FF
18% TU
3% FV
2% TY
11% L
2% R
1% Z
3% FGBL
1% SB
12% ED
2% CD
1% BP
1% AD
1% W
1% S
2% BO
1% SM
1% C
1% US
2% JY
1% KW
1% LN
1% LC
1% NK
1% SP
1% SF