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Asymptotic Notations

See ‘Asymptotic notations.mp4’

Big Oh = O notation = gives upper bound of time complexity

Big Omega = Ω notation = gives lower bound of time complexity

Theta = θ notation = gives avg time complexity

There are some methamatical equations to find out wither some value can be a big oh/big omega/thetha of actual time complexity.

Big Oh gives worst case time complexity. Big Omega gives best cast time complexity Thetha gives avg time complexity.

Generally, we are interested in worst case time complexity. So, we use Big Oh notation for algorithms.

5,6,3,7,9,0

If you need to find an element in this array, worst case can be n comparisons. Best case can be only 1 comparison and avg case can be n/2 comparisons.

So, O(n), Ω(1), θ(n/2)

## Time and Space complexity

### Loops

For(int i=0; i< n; i++) – O(n)

For(int i=n; i> 0; i--) – O(n)

For(int i=0; i < n; i = i+2) – O( n/2 )

For(int i=n; i > 0; i = i-2) – O( n/2 )

For(int i=0; i < n; i = **i\*2**) – O( log(n) )

For(int i=n; i > 0; i = **i/2**) – O( log(n) )

O(n^2) operations

For(int i=0; i< n; i++) {

For(int j=0; i< n; j++) {

}

}

For(int i=0; i< n; i++) {

For(int j=i+1; i< n; j++) {

}

}

O(n log n) operation

For(int i=0; i< n; i++) {

For(int j=0; i< n; j = **i+j**) {

}

}

(1+2+3+.....+n) = (n(n+1) / 2) = O(n^2)  
n+(n-1)+(n-2)+....+1 = (n(n+1) / 2) = O(n^2)  
n^K + (n^k-1) + (n^k-2) + ......+ = O(n^k)  
1 + 1/2 + 1/3 + 1/4 + ...... 1/n = O(log n)

2^0+2^1+…..+2^n = 2^n+1 - 1 =O(2^n)

<https://www.interviewbit.com/problems/reccmpl1/>

**int** **searchNumOccurrence**(vector**<int>** **&**V, **int** k, **int** start, **int** end) {

**if** (start **>** end) **return** 0;

**int** mid **=** (start **+** end) **/** 2;

**if** (V[mid] **<** k) **return** searchNumOccurrence(V, k, mid **+** 1, end);

**if** (V[mid] **>** k) **return** searchNumOccurrence(V, k, start, mid **-** 1);

**return** searchNumOccurrence(V, k, start, mid **-** 1) **+** 1 **+** searchNumOccurrence(V, k, mid **+** 1, end);

}

Takes O(n), for [3,3,3,3,3,3,3,3,3] and if you are finding k=3

<https://www.interviewbit.com/problems/amortized1/>

**int** j **=** 0;

**for**(**int** i **=** 0; i **<** n; **++**i) {

**while**(j **<** n **&&** arr[i] **<** arr[j]) {

j**++**;

}

}

At the first look, it may seem like it depends on value of arr, but if use see regardless of value of arr, outer loop will always execute.

At the second look, it may seem like O(n^2), if array is in ascending order.

But if u see properly, j is not reset to 0 for every single i. So, answer is O(n).

### When number of comparisons at each step doubles

1+2+4+8+16+……X

Runtime complexity = 2^n+1 -1 = O(2^n)

The sum of the sequence of powers of two is roughly equal o the next value in the sequence.

2^0 +2^1 +2^2 +2^3+2^4 = 2^5 - 1

### When number of comparisons at each step halvs

Let’s take Binary Search algorithm

At every step, number of elements available to compare halvs

N=16

N=8

N=4

N=2

N=1

N = 2^k

Log2 N = k

e.g.

16 = 2^4

log2 16 = 4

This is a bit hard to understand. Understand the explanation given in [Binary Search](#_Binary_Search) section.

Basically, when number of elements halvs at each step and number of comparisions at each step is 1 only, then runtime complexity is O(log n). This is the case with Binary Search.

### When number of comparisions increases by 1 at each step

(pg 46, 47 of CCA book)

for(int i=0; i<n; i++) {

for(int j=0; i<n; j++) {

…

}

}

Number of operations – n + n-1+ n-2 + n-3 +…..+ 1

So it is 1 + 2 + 3 + 4 + ….. n

It will come to n(n+1)/2, which will be O(n^2). This is how you calculate the total

<https://www.wikihow.com/Sum-the-Integers-from-1-to-N>



Similarly,

for(int i=0; i<n; i++) {

for(int j=i+1; i<n; i++) {

…

}

}

number of comparisons n-1 + n-2 + n-3 +…… 1

so, it is 1 + 2 + 3 + 4 + ….. n-1

In above fomula (n(n+1)/2), replace n by n-1

So, it will be (n-1)n/2, which will be O(n^2)

How O(C + log n) is different than O(C log n) ?  
  
SearchElementInSortedMatrix.java is an important algorithm to understand how O(C + log n) is calculated. In recursive tree of height (log n), if only one node id doing some operation taking O(C) time, then it comes to O(C + log n). if every node is doing that operation, then it comes to O(C log n).  
  
 This is classic Binary Search recursive calls.  
  
 BS(n)  
 BS (n/2)  
 BS(n/4)  
 ...  
 e.g.  
 In Binary Search, every node does O(1) operation of comparing required element with mid. So, it is O(1 log n).  
 If some algorithm does O(log m) taking operation on each node, then it is O((log m) (log n)).  
 If some algorithm does O(log m) taking operation on only one node of entire tree, then it is O(log m + log n).

### Matrix Multiplication

Remember, Matrix Multiplication takes O(n^3).

MatrixMultiplication\_iterative.java ---- O(n^3)

MatrixMultiplication\_divide\_and\_conquer.java --- O(n^3) + stack space is used for recursion

MatrixMultiplication\_Strassens\_Algorithm.java --- O(n^2.81) + stack space is used for recursion

### How long it takes to Sort Strings (not integers)?

(pg 49 of CCA book)

To sort an array of integers, quick sort takes O(n log n), we know that. During quick sort, when comparison of 2 integers happens, it takes O(1). Look at Integer class’ compareTo method.

But in case of Strings, to compare two strings of size s takes O(s). So, sorting of strings will take O(sn log n).

### Tricking question

for(int x=2; x\*x <= n; x++) {

….

}

What is runtime complexity?

x\*x <= n

x^2 <= n

x <= sqrt(n)

So, runtime complexity= O(sqrt(n))

### Momizaiton example

Pg 53 of CCA book

Fibonacci example with memoization

You can read more about Memoization in [Tree’s Recursion Concepts](#_Recursion_Concepts).

### O(n!) example

Pg 51 of CCA book

### What makes the running time n! instead of n^2?

For(int i=0; i<n; i++) {

For(int j=0; j<n; j++) {

….

}

}

This is n^2 operations.

Let’s say, n=10 (0 to 9). You have 10 dots in a graph. Starting from 0, you need to find all combinations to reach to 9. There can be many combinations. 0->1->2…->9, 0->2->1->…9 etc.

This kind of behavior needs n! operations.

Greedy Algorithm or Dynamic Programming is a solution for n! problem. Greedy Algorithm gives close to optimal solution at the end and Dynamic Programming give most optimal solution at the end, but you cannot Dynamic Programming all the time.

**Read** [When to use Greedy Programming and When to use Dynamic Programming?](#_Greedy_and_Dynamic)

### Recursive Method - Time Complexity

There are mainly two ways to figure out time complexity of an algorithm. Third way (Master’s Theorem) is a short cut.

#### Master’s Theorem

This theorem is a short cut of Back Substitution Strategy.

<https://www.geeksforgeeks.org/analysis-algorithm-set-4-master-method-solving-recurrences/>

T(n) = a T(n/b) + n^c, where a>=1 and b>1  
  
 if c < log a, then result is n ^ log a  
 b b  
 if c = log a, then result is log a \* n^c  
 b b  
 if c > log a, then result is n^c  
 b  
  
 e.g. Merge Sort, MatrixMultiplication\_divide\_and\_conqure.java

T(n) = 8 T(n/2) + n^2  
  
 a = 8, b = 2, c = 2  
  
 so, log a = log 8 = 3  
 b 2  
  
 so, T(n) = n ^ log 8  
 2  
 = n ^ 3  
  
  
You can prove Merge Sort's time space also using Master's theorem because Merge Sort's time complexity formula matches with Master's Theorem formula.

#### Back Substitution strategy

Watch ‘Back Substitution Method.mp4’ and see an example in TowerOfHenoi.java, Fibonacci.java.

#### Recursion Tree strategy

Cracking in Coding Algorithm book suggest this approach.

I like this strategy more than ‘[Back Substitution Strategy](#_Back_Substitution_strategy)’ because it helps to identify whether there is a chance to use Dynamic Programming top-bottom strategy (Memoization) to improve the time complexity.

You need to memorize some of the calculations. They are explained in below examples, but it’s better to memorize them.

e.g.

1+2+3+….+ n = n(n+1)/2 = O(n^2)

1+2+3+….+ n-1 = (n-1)n/2 = O(n^2)

n+n/2+n/4+…..+1 ~= 2n

2^0 + 2^1 + 2^2 + 2^3+…+2^n = 2^n+1 -1 = O(2^n)

**Height of balanced tree is O(log n) where base of the log will be same as number of branches each node has.** Balanced Binary Tree’s height is O(log2 n). Balanced Ternary Tree’s height is O(log3 n), where n is the number of nodes in a tree.

When you need to find out time complexity of a recursive method, you need to ask yourself

* How many nodes are there in recursive method tree?
* How many operations are happening on each node of recursive method tree? Are they same on each node?

If not, then

how many operations are happening on each level of the recursive method tree? Are they same on each level?

If you see method call with same input multiple times in recursive method tree, then there is a possibility of improving the time complexity by memorizing the output of the method (using Top-Bottom approach of Dynamic Programming).

##### Fibonacci Series O(2^n) and O(n), LIS O(2 ^ n) and O(n^2), Balanced Expression O(n \* 2^n), SubSet Sum O(2^n) and O(sum\*n)

###### Fibonacci Series

int f(int n) {

if(n=0 || n==1) return 1; ----- there is O(1) operation on each recursive call

return f(n-1)+f(n-2);

}

f(4)

f (3) f(2)

f(2) f(1) f(1) f(0)

f(1) f(0)

Height of the tree is n here and on each level, number of nodes are doubling. So, total number of nodes are around 2^n+1 -1 = O(2^n). On each node, exactly 1 operation is happening. So, time complexity is O(2^n).

there are 2^0 + 2^1 + 2^2 + 2^3+…+2^n = 2^n+1 -1 nodes in this recursive call tree.

Each node is doing just O(1) operation. So, time complexity is O(2^n).

Actual number of nodes will be little less than 2^n as you see right tree has a few less number of nodes than left tree. But you can go with 2^n

Height of tree is 4. Number of stack slots will be used is 4 = **O(n) is the space complexity that is same as height of the tree**.

When you use Dynamic Programming, only 2n nodes will be created

fib(5)

fib(4) fib(3)

fib(3) fib(2)

fib(2) fib(1)

fib(1) fib(0)

**TripleSteps.java and TowerOfHenoi.java** also falls under similar category.

###### Longest Increasing SubSequence (LIS)

LIS(…) {

exit condition(s) ---- executed once per method call

for(….) { // n-1 times

LIS(…)

…..

}

…..

}

LIS(5)

LIS (4) LIS(3) LIS(2) LIS(1)

LIS(3) LIS(2) LIS(1) LIS(2) LIS(1) LIS(1)

LIS(2) LIS(1) LIS(1) LIS(0) LIS(1)

LIS(1)

This kind of algorithm also has 2^n nodes. E,g. **LongestIncreasingSubSequenceInArray.java**

When you use Dynamic Programming, only n^2 nodes will be created

LIS(8)

LIS (7) LIS(6) LIS(5) LIS(4) LIS (3) LIS(2) LIS(1)

LIS(6) LIS(5) LIS(4) LIS (3) LIS(2) LIS(1)

LIS(5) LIS(4) LIS (3) LIS(2) LIS(1)

LIS(4) LIS (3) LIS(2) LIS(1)

LIS (3) LIS(2) LIS(1)

LIS(2) LIS(1)

LIS(1)

7 nodes + 6 nodes + 5 nodes + 4 nodes +3 nodes + 2 nodes + 1 node ~= n^2 nodes

###### Balanced Expression With Replacement – O(n \* 2^n)

There is one more example **BalancedExpressionWithReplacement.java** that doubles the number of recursions in a for loop and exit condition also loops n times.

isBalance(arr,start,end) {

**for** (**int** i = start; i <= end; i++) {

exit condition ----- loops n times

boolean res1 = isBalance(arr,start+1,end);

…..

boolean res2 = isBalance(arr,start+1,end);

return res2;

}

return …;

}

There are two differences between LIS algorithm and this algorithm.

* Method is recursed more than once in a for loop
* Exit condition is also executed n times

Rule of Thumb:

No matter how many times method is recursed in a for loop, number of nodes in a recursive tree will still remain ~ O(2^n).

In LIS, number of nodes are

2^2 + 2^3 + 2^4 +….

In this algorithm number of nodes will be

2^3 + 2^4 +2^5 +….

So, still ~ O(2^n) nodes.

Exit condition is executed n times on each method call. So, each node will do a task that takes O(n).

So, Time Complexity = O(n \* 2^n)

If you can use Dynamic Programming, it will have 2 \* n^2 nodes. So, it will take O(n \* (2\*n^2)) = O(2 \* n^3)

###### SubSet Sum

FindIfASubsetWithGivenSumExistsInGivenArray.java

Inputs = array(1,2,3,4,5), expected sum=12

If you see, you need to iterate through 2^n combinations of array elements whether any of those combinations make the sum=12.

So, time complexity of Brute-Force approach will be O(2^n)

(1,2,3,4,5),sum=12

(1,2,3,4),sum=12 (1,2,3,4),sum=7

(1,2,3),sum=12 (1,2,3),sum=8 (1,2,3),sum=7 (1,2,3),sum=3

(1,2),sum=12 (1,2),sum=9 (1,2),sum=8 (1,2),sum=5 (1,2),sum=7 (1,2),sum=4 (1,2),sum=3 (1,2),sum=0

(1),sum=12 (1),sum=10 (1),sum=9 (1),sum=7 (1),sum=8 (1),sum=6 (1),sum=5 (1),sum=3 (1),sum=7 (1),sum=5

---------- ------------

Using Dynamic Programming, you can reduce time complexity to O(sum \* n). If you cannot figure out by removing duplicates from above recursive tree, then think of Bottom-Up Dynamic programming approach, in which, you draw a 2-D matrix(array elements vs sum). That 2-D matrix will have sum\*n cells and you need to fill up those cells.

Time Complexity using Back Substitution

<https://www.youtube.com/watch?v=pqivnzmSbq4>

T(n) = T(n-1) + T(n-2) + number of operations happens at root level

**This formula is a bit different than T(n) = 2T(n/2) + constant --- you see this in tree/array recursion where number of nodes/elements are becoming half at each recursion**   
   
 T(n) = T(n-1) + T(n-2) + C  
   
 **Assuming that T(n-2) ~= T(n-1**) ---- in reality T(n-2) is little less than T(n-1)  
 This assumption will find **upper bound** of time complexity.  
   
 T(n) = 2T(n-1) + C  
 T(n-1) = 2T(n-2) + C  
 T(n-2) = 2T(n-3) + C  
   
 T(n) = 2T(n-1) + C  
 = 4T(n-2) + 3C  
 = 8T(n-3) + 7C  
 = (2^3) T(n-3) + (2^3-1)C  
   
 replacing the value that is changing with k. here, it is 3.  
   
 = (2^k) T(n-k) + (2^k-1)C  
   
 applying base condition, n-k=0. So, k=n.  
   
 = 2^n T(0) + (2^k-1)C  
   
 T(0) = 0  
   
 = 0 + (2^k-1)C  
 ~= 2^k = **2^n --------- upper bound**  
   
   
 **Assuming that T(n-1) ~= T(n-2)** ---- in reality T(n-1) is little more than T(n-2)  
 This assumption will find **lower bound** of time complexity.  
   
 T(n) = 2T(n-2) + C  
 T(n-2) = 2T(n-4) + C  
 T(n-4) = 2T(n-6) + C  
   
 T(n) = 2T(n-2) + C  
 = 2(2T(n-4) + C) + C  
 = 4T(n-4) + 3C  
 = 4(2T(n-6) + C) + 3C  
 = 8T(n-6) + 7C  
 = 2^k T(n-(k\*2)) + (2^k - 1)C  
   
 n-(k\*2) = 0  
 n = k\*2  
 k = n/2  
   
 = 2^(n/2) T(0) + (2^(n/2) - 1)C  
 ~=**O(2 ^ n/2) --------- lower bound**

**TripleSteps.java and TowerOfHenoi.java**’s time complexity is almost same as Fibonacci.

##### Binary Search **O(C log n)**

**Time complexity O(C log n) :**

BS(8)

BS (4) BS (4)

(not used)

BS (2) BS(2)

(not used)

BS(1) BS (1)

(not used)

Unlike to Fibonacci Series, in binary search, at each level, only 1 node is created. Each node in a tree is doing some constant time(O(1)) operation. So, there will be log n nodes in a tree. As each node is doing constant time operation, then the number of operations are also log n. As binary search divides the number of elements in 2 halvs, log has base 2. So, time complexity of binary search is log2 n.

**Time Complexity O(C + log n) :**

SearchElementInSortedMatrix.java is an important algorithm to understand how O(C + log n) is calculated.  
In recursive tree of height (log n), if only one node id doing some operation taking O(C) time, then it comes to O(C + log n). if every node is doing that operation, then it comes to O(C log n).  
  
This is classic Binary Search recursive calls.  
  
 F(n)  
 F(n/2)  
 F(n/4)  
 ...  
 e.g.  
 In Binary Search, every node does O(1) operation of comparing required element with mid. So, it is O(1 log n).  
 If some algorithm does O(log m) taking operation on each node, then it is O((log m) (log n)).  
 If some algorithm does O(log m) taking operation on only one node of entire tree, then it is O(log m + log n).

##### Quick Sort

QS(8) --- 8 elements are visited

QS (4) --- 4 elements are visited(n/2 operations) QS (4) --- 4 elements are visisted(n/2 operations) --- total n operations

QS (2) QS (2) QS(2) QS(2)

QS (1) QS (1) QS (1) QS (1) QS (1) QS (1) QS (1) QS (1)

8=2^3

so n = 2^k

Total number of nodes = 2^0+2^1+2^2+2^3+2^4 = 2^(k+1) – 1

In Fibonacci Series, each node is doing constant time operation, but that’s not the case here. Here, each level is doing n operations. So, time complexity=n\*height of a tree = n log2 n. log has base 2 because each node has 2 branches.

Important:

If the elements are objects(String or any other object), then checking equality operation trakes s time, then time complexity will be O(sn log n).

##### Binary Search Tree

Take an example of an algorithm that checks whether a tree is balanced by checking the height of its left and right sub trees.

There are two different approaches to write this algorithm.

One that takes O(n log n) time and another that takes O(n) time.



isBalanced(root) --- n nodes are visited

getHeight(root.left) getHeight(root.right)

isBalanced(left) isBalanced(right) ---- n-1 nodes are visited

getHeight(left) getHeight(right) getHeight(left) getHeight(right)

isBalanced(left) isBalanced(right) isBalanced(left) isBalanced(right)

getHeight(left) getHeight(right) getHeight(left) getHeight(right) getHeight(left) getHeight(right) getHeight(left) getHeight(right)

At each level, approx. n nodes are visited. Total number of levels are log2 n. **So, time complexity is O(n log n).**

Unlike to number or array based algorithms (fibonacci and quick sort), you don’t count number of recursive calls like 2^0+2^1+2^2+….+2^n. Number of recursive calls in a tree based algorithms are always same as number of nodes in a tree. If n=number of nodes in a tree, then number of recursive calls are also n.

Improved version of this algorithm



isBalanced(root)

calls

getHeight(root)

getHeight(left) getHeight(right)

getHeight(left) getHeight(right) getHeight(left) getHeight(right)

Each getHeight visits 1 node only on each method call. So, there is a constant time operation in each recursive call.  
Total number of nodes in recursion tree is always same as number of nodes in a tree. If number of of nodes in a tree is n, then number of recursions is also n.  
  
Remember, when each recursive method call does x operations. time complexity is O(xn).  
Here it is doing 1 operation, so, O(n).

If it would have been doing n operations in each recursive call, then time complexity would be O(n\*n).  
if it would have been doing n operations at each level , then time complexity would be (n log n).

log will have base same as number of branches for a node.

###### Using Back Substitution for Tree problems

T(n) = 2T(n/2) + number of operations happens at root level

T(n) = 2T(n/2) + n

T(n/2) = 2T(n/4) + n/2

T(n/4) = 2T(n/8) + n/4

T(n) = 2(2T(n/4) + n/2) +n --- replacing T(n/2)

= 4T(n/4) + n + n

= 4(2T(n/8) + n/4) + n + n --- replacing T(n/4)

= 8T(n/8) + n +n + n

= 2^3 T(n/2^3) + 3n

**Replace the number that is changing at each step. Here, it is 3.**

T(n) = 2^k T(n/2^k) + kn

Replace T(n/2^k) with base condition. **You at least have to assume that there is at least one node exist in a tree., even though you may have base condition checking 0 nodes (root==null) in your code. Because if you replace n/2^k = 0 then k=0, which will give you wrong result. If it wouldn’t result k=0, then you could keep base condition as T(0).**

**This rule is only for trees.**

So, base condition is T(1)

n/2^k = 1

n = 2^k

log2 n = k

replace k by log2 n

T(n) = 2^ (log2 n) + (log2 n)n

~= (log2 n)n = O(n log n)

##### Matrix

Recursion Method Tree in matrix looks same but important thing is to remember is number of elements n=number of rows \* number of cols.

Read RobotInGrid.java carefully to understand how time complexity changes when you want to find all possible paths to reach from one to another cell compared to finding just one possible path.

T(r,c)  
 T(r-1, c) T(r, c-1)  
T(r-2, c) T(r-1, c+1) T(r-1, c-1) T(r, c-2)

Height of recursive tree will be r+c, for this kind of algorithm.

Time complexity will be O(2^ r+c)

If we memoization the result, then it can be reduced to O(rc) by visiting every cell just once.

RobotInGrid.java anyways visits every cell just once without need of memoization. It takes O(2(r+c)) time.

Another example

<https://www.interviewbit.com/problems/reccmpl2/>

**int** **findMinPath**(vector**<**vector**<int>** **>** **&**V, **int** r, **int** c) {

**int** R **=** V.size();

**int** C **=** V[0].size();

**if** (r **>=** R **||** c **>=** C) **return** 100000000; *// Infinity*

**if** (r **==** R **-** 1 **&&** c **==** C **-** 1) **return** 0;

**return** V[r][c] **+** min(findMinPath(V, r **+** 1, c), findMinPath(V, r, c **+** 1));

}

O(2^(R+C)) is the right answer.

<https://www.interviewbit.com/problems/reccmpl3/>

**int** memo[101][101];

**int** **findMinPath**(vector**<**vector**<int>** **>&** V, **int** r, **int** c) {

**int** R **=** V.size();

**int** C **=** V[0].size();

**if** (r **>=** R **||** c **>=** C) **return** 100000000; *// Infinity*

**if** (r **==** R **-** 1 **&&** c **==** C **-** 1) **return** 0;

**if** (memo[r][c] **!=** **-**1) **return** memo[r][c];

memo[r][c] **=** V[r][c] **+** min(findMinPath(V, r **+** 1, c), findMinPath(V, r, c **+** 1));

**return** memo[r][c];

}

In this example, outcome is memoized. So, time complexity = O(rc)

##### Something different than above problems

If you have an algorithm where root processing takes n^2 and recursive calls are made on half-half elements.

T(n) --- number of operations n^2

T(n/2) T(n/2) --- number of operations on each node (n/2)^2

T(n/4) T(n/4) T(n/4) T(n/4) --- number of operations on each node (n/4)^2

…

**This problem is different than all above problems because number of operations on each level is different and number of operations on each node is also different.**

Calculation:

n^2 + (n/2)^2+(n/2)^2 + (n/4)^2+(n/4)^2 +….

n^2 + n^2/4+ n^2/4 + n^2/16+n^2/16+…

n^2 (1+ ¼ + 1/16 +…)

approx. 2(n^2)

So, O(n^2)

###### Using Back Substitution

T(n) = 2T(n/2) + n^2 ---- number of operations at root level is n^2

T(n/2) = 2T(n/4) + (n/2)^2

T(n/4) = 2(n/8) + (n/4)^2

T(n) = 2 (2T(n/4) + (n/2)^2) + n^2

= 4T(n/4) + (n^2)/2 + n^2

= 4( 2T(n/8) + (n/4)^2 ) + (n^2)/2 + n^2

= 8T(n/8) + (n^2)/4 + (n^2)/2 + n^2

= 2^3T(n/2^3) + (n^2)/2^2 + (n^2)/2^1 + n^2

let’s replace a number that is changing at each step

= 2^k T(n/2^k) + (n^2)/2^(k-1) + (n^2)/2^(k-2) + n^2

base condition T(0) = 0**.**

n/2^k = 0

k=0

= 2^0 T(0) + (n^2)/2^(0 -1) + (n^2)/2^(0-2) + n^2

= (n^2) (1 / 2^-1) + (n^2) (1 / 2^-2) + n^2

= n^2 (1+2+4)

~= n^2

## Mathematical Stuff

### Quotient

0/2 = 0

1/2 = 0

2/2 = 1

### Remainder

0%2 =0

1%2 =1

2%2 =0

### Prime number

A number that is divisible by 1 or itself is called a prime number.

For(int i=1; i<**sqrt(n**); i++) { --- you don’t have to go till i<n

If(n % i > 0) continue;

return false; // not a prime number

}

## Array

### How to work with 1-D, 2-D array for Recursion?

#### 1-D Array

Watch ‘Recursion of Array.mp4’.

binarySearchRecursive(array, 0, array.**length** - 1, elementToSearch)

* Always pass start and end element position in array to recursive method.
* One of the Exit condition will be if(start<end)…
* When you need to convert recursive method into iterative method, extra passed parameters to recursive method becomes local variables and after that that you need to add a while loop for reoccurring code.

##### How to convert O(n^3) to O(n^2) or O(n^2) to O(n)?

Read ArrayFundamentals.java

#### 2-D Array (Matrix)

Matrix:

Matrix is nothing but 2-D array.

As you pass start and end indices in 1-D array, you need to pass startRow,startCol,endRow,endCol in matrix traversal related problems.

e.g. RobotInGrid.java

### How to find mid of array?

binarySearchRecursive(array, 0, array.**length** - 1, elementToSearch)

void binarySearchRecursive(int[] array, int start, int end) {

…

mid = (end+start)/2

…

}

### How to choose Random number from array?

quickSort (A, 0, A.length – 1);

void quickSort(**int**[] A, **int** start, **int** end) {

…

**int** pivot = **new** Random().nextInt((end - start) + 1) + start;

or

**int** pivot = **new** Random().nextInt(A.length - start) + start;

…

}

### How to do shuffling?

void shuffle(int[] A) {

// if you start from i=0, new Random(0,0) will error out

for(int i=1; i < A.length;i++) {

// pick random number between 0 and i

// People sometimes choose random number between 0 and n-1, but it doesn't give uniformly random result

int randomIndex = new Random().nextInt(0, i);

exchange(A, i, randomIndex);

}

}

### How to write an in-place algorithm?

Think of the difference between Quick Sort using Aux arrays and in-place.

Just like Merge Sort, Quick Sort is also Divide and Concur algorithm.

Just like Merge Sort, you can create aux arrays in Quick Sort also, but better approach is to do quick sort in-place.

This teaches us a trick:

whenever you need to do something in-place, think of using an additional pointer. One pointer is for normal traversal of an array and another pointer increments on some special condition. Hard thing is to find this special condition.

### How to find the size of a List in O(log n) if size() method is not available

e.g. SortedSearch.java

You have a class Listy that has all sorted positive elements returns -1, if element at passed index is not available. It doesn’t have size() method. What is the best way to find an index of an element in this data structure?

**static class** Listy {  
 **int**[] **elements** = {1,2,3,4,5,6,7,8,9};  
  
 **public int** elementAt(**int** index) {  
 **if**(index > **elements**.**length**-1) **return** -1; *// this is the condition imposed by requirement* **return elements**[index];  
 }  
}

You start from index=1 and keep doubling the index till elementAt method returns -1. This way you will not be able to find exact size in O(log n), but you will be able to find approximate size and it makes it easier to search an element using binary search.

You have a class Listy that has all sorted positive elements. It returns -1, if element at passed index is not available. It doesn’t have size() method. What is the best way to find an index of an element in this data structure?

You can do a linear search in searching all elements one by one in Listy. If this would have been an ideal solution, interviewer will not tell you that all elements in Listy are in sorted order.  
When you see sorted array, you should think of binary search.  
  
But Listy doesn't have size() method. Can you find its approx size in O(log n)?  
You jump double the index every time and stop when listy.elementAt(index) returns -1.  
In worst case, you may end up finding the size that will be double the actual size of Listy, but that's ok because binary search will reduce the size to half immediately.  
  
You are jumping double index  
So, for some k in 2^k, you will hit >=n  
What is k?  
2^k=n  
log2 2^k = log2 n  
k = log2 n  
  
  
e.g.  
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]  
  
This will result in  
  
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, -1, -1, -1, -1,-1, -1, -1, -1,-1, -1, -1, -1,-1, -1, -1, -1,-1, -1, -1]  
  
and size will be 32

## Graph

### Basic understanding of Graph properties and searching algorithms

README\_Graphs.docx

BfsDfsGrokkingAlgorithms.java

DijkstraAlgorithmForPositivelyWeightedGraphGrokkingAlgorithmBook.java

### How many ways to create a graph?

Read README\_Graphs.docx

1. Edge List

Each Edge object has startVertex, endVertex and weight.

These Edge objects are stored in a list

**class** Graph {  
 **private** Set<Vertex> **vertices** = **new** HashSet<>();  
 **private** Set<Edge> **edges** = **new** HashSet<>();

}

**class** Edge {**private** Vertex **fromVertex**;  
 **private** Vertex **toVertex**;  
 **private** Integer **weight**;

}

**class** Vertex {  
**private** int value;

}

1. Adjacency Matrix

Good for Dense graph

1. Adjacenncy List

Good for sparse graph. Normally, graphs in real life are Sparse only.

Adjacency list can be represented in multiple ways. One of the ways is HashMap to store relation between two vertices.

Graph with adjacency list can be represented as

Map<Vertex, LinkedList<Vertex>> or

Map<Vertex, Set<Vertex>> or

Map<Vertex, Vertex[]> or

Map<Vertex, BST> --- for faster search of a specific neighbour

If you want to show weight also, then

Map<Vertex, LinkedList<VertexWeight>>

Normally, I would use Map<Vertex, LinkedList<Vertex>>.

Grokking Algorithm book uses Map<Vertex, Vertex[]>.

**class** Graph {  
 **private** Map<Vertex, Set<Vertex> adjacencyList;

}



****

or

Class Graph {

Vertex[] vertices;

LinkedList[] adjacentList; --- this can be array of bsts also.

}



## When to use Linear Programming?

<https://www.youtube.com/watch?v=M4K6HYLHREQ>

Under some constraint, you need to maximize something (e.g. profit) As mentioned in above youtube video, constraint is number of available hours for farming. In that constraint, you need to maximize the profit by putting some corns and some oats plants giving different amount of profits.

## When to use Binary Search, Binary Search Tree and Min/Max Heap(Priority Queue)?

**When to use Binary Search?**

Binary Search works best on already sorted array (SortedSearch.java, SparseSearch.java) or matrix (SortedMatrixSearch.java)

Binary Search can be used on unsorted array to find peaks and valleys (PeakAndValleyInUnOrderedArray.java)

Remember, Binary Search needs access by index, so it needs an array as an input, it will perform bad on sorted linkedlist.  
You can create an array from sorted linkedlist first and apply binary search on it.

**When to use BST?**

If you want to use Binary Search to search an element in unsorted array, you need to sort it first before you can search. This takes at least O(n log n) for sorting and O(log n) for binary search.

You can do better by searching an element in BST. Inserting elements will take O(n) and searching an element will take O(log n) provided created BST is close to balanced.

BST is worse for sorted array. It will created totally unbalanced tree.

BST is a symmetric tree. Means left nodes of any nodes are always lesser and right nodes are always bigger.

BST takes O(log n) for insert/delete. It takes avg O(log n) and worse O(n) for search. Worst case is when BST is created from sorted array. BST will have all elements on one side of the tree only. That's why height of the tree will be n because it won't be balanced.

Unbalanced Tree:



Comparing BST to Binary Search:

For Binary Search, you need sorted array, so Inserting and deleting in array to keep it sorted may take O(n). Search takes same as Binary Search.

So BST has an advantage over Binary Search for insert/delete/search when array is unordered.



**Which algorithm is used to create perfectly balanced tree?**

Red-Black Tree

**When to use Min/Max Heap(Priority Queue)?**

When you need to search min/max element in O(1), then you use Min/Max-Heap(Priority Queue) because min element is always on the top of of the min-heap and similary max element is always on the top of max-heap.

Remember, min/max-heap are not trees. It just keeps track of indices in the array to keep track of min/max element.

## Which algorithm is used for finding min or max?

Min-Heap, Max-Heap (prioritiy queue)

Priority Queue uses Heap Sort.

Heap Sort is very useful when you need to find min/max in O(1) time and insert an element in O(log n) time. It requires an aux array through. so O(n) space and total execution time is O(nlogn).

Priority Queue is based on Binary Heap (BinaryHeap.java in algorithms package).  
There is Min BinaryHeap and Max BinaryHeap.  
BinaryHeap look like a tree, but it is just a reordering of elements in an array. Based on index you can find higher priority the element.  
You can find min priority element on the top of Min BinaryHeap, you don't need to search for it like BST.

## Which algorithm is used by Databases?

B-Tree

## When you encounter a problem that has inputs from multiple arrays (multiple sources), what should you think of?

Using Priority Queue to store inputs coming from multiple sources. Priority Queue.

## 

## String Operations

For any String operation, remember below points

* String contains char[] and you can use char[] chars = str.toCharArray()  
  - str.charAt(i) is very useful
* Ask interviewer whether you should support ascii/extended ascii/unicode chars.  
   ASCII chars are english numbers+letters+special chars = 128. Extended ascii chars (total 256) contains many other special chars.  
   If interviewer say ascii is good, then use aux array of size 128 (char[] chars = new char[128])
* Default value of char[] is Character.MIN\_VALUE ('\u0000')
* Remember this pattern  
    
   char[] result = new char[128]

for (int i = 0; i < str.length(); i++) {  
 char c = str.charAt(i);  
 result[c] = c; // or result[c]++ every character has some int value attached to it in ASCII  
 ...  
 }

ASCII A-Z = 65-90, a-z = 97-122. There are some special chars in between 90 and 97.

<http://www.asciitable.com/>

## Number Conversion

1) Very important: Do not try to convert a integer char into int.

e.g. char[] cs = {‘1’,’2’,’4’}

int[] ints = new int[cs.length];

for(int i=0; i< cs.lenght;i++) {

ints[i] = cs[i];

}

you might be expecint output as ints={1,2,4}, but you will get {49,50,52}.

Remember when you try to convert char to int, it gives it ascii value.

2) Do not try to subtract long value from double. It gives unexpected result sometimes.

double D = 1.2

long L = 1

double res = D-L

**you must be expecting res=0.2, but it will be 0.19999999999**

3) double D = 1.2

**new Double(D).longValue()**

**is same as**

**(long)D**

it will give long value: 1

4) Difference between **new Double(D).longValue() / (long)D** and **Double.valueOf(string/double)**

first one truncates the value and second one is used to convert string to double. Second one will give NumberFormatException, if you try to convert a string that is not a number. So, be careful.

## LinkedList Operations

Major difference between String and LinkedList is String has charArray. It is easy to iterate an indexed array compared to LinkedList. So, you need extra intelligence to travers a LinkedList.

e.g. Palindrome Algorithm of String vs LinkedList.

It is so easy to work with String. You can traverse charArray from left and right together till you come in the middle and compare the elements.

In case of LinkedList, you need to use runners and stack to achieve the same thing.

### Can you use Doubly LinkedList?

This is a question for an interviewer. To make your computation easier, you can ask an interviewer whether you can use a doubley linkedlist for solving a problem. You can also ask whether you can keep length variable in LinkedList. You can increment this variable on each insert and decrement on each deletion. This will help you not to iterate through entire linkedlist when you need find a length of it.

LinkedList class is just a wrapper of Head node  
  
 public class LinkedList {  
 private Node head;  
   
 public Node addToTail(Node newNode) {...}  
   
 public Node addAsHead(Node newNode) {...}  
   
 public Node delete(Node node) {...}  
   
 // peek just reads the head node and returns it. It doesn't remove the head node  
 public Node peek() {...}  
   
 // pop just reads the head node, removes it and returns it.  
 public Node pop() {...}  
 }  
   
 public class Node {  
 private int data;  
 private Node next;  
   
 public Node(int data) {  
 this.data = data;  
 }  
 }  
  
Runner Node(s)

#### Use Runner to traverse through a LinkedList.

Don't do head=head.next. You will end up moving head pointer to some other node in the LinkedList.  
   
 head  
 runner  
 |  
 v  
 --------------------  
 | 1 | 2 | 3 | 4 | 5 |  
 --------------------

Whenever you need to iterate through a linked list, always create a runner node.  
Do not iterate by moving head=head.next, otherwise you will end up moving head pointer to somewhere else in the linked list.

You should do

Node runner = head;  
 while(...) runner = runner.next;  
   
   
(VERY IMP) More than one Runner Technique:  
   
The runner technique means that you iterate through the linked list with two pointers simultaneously, with one ahead of the other.

The "fast" runner might be ahead by a fixed amount, or it might be hopping multiple nodes for each one node that the "slow" node iterates through.  
   
For example, suppose you had a linked list a1->a2->.....->an->b1->b2->.....->bn and you wanted to arrange it into a1->b1->a2->b2->...->an->bn. You do not need to know the length of the linked list(but you do know that the length is an even number).  
   
You could have one pointer p1(the fast pointer) move every two elements for every one move that p2 makes.  
When p1 hits the end of the linked list, p2 will be at the midpoint. Then, move p1 back to the front and begin "weaving" the elements. On each iteration, p2 selects and element and inserts it after p1.

Try to design algorithm in such a way that you can add the node in the front of a list instead of somewhere in middle or last

PartitionLinkedListFromSomeNode.java

Recursion  
  
You can write a normal Iterative traversal for linkedlist based algorithms, but if you want to use the recursion, then you can do following.  
  
 public Node search(int data) {  
 return search(head, data)  
 }  
 public Node search(Node runner, int data) {  
 if(runner == null || runner.data == data) return runner;  
 return(runner.next, data);  
 }

Here, you automatically created a runner because when pass ‘head’ as a parameter, that method parameter ‘runner’ is a separate reference to the head of the linkedlist. So, you don’t have create another ‘runner’ inside search method.

### 

### Using extra buffer for linkedlist algorithms?

#### Using map or set as extra buffer

Many times when you traverse a linkedlist using runners (pointers), you may end up with runtime complexity O(n^2).

e.g. RemoveDups.java

**(IMP) Ask interviewer, are you allowed to use extra buffer?**

If he says yes, you can use map/set as extra buffer.

**NOTE: Set internally uses Map.** So, searching anything in Set will take O(1) only.

e.g. Remove Duplicates from LinkedList algorithm (RemoveDups.java)

#### Using stack extra buffer

e.g. PalindromeLinkedList.java, ReturnKthToLastElement.java

NOTE:

In case of String’ Palindrome StringPalindrome.java, you don’t need any complexity because String provides you indexed charArray using str.toCharArray(). It’s easy to iterate indexed array compared to a LinkedList.

### Do Not modify an object sent as parameter

e.g. DeleteMiddleNode.java

**private static void** delete(Node head1, Node nodeToBeDeleted) {  
 **if** (head == **null**) **return**;  
  
 Node R = head1;  
 Node prevOfR = **null**;  
 **while** (R != **null**) {  
 **if** (R.equals(nodeToBeDeleted)) {  
 **if** (prevOfR != **null**) {  
 prevOfR.**next** = R.**next**;  
 } **else** {  
 head1 = R.**next**; *// This doesn't work. head1 ref is different than head ref.Actual head of linkedlist is still pointing on first element of linkedlist.head1 is moving to next element.* R.**next** = **null**;  
 }  
 **break**;  
 }  
 prevOfR = R;  
 R = R.**next**;  
  
 }  
}

linkedlist(Node=1,Node=2,Node=3,Node=4,Node=5)

delete(linkedlist.head, new Node(1))

System.out.println(linkedlist.head);// [Node=1,Node=2,Node=3,Node=4,Node=5]

You want to print [Node=2,Node=3,Node=4,Node=5]

In above code, you are trying to manipulate sent object (head), but you are forgetting that  
When caller calls a method, situation is like below  
  
 sent head from caller -----|  
 | -> 5  
 param head-------------------|  
  
When you modify incoming parameter, situation will be as follows:  
  
 sent head from caller --------> 5  
 param head----------------------> 2  
  
It won't change the actual ‘head’ object sent by a caller  
  
Solutions:

1) Node delete(Node head, Node nodeToBeDeleted) {  
 ...  
 Node newHead = 2;  
 ...  
  
 return newHead;  
 }

**private static** Node delete\_2(Node head, Node nodeToBeDeleted) {  
 **if** (head == **null**) **return** head;  
  
 Node R = head;  
 Node prevOfR = **null**;  
 Node newHead = head;  
 **while** (R != **null**) {  
 **if** (R.equals(nodeToBeDeleted)) {  
 **if** (prevOfR != **null**) {  
 prevOfR.**next** = R.**next**;  
 } **else** {  
 newHead = R.**next**;  
 R.**next** = **null**;  
 }  
 **break**;  
 }  
 prevOfR = R;  
 R = R.**next**;  
 }  
 **return** newHead;  
}

2) Whenever you need to do that, you wrap that param with some other class and send that class object as a param.  
 e.g. SinglyLinkedList  
 private static void delete(SinglyLinkedList LL, Node nodeToBeDeleted) {  
  
 Now, when you do LL.head = 2. It will actually update the content sent LL object.

**private static void** delete\_1(SinglyLinkedList LL, Node nodeToBeDeleted) {  
 **if** (LL.**head** == **null**) **return**;  
  
 Node R = LL.**head**;  
 Node prevOfR = **null**;  
 **while** (R != **null**) {  
 **if** (R.equals(nodeToBeDeleted)) {  
 **if** (prevOfR != **null**) {  
 prevOfR.**next** = R.**next**;  
 } **else** {  
 LL.**head** = R.**next**;  
 R.**next** = **null**;  
 }  
 **break**;  
 }  
 prevOfR = R;  
 R = R.**next**;  
  
 }  
}

### How to check whether LinkedList has odd or even size?

1 -> 2 -> 3 -> 4 -> null

a

move runner ‘a’ two steps at a time till (a==null or a.next==null).

If(a==null) then it’s a even size.

If(a.next==null) then it’s odd size

e.g. PalindromeLinkedList.java

## 

## Stack And Queue

Stack and Queue are created using linkedlist.

Important:

* Stack is a LinkedList where items are added and removed to/from head(top). 'head' in Stack is called 'top'.
* Queue is a LinkedList where items are added at tail and removed from head.
* Stack is useful for recursions.
* Queue is useful for BFS (Breadth First Search) and for implementing a Cache. LRUCache.java is an example of using a Queue for Caching.

class MyStack<T> {

Node<T> top;

public T pop(){…}

public T peek(){…}

public T push(T item){…}

public boolean isEmpty(){…}

}

class MyQueue<T> {

Node<T> first;

Node<T> last;

public T remove(){…}

public T peek(){…}

public T add(T item){…}

public boolean isEmpty(){…}

}

Stack is LIFO and Queue is FIFO.

LinkedList doesn't create an array to store elements. It maintains references between two nodes of elements.

Popping activity is same in both in stack and queue, first element is popped and new first element is set as old first element's next

Important thing is base class for LinkedList. If you remember Node class, then Stack and Queue algorithms are easy to create.

Why can't we use Array instead of LinkedList?

Because Array has to be declared with fixed size and if you don't know how many elements you are dealing with then it's very hard to use Array.

You can use Resizable Array instead of Array. Read document for more details.

java.util.Stack extends Vector which is based on Resizable Array

java.util.Queue has many forms BlockingQueue, ArrayBlockingQueue, LinkedBlockingQueue etc. It provides client a choice to use Fixed size Array or LinkedList.

Important Stack methods:

pop() - Removes the top item from the stack.

push(item) - Add an item to the top of the stack.

peek() - Return the top of the stack (does not remove an item like pop())

isEmpty() - Returns tru if and only if the stack is empty.

Important Queue methods:

add(item) - Add an item to the end of the list.

remove() - remove the first item in the list.

peek() - Return the top of the stack.(does not remove an item like remove())

isEmpty() - Return true if and only if the stack is empty.

## Tree

### Recursion Concepts

Read *\_0RecursionConcepts.java*

These are very important concepts. Do not start recursion problems without understanding them.

Dynamic Programming Concepts

This Dynamic Programming is a bit different than what you understood from Grokking Algorithms book.

It has two important concepts

* Top-Bottom Dynamic Programming (Memoization)
* Bottom-Up Dynamic Programming

Read *DynamicProgrammingConcepts.java*

### Tail-Recursion Concepts

See *TailRecursionConcepts.java*

### How to delete a node from BST?

Insertion/finding of a node in BST is easy, but deletion is tricky.

/\*  
 As mentioned in  
 http://www.algolist.net/Data\_structures/Binary\_search\_tree/Removal  
 https://www.youtube.com/watch?v=gcULXE7ViZw,  
  
 if node toBeDeleted is found in a tree, then  
 there are three possibilities for a node that needs to be deleted  
 - node is a leaf node --- just de-link this node from its parent  
 - node has only one child node --- child node becomes toBeDeleted node's parent node's left/right child. In this way, toBeDeleted node will be de-linked.  
 - node has two children nodes --- find a minValueNode from right subtree of a toBeDeleted node.  
 set the value of toBeDeleted node as this min value.  
 delete that minValueNode.  
  
 first two cases are easy to handle, third one is a bit complicated.  
  
 \*/  
**private** TreeNode deleteNode(TreeNode root, Integer data) {  
 **if** (root == **null**) **return** root; *// exit condition* **if** (data.compareTo(root.**data**) == 0) { *// exit condition* **return** deleteRootAndMergeItsLeftAndRight(root);  
 }  
 **if** (data.compareTo(root.**data**) < 0) {  
 **return** deleteNode(root.**left**, data); *// returned result will be same as one of the exit condition's result. If you see, in deleteRootAndMergeItsLeftAndRight method, we already merging the changes with root (passed node). So, here we don't have to merge the result of recursive method with the root.* } **else** {  
 **return** deleteNode(root.**right**, data); *// returned result will be same as one of the exit condition's result. If you see, in deleteRootAndMergeItsLeftAndRight method, we already merging the changes with root (passed node). So, here we don't have to merge the result of recursive method with the root.* }  
  
}  
  
*// If you see FunctionalProgrammingInJava's Tree.java,  
// we don't modify the current tree. On every modification, we create a new tree.  
// Advantage of that is recursive method's logic becomes very simple. You don't have to think about all these if-elseif conditions that we have implemented here in this method and also you don't need to remember parent.***private** TreeNode deleteRootAndMergeItsLeftAndRight(TreeNode root) {  
 **if** (root.isLeafNode()) { *// has no children - just de-link this node from its parent* **if** (root.hasParent()) {  
 **if** (root.amILeftChildOfMyParent()) {  
 root.getParent().setLeft(**null**);  
 } **else** {  
 root.getParent().setRight(**null**);  
 }  
 }  
 } **else if** (root.hasOnlyOneChild()) { *// child node becomes toBeDeleted node's parent node's left/right child. In this way, toBeDeleted node will be de-linked.* **if** (root.getLeft() != **null**) {  
 root.**parent**.setLeft(root.getLeft());  
 } **else** {  
 root.**parent**.setRight(root.getRight());  
 }  
 } **else** {  
 *// find a minValueNode from right subtree of a toBeDeleted node.* TreeNode minValueTreeNode = min(root.getRight());  
  
 *// set the value of toBeDeleted node as this min value.* root.setData(minValueTreeNode.getData());  
  
 *// delete that minValueNode.* deleteNode(minValueTreeNode, minValueTreeNode.getData());  
  
 }  
 **return** root;  
}

### How to measure memory and time complexity of a binary tree algorithm?

m(n)  
 m(n/2) m(n/2)  
m(n/4) m(n/4) m(n/4) m(n/4)  
  
This is how tree execution looks like when you use recursive method. When you recurse a method with left and right children of a tree node, basically you are recursing with n/2-n/2 nodes of a tree.  
  
Now,  
  
at each recursion of method 'm', 'gh' method is called. If you see at each level of a tree, 'gh' method visits n nodes.  
So, time complexity of this kind of algorithm will be **O(n log n**). log n is the height of the tree (and so number of levels)  
  
 m(n)  
 gh(n/2) get(n/2)  
  
 m(n/2) m(n/2)  
 gh(n/4) get(n/4) gh(n/4) get(n/4)  
  
 m(n/4) m(n/4) m(n/4) m(n/4)  
 gh(n/8) get(n/8) gh(n/8) get(n/8) gh(n/8) get(n/8) gh(n/8) get(n/8)  
  
  
In below case, every node of m tree calls gh that visits p number of nodes. Unlike to above example, here gh is not reducing the number of nodes to visit to half for next level down.  
You can think of like n belongs to tree T1 and p belongs to tree T2. n and p are number of nodes in T1 and T2 respectively. You are trying to find whether T2 is a subtree of T1.  
So, when you see this kind of case, then its time complexity is **O(np)**  
e.g. CheckSubTree.java  
  
 m(n)  
 gh(p/2) get(p/2)  
  
 m(n/2) m(n/2)  
 gh(p/2) get(p/2) gh(p/2) get(p/2)  
  
 m(n/4) m(n/4) m(n/4) m(n/4)  
 gh(p/2) get(p/2) gh(p/2) get(p/2) gh(p/2) get(p/2) gh(p/2) get(p/2)

Making it simpler

m(n)  
 m(n/2) m(n/2)  
m(n/4) m(n/4) m(n/4) m(n/4)

At each node of this tree, nothing major is happening but node passed to the method (e.g. left node), is just visited. It means that at each node of above recursive tree, O(1) operation is happening. So, time complexity will be O(1 \* n).

You can see this example in FindWhetherTreeIsBalanced.java’s isBalanced\_Better method.

### How to find the height of a tree?

**private static int** getHeight(TreeNode root) {  
 **if** (root == **null**) {  
 **return** 0;  
 }  
 **int** leftSubTreeHeight = *getHeight*(root.**left**);  
 **int** rightSubTreeHeight = *getHeight*(root.**right**);  
  
 **if** (leftSubTreeHeight > rightSubTreeHeight) **return** leftSubTreeHeight + 1;*// adding parent parent's height* **else return** rightSubTreeHeight + 1;*// adding parent parent's height*}

### You may need to have link to parent node in BST

See FindInOrderSuccessor.java

### You may need to have size attribute for each node in a tree

See RandomNode.java

### How to evaluate call stack of recursive calls of a tree?

See FindLowestCommonAncestorInBinaryTree.java  
  
Any recursive algorithm is made of one or more of below steps  
  
- exit condition on entry (mandatory)  
- optimization condition that decides whether to traverse left subtree or not for better time complexity of the algorithm(optional)  
- recursive call to left subtree (mandatory)  
- optimization condition that decides whether to traverse right subtree or not for better time complexity of the algorithm (optional)  
- recursive call to right subtree (mandatory)  
- exit condition on exit (optional)  
 if this one is there, then it shows that you are using post-traversal method to traverse a binary tree.  
  
 Let's look at FindLowestCommonAncestorInBinaryTree.java algorithm  
 CA(5,2,9)  
 CAL=CA(3,2,9) --- CAL=CA(2,2,9) --- CAL=(null,2,9)  
 CAR=(null,2,9)  
 CAR=CA(4,2,9) --- CAL=(null,2,9)  
 CAR=(null,2,9)  
  
 CAR=CA(9,2,9) --- CAL=CA(8,2,9) --- ...  
 CAR=CA(10,2,9) --- ...  
  
 When you are tracing a call stack on paper, you can do it in tree form.  
  
 CA(5,2,9) {  
 exit\_condition\_on\_entry  
 CAL=CA(3,2,9)  
 exit\_condition\_on\_entry  
 CAL=CA(2,2,9)  
 ...  
 CAR=CA(4,2,9)  
 ...  
 exit\_condition\_on\_exit  
 CAR=CA(9,2,9)  
 exit\_condition\_on\_entry  
 ...  
 exit\_condition\_on\_exit  
 exit\_condition\_on\_exit  
 }  
  
 If value is returned from exit\_condition\_on\_entry or exit\_condition\_on\_exit of  
 - CA(3,2,9) call, then it is assigned to CAL of CA(5,2,9)  
 - CA(9,2,9) call, then it is assigned to CAR of CA(5,2,9)

## Greedy and Dynamic Programming

### What is Dynamic Programming?

Read RecursionConcepts.java

### When can you use dynamic programming?

Read RecursionConcepts.java

### When can you use Greedy programming?

Read RecursionConcepts.java

How to decide what should be the key for memoization table(array/map) for Top-Down Dynamic Approach?

Read RecursionConcepts.java

### Brute-Force followed by Top-Bottom Dynamic approach and thnking directly using Bottom-Up Dynamic Approach

Read RecursionConcepts.java

1-D and 2-D problems for Bottom-Up approach

Read RecursionConcepts.java

### How to think think differently for solving 2-D problem?

Read RecursionConcepts.java

## Sorting

### Arrays.sort, Collections.sort

* Arrays.sort uses 3-Way-QuickSort for int[], float[] etc. But it uses Merge Sort/Insertion Sort for Object[].

If positions for primitives are changed during sorting, then it's ok, but it's not ok for Objects.

* Collections.sort uses Arrays.sort internally.
* Heap Sort uses Binary Heap algorithm and Priority Queue uses Heap Sort.

Remember, Quick Sort is a in-place sorting algorithm but it is unstable.

From BSIS and M-HQ sorts, HQ are unstable.

3-way sort is useful for array with many duplicates.

### What is stable and unstable sort?

[**http://programmers.stackexchange.com/questions/247440/what-does-it-mean-for-a-sorting-algorithm-to-be-stable**](http://programmers.stackexchange.com/questions/247440/what-does-it-mean-for-a-sorting-algorithm-to-be-stable)

A stable sort is one which preserves the original order of the input set, where the comparison algorithm does not distinguish between two or more items.

Consider a sorting algorithm that sorts cards by rank, but not by suit. The stable sort will guarantee that the original order of cards having the same rank is preserved; the unstable sort will not.



From BSIS(bubble,selection,insertion,shell) and M-HQ(merge, heap,quick) sorting algorithms, HQ are unstable.

Important:

Java uses Quick Sort for literals like int, long etc.

For objects like String or any other, it uses insertion sort, if number of elements are <=10, otherwise it uses Merge Sort for stable sorting.

See [Arrays.sort, Collections.sort](#_Arrays.sort,_Collections.sort).

Both Quick Sort and Heap Sort works in O(n log n), but java uses Quick Sort because Heap Sort uses Binary Heap that needs Aux array.

You can also think differently – In whichever Sort, swaps(exchanges) of elements happen between distance elements (not adjustant elements), they are unstable. This is explained in Coursera video of ‘6-5 Stability (5-39).mp4’.

### How to get random number from an array?

new Random().nextInt(number+1) gives a random number from 0 to number.

To have similar functionality in case of array with array’s start and end index provided,

**int** pivot = **new** Random().nextInt((end - start) + 1) + start;

This functionality is used to choose a pivot in quick sort.

### How to Shuffle an array?

**private static** <T> **void** shuffle(Comparable<T>[] numbers) {

Random random = **new** Random();

**for** (**int** i = 1; i < numbers.**length**; i++) {

// pick random number between 0 and i  
 // People sometimes choose random number between 0 and n-1, but it doesn't give uniformly random result

**int** randomArrayIndex = random.nextInt(i);

exchange(numbers, i, randomArrayIndex);  
 }

System.***out***.println(**"Shuffled Array:"** + Arrays.*asList*(numbers));

}

## When to Use What

### Arrays

In many situations the array is the first kind of structure you should consider when storing

and manipulating data. Arrays are useful when

• The amount of data is reasonably small.

• The amount of data is predictable in advance.

If you have plenty of memory, you can relax the second condition; just make the array big

enough to handle any foreseeable influx of data.

If insertion speed is important, use an unordered array. If search speed is important, use

an ordered array with a binary search. Deletion is always slow in arrays because an

average of half the items must be moved to fill in the newly vacated cell. Traversal is fast

in an ordered array but not supported in an unordered array.

Vectors, such as the Vector class supplied with Java, are arrays that expand

themselves when they become too full. Vectors may work when the amount of data isn't

known in advance. However, there may periodically be a significant pause while they

enlarge themselves by copying the old data into the new space.

### Linked lists

Consider a linked list whenever the amount of data to be stored cannot be predicted in

advance or when data will frequently be inserted and deleted. The linked list obtains

whatever storage it needs as new items are added, so it can expand to fill all of available

memory; and there is no need to fill "holes" during deletion, as there is in arrays.

Insertion is fast in an unordered list. Searching and deletion are slow (although deletion is

faster than in an array), so, like arrays, linked lists are best used when the amount of data

is comparatively small.

A linked list is somewhat more complicated to program than an array, but is simple

compared with a tree or hash table

### Binary Search Trees

A binary tree is the first structure to consider when arrays and linked lists prove too slow.

A tree provides fast O(logN) insertion, searching, and deletion. Traversal is O(N), which

is the maximum for any data structure (by definition, you must visit every item). You can

also find the minimum and maximum quickly, and traverse a range of items.

An unbalanced binary tree is much easier to program than a balanced tree, but

unfortunately, ordered data can reduce its performance to O(N) time, no better than a

linked list. However, if you're sure the data will arrive in random order, there's no point

using a balanced tree.

### Balanced Trees

Of the various kinds of balanced trees, we discussed red-black trees and 2-3-4 trees.

They are both balanced trees, and thus guarantee O(logN) performance whether the

input data is ordered or not. However, these balanced trees are challenging to program,

with the red-black tree being the more difficult. They also impose additional memory

overhead, which may or may not be significant.

The problem of complex programming may be reduced if a commercial class can be

used for a tree. In some cases a hash table may be a better choice than a balanced tree.

Hash-table performance doesn't degrade when the data is ordered.

There are other kinds of balanced trees, including AVL trees, splay trees, 2-3 trees, and

so on, but they are not as commonly used as the red-black tree.

### Hash Tables

Hash tables are the fastest data storage structure. This makes them a necessity for

situations in which a computer program, rather than a human, is interacting with the data.

Hash tables are typically used in spelling checkers and as symbol tables in computer

language compilers, where a program must check thousands of words or symbols in a

fraction of a second.

Hash tables may also be useful when a person, as opposed to a computer, initiates dataaccess

operations. As noted above, hash tables are not sensitive to the order in which

data is inserted, and so can take the place of a balanced tree. Programming is much

simpler than for balanced trees.

Hash tables require additional memory, especially for open addressing. Also, the amount

of data to be stored must be known fairly accurately in advance, because an array is

used as the underlying structure.

A hash table with separate chaining is the most robust implementation unless the amount

of data is known accurately in advance, in which case open addressing offers simpler

programming because no linked list class is required.

Hash tables don't support any kind of ordered traversal or access to the minimum or

maximum items. If these capabilities are important, the binary search tree is a better

choice..



## Bit Manipulation

See BitMaipulationFundamentals.java

## Threads and Locks

See ThreadAndLocksFundamentals.java