Practica 5 expresiones regulares

1) Ej 1 practica 2)

a) $\Sigma = \{0\}$ de longitud par $(00)^*$

b) $\Sigma = \{0,1\}$ cant par de 0 $1^*(01^*0)^*1^* \mid (01^*0|1)^*$

c) $\Sigma = \{0,1\}$ cant impar de 1 $0^*10^*(10^*10^*)^*$

~~d) $\Sigma = \{0,1\}$ cant par 0 cant impar 1~~

Ej 2 practica 2) $\Sigma = \{0,1\}$

a) Cadenas que comienzan con 010 $010(1|0)^*$

b) Cadenas que terminen con 010 $(1|0)^*010$

c) contenga 000 $(1|0)^*000(1|0)^*$

d) que no contenga 000

e) que contenga 000 exactamente una vez

f) que no contenga subcadena 000 ni 010

?

Ej 3 practica 2)

a) $(a|\cdots|z|A|\cdots|Z|\_).(a|\cdots|z|A|\cdots|Z|0|\cdots|9|\_)^*$

b) $(+|-)(0\cdots9)^+$

c) $(+|-)(0\cdots9)^+ \mid (0\cdots9)^+$

$(+|-|\lambda)(0\cdots9)^+$

d) $(+|-)(0\cdots9)^+.(0\cdots9)^+$

e) $(+|-|\lambda)(0\cdots9)^*.(0\cdots9)^*$

f) $(+|-|\lambda)(0\cdots9)^*.(0\cdots9)^*E(+|-|\lambda)(0\cdots9)^+$

2) a) $\partial_1(10^*1) = \partial_1(1)0^*1 \mid \epsilon(1)\,\partial_1(0^*1)$

$= \lambda\, 0^*1 \mid \emptyset \cdot \partial_1(0^*1)$

$= 0^*1 \mid \emptyset = 0^*1$

b) $\partial_a(ab^* \mid ac \mid c^+) = \partial_a(ab^*) \mid \partial_a(ac) \mid \partial_a(c^+)$

$= \partial_a(a)\,b^* \mid \partial_a(a)\,c \mid \partial_a(c)\,c^*$

$= \lambda b^* \mid \lambda c \mid \emptyset \cdot c^*$

$= b^* \mid c$

b) $\partial_0(10^*1) = \partial_0(1) \cdot 0^*1 \mid \epsilon(1) \cdot \partial(0^*1)$

$= \emptyset \cdot 0^*1 \mid \emptyset = \emptyset \mid \emptyset = \emptyset$

d) $\partial_a(a^+ba) = \partial_a(a^+) \cdot ba \mid \epsilon(a^+) \cdot \partial_a(ba)$

$= \partial_a(a \cdot a^*) \cdot ba \mid \emptyset$

$= \partial_a(a) \cdot a^* \cdot ba \mid \epsilon(a)\,\partial_a(a^*)\,ba$

$= \lambda a^* ba \mid \emptyset = a^* ba$

e) $\partial_a(a^*ba) = \partial_a(a \cdot a^*) \cdot ba \mid \epsilon(a^*)\,\partial_a(ba)$

$= \partial_a(a)\,a^* \cdot ba \mid \lambda \cdot \partial_a(b) \cdot a$

$= \lambda a^* ba \mid \emptyset = a^* ba$

f) $\partial_1(\partial_0(0(1 \mid \lambda) \mid 1^+)) = \partial_1(\partial_0(0(1 \mid \lambda)) \mid \partial_0(1^+))$

$= \partial_1(\partial_0(0) \cdot (1 \mid \lambda) \mid \emptyset)$

$= \partial_1(\lambda \cdot (1 \mid \lambda))$

$= \partial_1(1 \mid \lambda) = \partial_1(1) \mid \partial_1(\lambda)$

$= \lambda \mid \emptyset = \lambda$

$\partial_a(\underbrace{R}_{ER}) = \underbrace{R'}_{ER} : L(R') = a^{-1}L(R)$

3) a) $E_0 = (0|1)^* 01$

$$\partial_0((0|1)^* 01) = \partial_0((0|1)^*).01 \mid \epsilon((0|1)^*)\ \partial_0(01)$$
$$= \partial_0((0|1))(0|1)^* 01 \mid \lambda 1$$
$$= (\partial_0(0) \mid \partial_0(1))\ (0|1)^* 01 \mid 1$$
$$= (\lambda \mid \emptyset)\ (0|1)^* 01 \mid 1$$
$$= (0|1)^* 01 \mid 1 = E_1$$

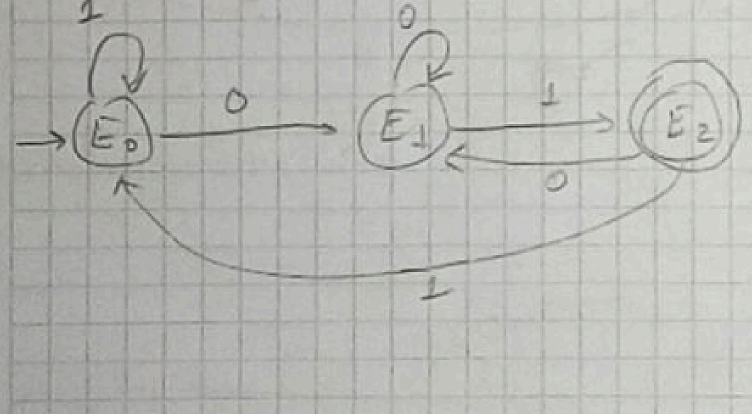
$$\partial_1((0|1)^* 01) = \partial_1((0|1)^*).01 \mid \partial_1(01)$$
$$= \partial_1((0|1))(0|1)^* 01 \mid \emptyset$$
$$= (\partial_1(0) \mid \partial_1(1))(0|1)^* 01$$
$$= \lambda (0|1)^* 01 = (0|1)^* 01 = E_0$$

$$\partial_0((0|1)^* 01 \mid 1) = \partial_0((0|1)^* 01) \mid \overset{\emptyset}{\widetilde{\partial_0(1)}}$$
$$= (0|1)^* 01 \mid 1 = E_1$$

$$\partial_1((0|1)^* 01 \mid 1) = \partial_1((0|1)^* 01) \mid \partial_1(1)$$
$$= (0|1)^* 01 \mid \lambda = E_2$$

$$\partial_0((0|1)^* 01 \mid \lambda) = (0|1)^* 01 \mid 1 = E_1$$

$$\partial_1((0|1)^* 01 \mid \lambda) = (0|1)^* 01 = E_0$$



b) $E_0 = (a(b|\lambda) \mid b^+)$

$\partial_a(a(b|\lambda) \mid b^+) = \partial_a(a(b|\lambda)) \mid \partial_a(b^+)$

$= \lambda(b|\lambda) \mid \emptyset = (b|\lambda) = E_1$

$\partial_b(a(b|\lambda) \mid b^+) = \partial_b(a(b|\lambda)) \mid \partial_b(b^+)$

$= \emptyset \mid b^* = b^+ = E_2$

$\partial_a(E_1) = \partial_a(b|\lambda) = \partial_a(b) \mid \partial_a(\lambda) = \emptyset = E_T$

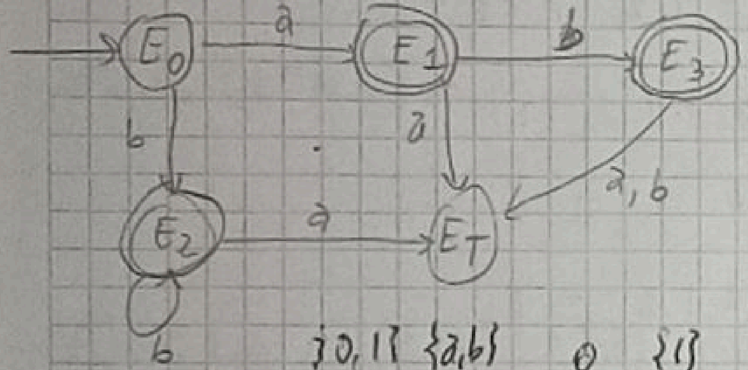
$\partial_b(E_1) = \partial_b(b) \mid \partial_b(\lambda) = \lambda \mid \emptyset = \lambda = E_3$

$\partial_a(E_2) = \partial_a(b^*) = \emptyset = E_T$

$\partial_b(E_2) = \partial_b(b^*) = b^* = E_2$

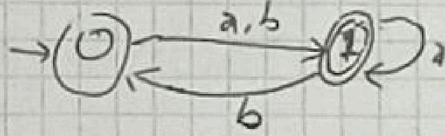
$\partial_a(E_3) = \partial_a(\lambda) = \emptyset = E_T$

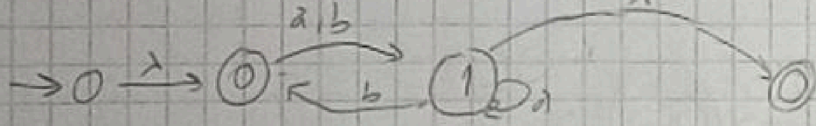
$\partial_b(E_3) = \emptyset = E_T$

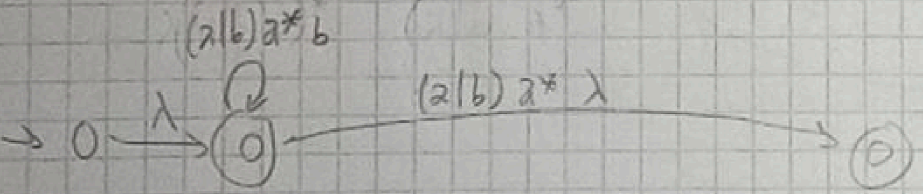


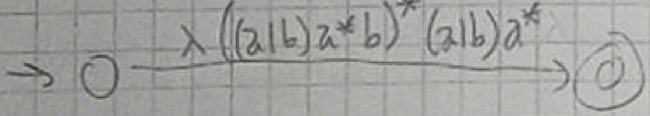
$\{0,1\}$ $\{a,b\}$ 0 $\{1\}$

4) $M_1 = \langle Q_1, \Sigma_1, \delta_1, q_1, F_1 \rangle$









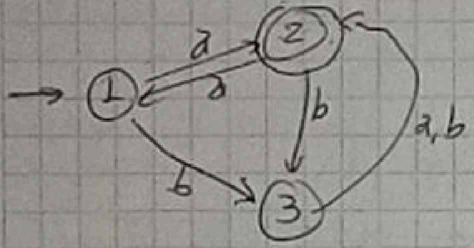
$\boxed{((a|b)a^*b)^*(a|b)a^*}$

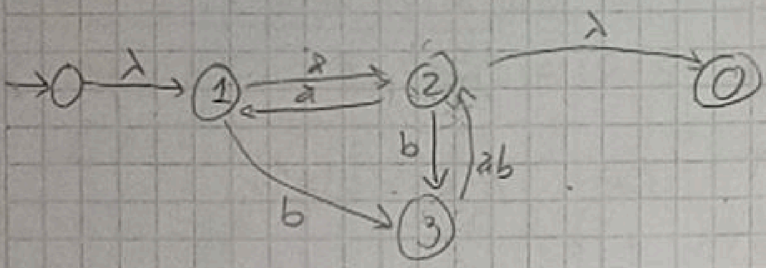
NOTA

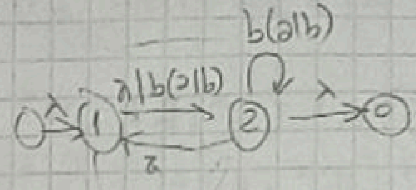
b) $M_2 = \langle \{1,2,3\}, \{a,b\}, \delta_2, 1, \{2\} \rangle$

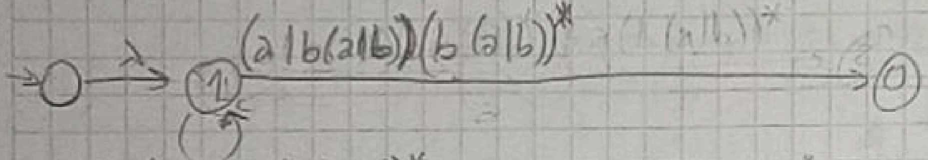
$\delta_2 =$

	a	b
1	2	3
2	1	2
3	2	2





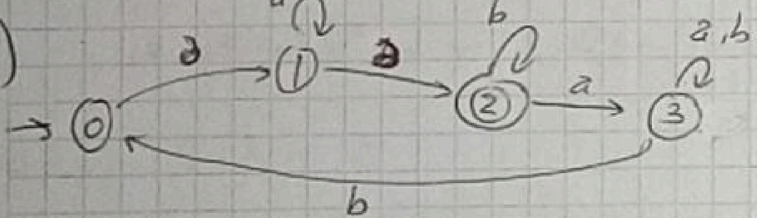


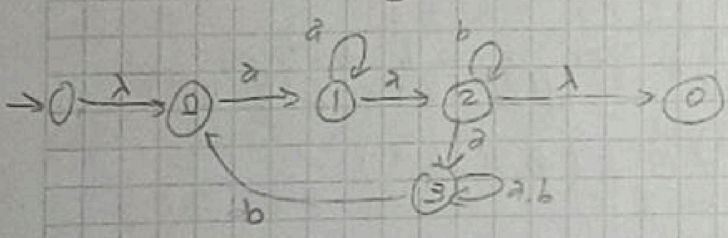


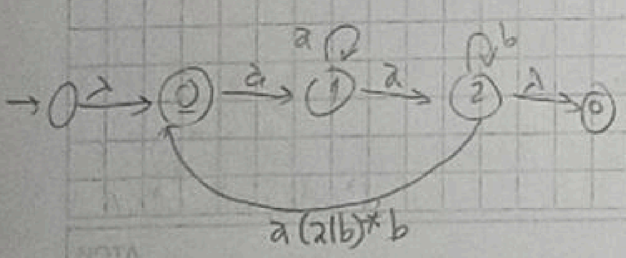
$(a|b(a|b))\,(b(a|b))^* a$

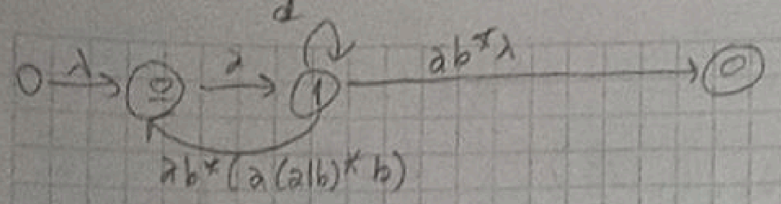
$(a|b(a|b))\,(b(a|b))^* a \;\; (a|b(a|b))\,(b(a|b))^*$

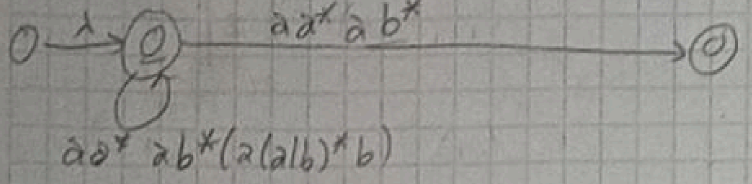
c)

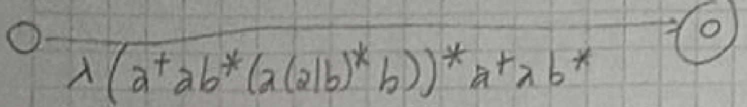












$\boxed{(a^+ab^*(a(a|b)^*b))^*a^+ab^*}$

5) a) $(R^*|R) = R^*$

$L(R^*|R) = L(R^*) \cup L(R)$ como $L(R) \subseteq L(R^*)$

$= L(R)^* \cup L(R) = L(R)^*$

que es justamente el lenguaje expresado por R^*

b) $R.R^* = R^*.R$

$L(R.R^*) = L(R).L(R^*) = L(R).L(R)^*$

Cadenas de la forma $\omega = r_1 \cdots r_n \quad n \geq 1 \quad r_i \in L(R)$

$L(R^*.R) = L(R)^*.L(R)$

Cadenas de la forma $\omega = r_1 \cdots r_n \quad n \geq 1 \quad r_i \in L(R)$

Ambos lenguajes tienen las mismas palabras, por lo que son iguales. Luego las ER son equivalentes.

c) $R.\underline{R^*R} = R.\underline{R.R^*}$

$L(R.R^*.R) = L(R).L(R^*.R)$ por punto b sabemos que $L(R^*.R) = L(R.R^*)$

$= L(R).L(R.R^*) = L(R.R.R^*)$

d) $(R^*)^* = R^*$

$L((R^*)^*) = L(R^*)^* = (L(R)^*)^* = L(R)^*$ por ej 10g guia 1

Luego el lenguaje asociado a $(R^*)^*$ es $L(R)^*$ que es el lenguaje asociado a R^*

$L(R^*) = L(R)^*$

e) ~~$R(S.R)^*$~~ $R(S.R)^* = (R.S)^* R$

$L(R(S.R)^*) = L(R).L((S.R)^*) = L(R).(L(S).L(R))^*$

$\alpha = \gamma\beta \qquad \gamma \in L(R) \qquad \beta \in (L(S)\,L(R))^*$

$\cancel{L((R.S)^* R) = (L(R).L(S))^* . L(R)}$

$\cancel{\alpha' = \beta'\gamma' \qquad \gamma' \in (L(R)\,L(S))^* \qquad \beta' \in L(R)}$

- Sup $\beta = \lambda \rightarrow \alpha \in L((R.S)^* R)$
- Sup $\beta \neq \lambda \rightarrow \beta = \beta_1 \dots \beta_n \quad n \geq 0 \quad \beta_i = \beta_i^1 \beta_i^2$

$\beta_i^1 \in L(S) \quad \beta_i^2 \in L(R)$

$$\alpha = \underbrace{\overset{\in L(R)}{\gamma} . \overset{\in L(S)}{\beta_1^1} \overset{\in L(R)}{\beta_1^2} \dots \overset{\in L(S)}{\beta_n^1}}_{\in (L(R)L(S))^*} \underbrace{\beta_n^2}_{\in L(R)} \in (L(R)L(S))^* L(R)$$

6) a) $R|\lambda \neq R$

$R = a \qquad L(R|\lambda) = \{a, \lambda\} \qquad L(R) = \{a\}$

b) $R.S \neq S.R \qquad R = a \qquad S = b$

$L(R.S) = L(R).L(S) = a.b \neq L(S.R) = L(S).L(R) = b.a$

c) $R.R \neq R \qquad R = a$

$L(a.a) = L(a).L(a) = a.a \neq L(a) = a$

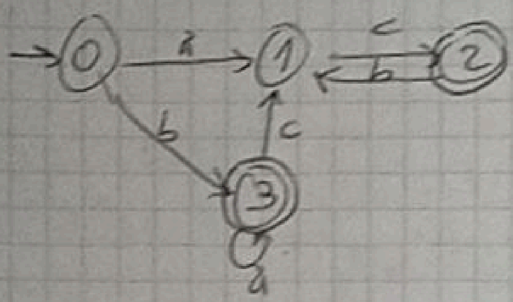
d) $R|(S.T) \neq (R|S).(R|T)$

$R = a \quad S = b \quad T = c$

$E_1 = R|(S.T) = a|(bc) \qquad a \in L(E_1) \quad a \notin L(E_2)$

$E_2 = (R|S)(R|T) = ab|ac$

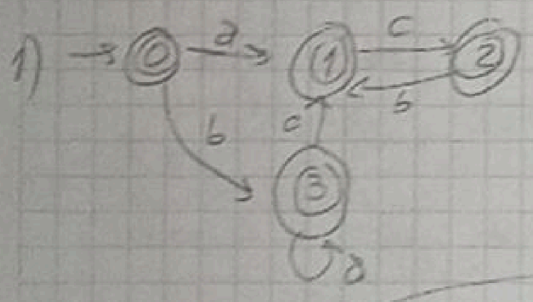
7) $M = \langle \{0,1,2,3\}, \{a,b,c\}, \delta, 0, \{2,3\} \rangle$

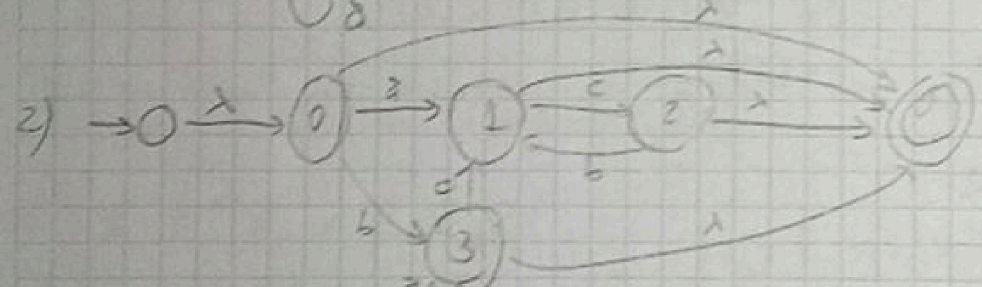


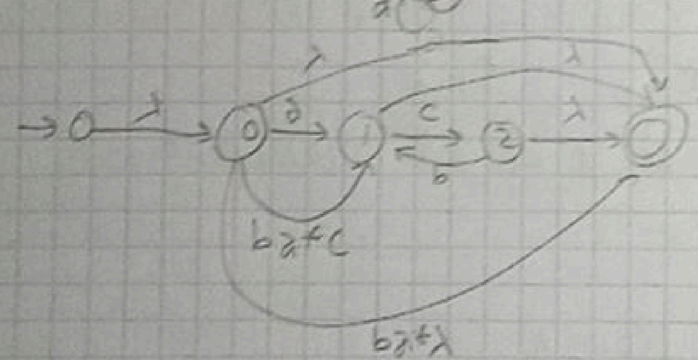
Dar ER para $(\overbrace{Ini(L(M))}^{\text{concatenaciones de prefijos de } L(M)})^*$

$Ini(L) = \{\alpha \in \Sigma^* \mid \exists \beta \in \Sigma^* : \alpha\beta \in L\}$
(prefijos)

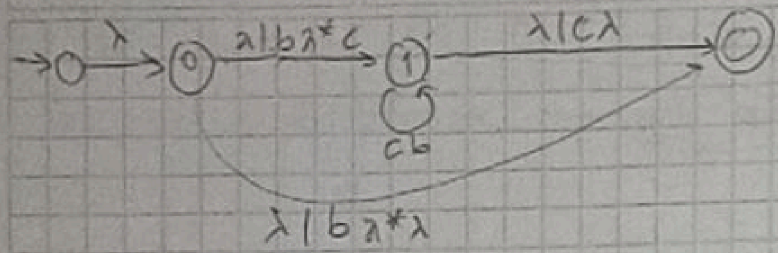
1) hacer todos los estados como final
2) hacer RE para esa AFD
3) Aplicar la $(.)^*$

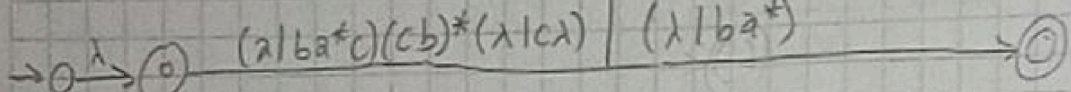
1)


2)




NOTA





$$\boxed{(\lambda|ba^*c)(cb)^*(\lambda|c) \mid (\lambda|ba)^*}$$

8) $L = L(\underbrace{((12|2)^*(\lambda|1))}_{E_0})$ Dar RE pra L^c $\Sigma=\{1,2\}$

$\partial_1((12|2)^*(\lambda|1)) = \partial_1((12|2)^*).(\lambda|1) \mid \partial_1(\lambda|1)$

$= \partial_1(12|2)(12|2)^*(\lambda|1) \mid (\partial_1(\lambda) \mid \partial_1(1))$

$= (\partial_1(12)\mid\partial_1(2))(12|2)^*(\lambda|1) \mid \emptyset \mid \lambda$

$= 2(12|2)^*(\lambda|1) \mid \lambda = E_1$

$\partial_2((12|2)^*(\lambda|1)) = \partial_2(12|2)(12|2)^*(\lambda|1) \mid \partial_2(\lambda|1)$

$= \lambda(12|2)^*(\lambda|1) \mid \emptyset = (12|2)^*(\lambda|1) = E_0$

$\partial_1(2(12|2)^*(\lambda|1)\mid\lambda) = \partial_1(2(12|2)^*(\lambda|1)) \mid \partial_1(\lambda)$

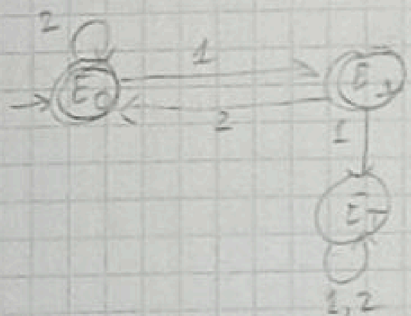
$= \emptyset \mid \emptyset = \emptyset = E_T$

$\partial_2(2(12|2)^*(\lambda|1)\mid\lambda) = \partial_2(2(12|2)^*(\lambda|1)) \mid \partial_2(\lambda)$

$= (12|2)^*(\lambda|1) \mid \emptyset = (12|2)^*(\lambda|1) = E_0$

	1	2
E_0	E_1	E_0
E_1	E_T	E_0
E_T	E_T	E_T

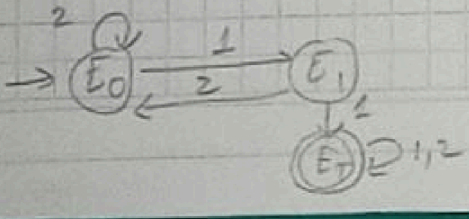
M:

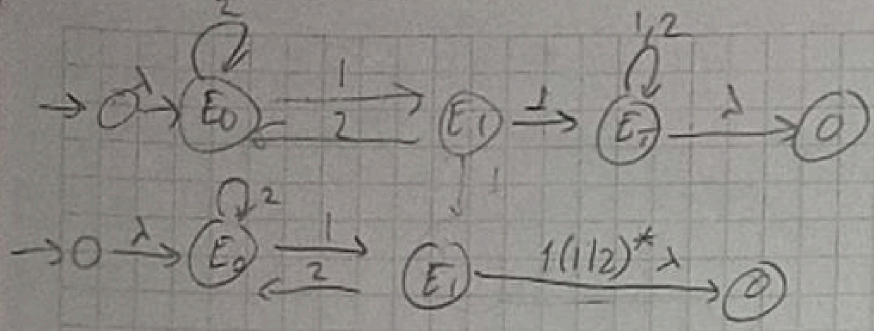


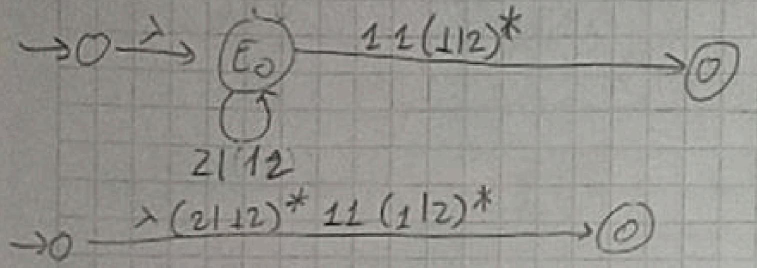
$L(M) = L(E_0)$

$L^c = L(M)^c = L(M^c)$

M^c:

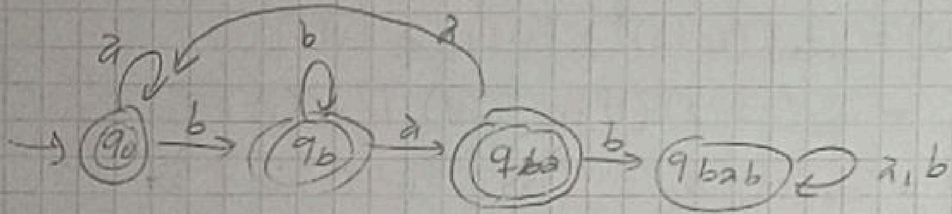


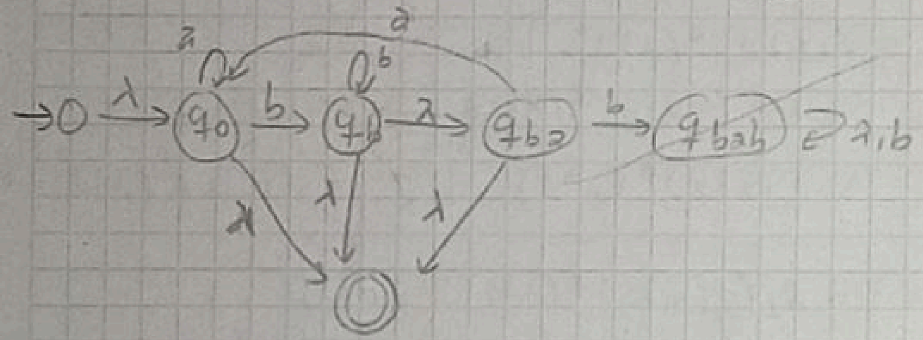


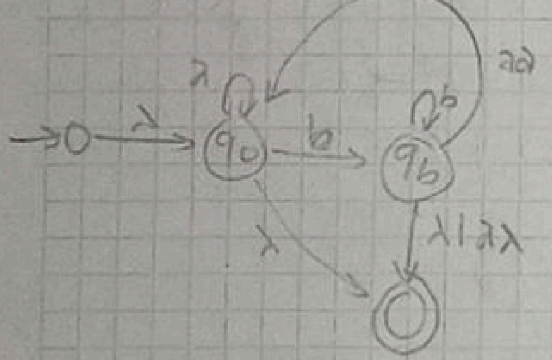


$$\boxed{L^c = L\left((2|12)^* 11 (1|2)^*\right)}$$

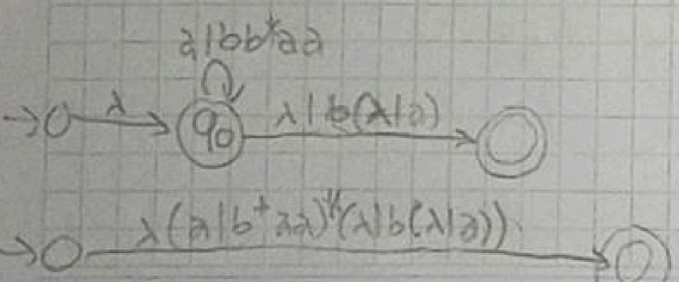
a) Dar RE pra $L = \{\omega \in \{a,b\}^* \mid bab$ no es subcadena de $\omega\}$







$$\boxed{(a|b^+aa)^*(\lambda|b(\lambda|a))}$$



10) a) Dado RE E. Dar método para obtener RE de $Ini(L(E))$ los prefijos del lenguaje expresado por E.

$Ini(\emptyset) = \emptyset$

$Ini(\lambda) = \lambda$

$Ini(a) = a$ para cada $a \in \Sigma$

Si S y R ER

$Ini(R|S) = Ini(R) | Ini(S)$

$Ini(R.S) = Ini(R) | R.Ini(S)$

$Ini(R^*) = R^* Ini(R)$

b) $Ini(L((aa|bb)^*)) =$

$$Ini((aa|bb)^*) = (aa|bb)^* \ Ini(aa|bb)$$
$$= (aa|bb)^* (Ini(aa) \,|\, Ini(bb))$$
$$= (aa|bb)^* ((a|aa) \,|\, (b|bb))$$
$$= \boxed{(aa|bb)^* (a|aa|b|bb)}$$

redundante?