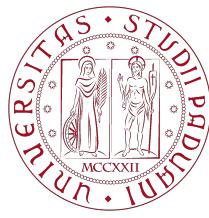


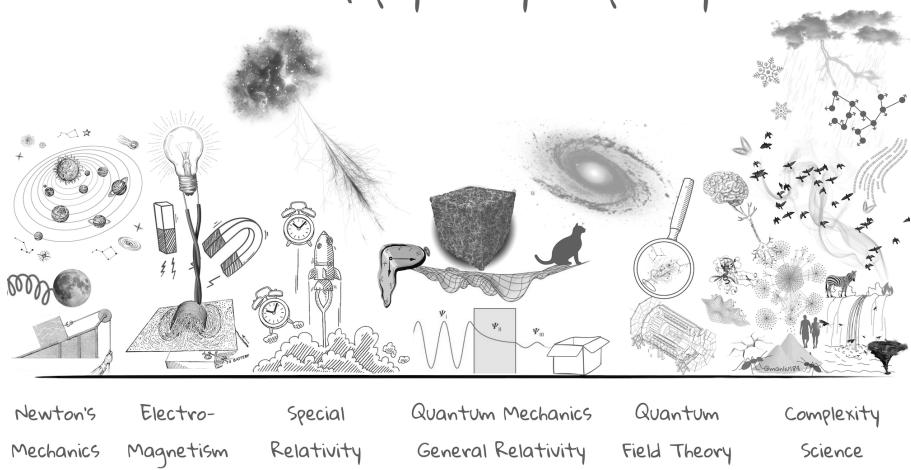
# Final Report

Physics of Complex Networks: Structure and Dynamics



UNIVERSITÀ  
DEGLI STUDI  
DI PADOVA

Areas of physics by complexity



Newton's Mechanics    Electro-Magnetism    Special Relativity    Quantum Mechanics    Quantum Field Theory    Complexity Science

## Projects Report: 15, 44

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# 1 | Task 15: Self-Organized Criticality on Networks

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## 1.1 | Framework (BTW sandpile on a graph)

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We simulate the Bak–Tang–Wiesenfeld (BTW) sandpile on a network with  $N$  nodes. Each node  $i$  carries an integer load  $z_i \in \{0, 1, 2, \dots\}$  and a threshold  $z_c(i)$ . At each driving step one grain is added to a uniformly random node; then the system relaxes via topplings. On a generic graph we take  $z_c(i) = k_i$  (degree threshold), so one toppling at  $i$  sends one grain to each neighbor. If  $z_i \geq z_c(i)$ , a toppling performs

$$z_i \leftarrow z_i - k_i, \quad z_j \leftarrow z_j + 1 \quad \forall j \in \partial i. \quad (1.1)$$

To reach a stationary regime on finite networks we include dissipation with small probability  $f$  (per-toppling) so that avalanches remain finite. An *avalanche* is the full relaxation triggered by one grain addition. We measure standard observables [2, 6]: size  $S$  (total topplings), duration  $T$  (number of update waves), and area  $A$  (distinct toppled nodes).

## 1.2 | Results

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**From SOC on networks to SOC with interdependence.** Power-law avalanche statistics on networks are a standard signature of self-organized criticality in sandpile models [2, 6]. Here we focus on a complementary question: how *interdependence* between two networked systems reshapes the probability of extreme events, following Brummitt et al. [3].

**Coupled sandpiles and large cascades.** We consider two modules  $A$  and  $B$  and add sparse interconnections controlled by a coupling parameter  $p$ . Here a *module* simply denotes one of the two subnetworks ( $A$  or  $B$ ) in the coupled system [3]. In this section,  $N$  denotes the number of nodes *per module* (so the coupled system has  $2N$  nodes). Because the BTW threshold is degree-based ( $z_c(i) = k_i$ ), adding bridges modifies both (i) pathways for load to propagate between modules and (ii) node capacities/total load that the system can hold [3].

Following [3], we classify events by whether a *large cascade* occurs in module  $A$ , using a fixed cutoff on the avalanche size in that module:  $S_A > C$  with  $C = \frac{N}{2} = 1000$ . We also track *global* cascades using  $S > C_g$  with  $C_g = N = 2000$ . Dissipation is

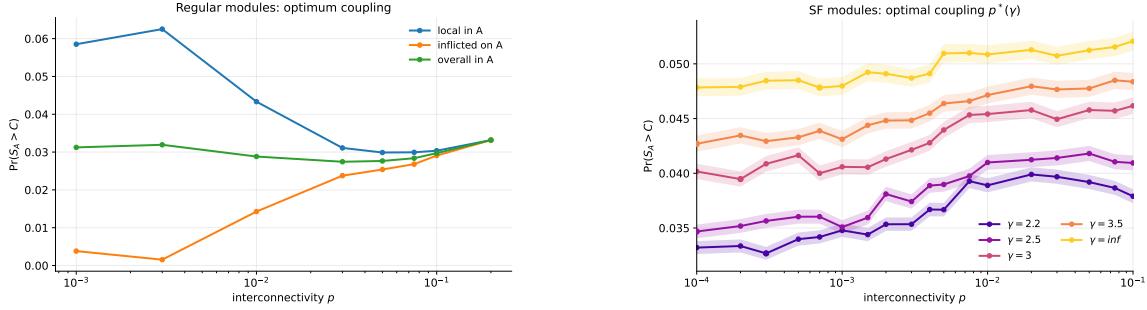


Figure 1.1: Coupled networks: probability of a large cascade in module  $A$  versus coupling  $p$  (regular vs scale-free modules). Scale-free modules show a stronger dependence on the generating exponent  $\gamma$ , while regular modules follow a smoother trend with  $p$ .

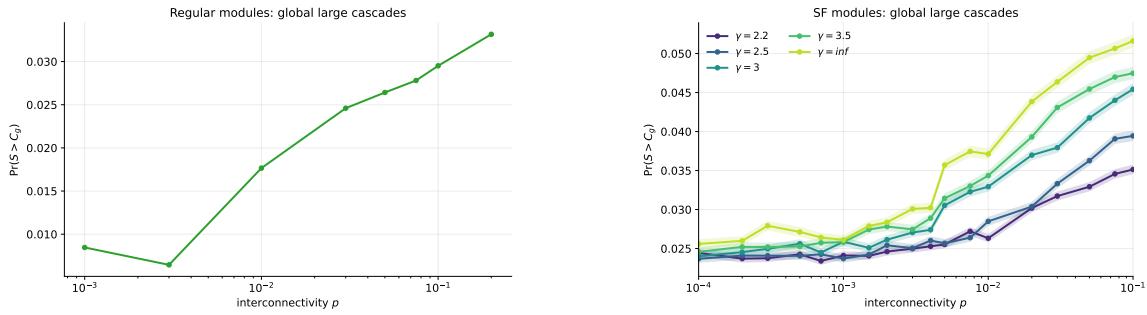


Figure 1.2: Robustness across event definitions: probability of a *global* large cascade ( $S > C_g$ , with  $C_g = 2000$ ) versus coupling  $p$ . The qualitative non-monotonic dependence on  $p$  persists, supporting the interpretation of an ‘‘optimal coupling’’ rather than a cutoff artifact [3].

implemented in per-toppling mode with probability  $f = 0.01$ . The ‘‘regular’’ modules are random regular graphs  $R(z_a)$  and  $R(z_b)$  (here  $z_a = z_b = 3$ ), whereas the scale-free modules are generated by a static model with tunable exponent  $\gamma$  (including  $\gamma = \infty$ , approaching an ER-like limit). In the scale-free panels, the shaded band shows a 95% Wilson score confidence interval (binomial) for the estimated probability from aggregated event counts. The three curves in Fig. 1.1 separate cascades that stay in  $A$  (local) from those triggered by activity arriving from the other module (inflicted), and their sum (any large in  $A$ ). For small  $p$ , interconnections are beneficial because they suppress the largest within-module cascades; for large  $p$ , they become detrimental because they enable inflicted events and also increase capacities/total load. The trade-off can produce an intermediate optimum in  $p$  [3].

Global large cascades (defined by  $S > C_g$ ) are rarer but more systemic events than ‘‘large-in- $A$ ’’ cascades. The non-monotonic dependence on coupling  $p$  persists: small  $p$  can suppress the largest within-module events, whereas large  $p$  enables inter-module propagation (inflicted cascades) and increases total capacity/load, which can fuel larger system-wide events. The similar qualitative shape across regular and scale-free modules supports the interpretation of an ‘‘optimal coupling’’ rather than a cutoff-specific artifact. For a detailed branching-process description and analytic estimates of the optimal coupling  $p^*$ , we refer to [3].

## 2 | Task 44: Social Connectedness Index II

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### 2.1 | Dataset and goal

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We use the Social Connectedness Index II (SCI) by Meta/HDX, which reports the intensity of Facebook friendship ties between pairs of administrative regions. Each record provides a pair of regions and a positive score (`scaled_sci`). This release covers 178 countries and corresponds to the reference period 2025-12-26 to 2026-01-25 (updated 2026-02-07, CC0) [7]. Following the task instructions, we exclude the USA from our outputs.

**Concept of SCI.** SCI is a normalized measure of friendship intensity between locations. Conceptually, for regions  $i, j$  one can think of

$$\text{SCI}_{ij} \propto \frac{F_{ij}}{U_i U_j},$$

where  $F_{ij}$  is the number of friendships connecting the two regions and  $U_i, U_j$  are the corresponding Facebook-user populations; published values are then rescaled within each layer to  $[1, 10^9]$  (`scaled_sci`) [7, 1]. In our analysis we treat `scaled_sci` as a *within-country* edge weight.

### 2.2 | Network construction

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**End-to-end pipeline.** From the raw SCI layer CSVs, we:

- select the set of countries to include;
- set the resolution per country: EU countries use NUTS3 (higher resolution), while all other countries use GADM level 1;
- keep within-country rows only (`user_country = friend_country`), drop self-loops, store one edge per unordered pair, and sum duplicate `scaled_sci`;
- optionally add coordinates by joining a separate region-centroids table (built from boundary polygons), then export the submission CSVs and run sanity checks.

**Nodes.** Nodes represent subnational administrative regions at the best available resolution. We export a single `nodes.csv` with global consecutive `nodeID` and `nodeLabel`; labels are prefixed with ISO3 (e.g., ITA:...) to avoid collisions.

**Edges.** Edges are built *within country* only. We treat ties as undirected, keep one edge per unordered region pair, and aggregate duplicates by summing `scaled_sci`; self-loops are removed. The submission file `edges.csv` stores endpoints and country tags; `edges_weighted.csv` is produced for analysis/debug.

**Geographic coordinates.** The SCI layer CSVs do not include coordinates. We therefore build a region-to-coordinate table by downloading administrative boundary datasets and extracting one representative point per polygon (EPSG:4326), using:

- **GADM v4.1** level-1 polygons (downloaded as shapefiles; region code `GID_1`) [5].
- **EU NUTS 2024** level-3 polygons (GISCO GeoJSON; region code `NUTS_ID`) [4].

The resulting (`latitude`, `longitude`) are merged into `nodes.csv` when available; boundary files are large and are downloaded on demand and cached locally.

## 2.3 | Sanity checks

For the selected top-100 countries (by available GADM1 regions, USA excluded), the global output contains  $N = 3040$  nodes and  $E = 134016$  edges. Coordinate coverage is high after including centroids ( $\sim 98\%$  of nodes). Within-country graphs are often very dense (close to complete), so unweighted structure is less informative than weighted summaries. To mitigate size effects we inspect mean edge weight (total `scaled_sci` divided by  $E$ ).

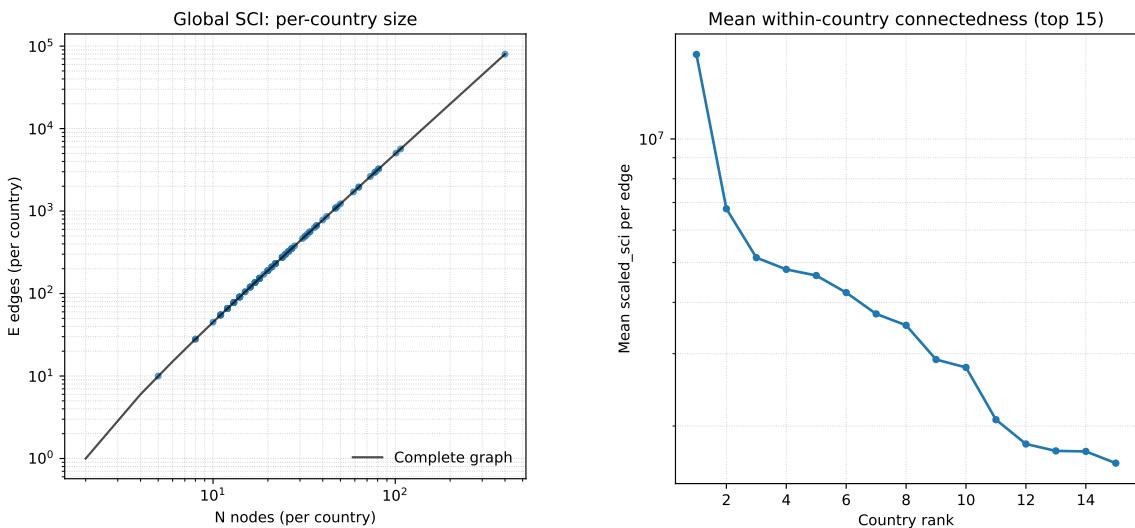


Figure 2.1: Left: number of edges versus number of nodes for each country network (log–log scale). The black curve is the complete-graph reference  $E = N(N - 1)/2$ . Right: mean within-country connectedness (total `scaled_sci` divided by  $E$ ) versus country rank (top 10).

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