Q 1)

i) Find P of them ending at same point after n step.

we can model this ~~as~~ a system where one of the drunks is stuck at ~~a~~ the origin, and the other drunk takes 2n steps.

Now for them to end up at the same point, the moving drunk has to move n steps right and n steps left in any order.

the number of such orders is

${}^{2n}C_n$,

Hence, the probability of this is

$${}^{2n}C_n \times \left(\frac{1}{2}\right)^{2n}$$

~~(as going either right or left has a probability of $\frac{1}{2}$)~~

(as total number of possibilities is 2^{2n})

2) P of drunk ending up at origin.

for this to happen, he has to move right exactly $\frac{n}{2}$ times, and move left exactly $\frac{n}{2}$ times. hence, if n is even, then the number of ways he can do this is,

$$^{n}C_{\frac{n}{2}}$$

So $$P = \begin{cases} ^{n}C_{\frac{n}{2}} \times \left(\frac{1}{2}\right)^{n} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$$

3) Mean displacement.

let us define X_i as a random variable, where if the i^{th} step is right, $X_i = 1$, and if it is left then $X_i = -1$.

Cleary

Mean displacement $= E(X_1 + X_2 + X_3 \dots X_n)$

as X_i are independant of each other,

$$E(x_1 + x_2 + x_3 \dots x_n) = \sum_{i=1}^{n} E(x_i)$$

now, $E(x) = \frac{1}{2}\cdot(-1) + \frac{1}{2}\cdot(1)$

$= 0$

So $\sum_{i=1} E(x_i) = 0$

So mean displacement $= 0$.

4) Averager ~~mean~~ Squared displacement

We use the same X_i as in the the previous part.

mean squared displacement $= E\left(\left(\sum_{i=1} x_i\right)^2\right)$

$= E\left(\left(x_1^2 + x_2^2 + x_3^2 \dots\right) + 2\left(x_1x_2 + x_2x_3 + x_3x_1 \dots\right)\right)$

$= E(x_1^2) + E(x_2^2) \dots + 2\left(E(x_1x_2) + E(x_2x_3) \dots\right)$

$E(x_i^2) = 1$ as $x_i =$ either 1 or -1

so $x_i^2 = 1$

so $E(x_i^2) = 1$

$E(x_ix_j) = 0$ as $X_iX_j = 1$ with $P = \frac{1}{2}$

-1 with $P = \frac{1}{2}$

so $E(x_ix_j) = 1\cdot\frac{1}{2} + -1\cdot\frac{1}{2}$

$= 0$

So..

mean squared displacement = $(1 + 1 + 1 \ldots n \text{ times})$

$+ (0 + 0 + 0 \ldots)$

$= \underline{n}$

Code Explanation

Question 1

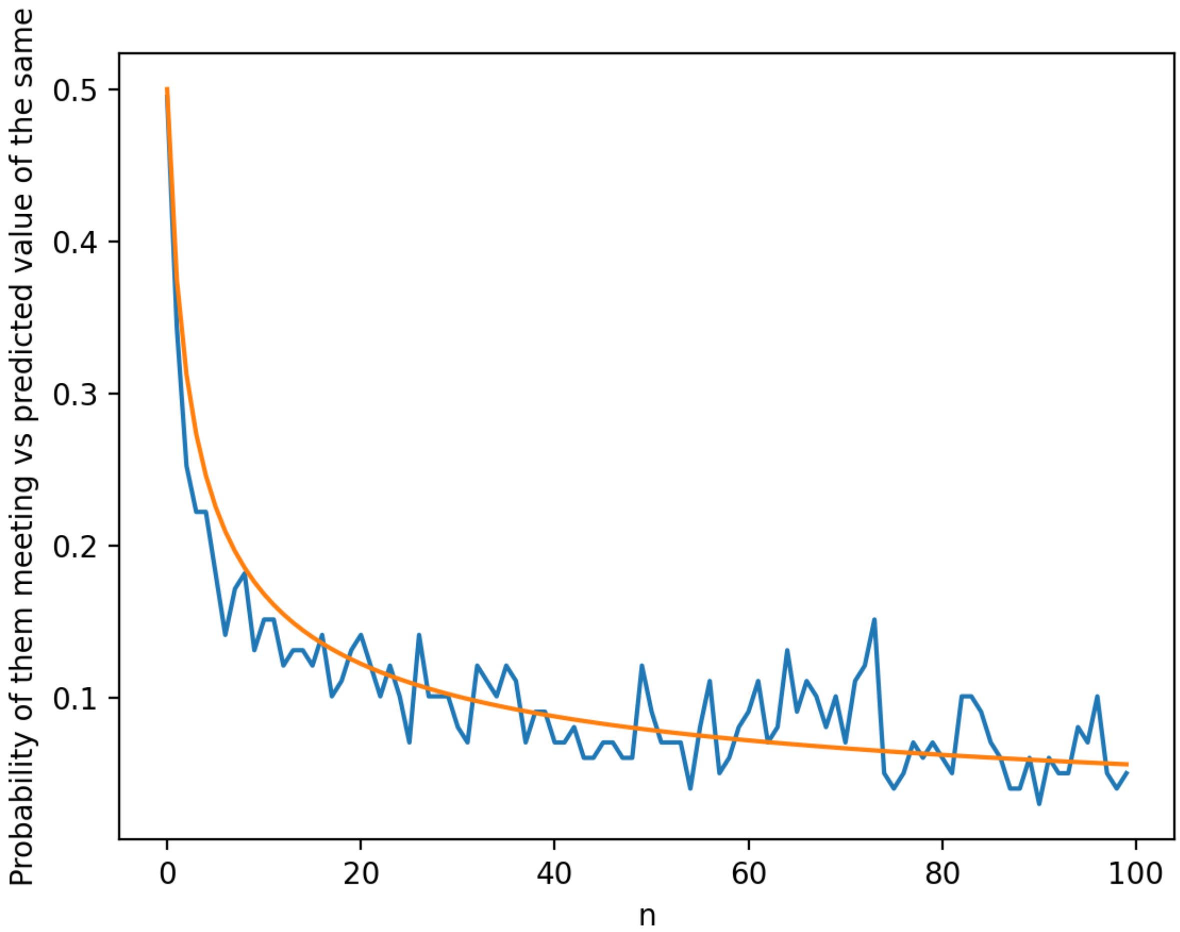
We basically ran a simulation multiple times, and found average values or displacement or displacement squared and found the probability of them ending at origin or of them meeting by dividing the number they ended at origin or met by the total number of trials.

We graph the findings below, comparing with the values we get mathematically. To see how we calculated the values, look at the pdf.

probability that they meet after n steps

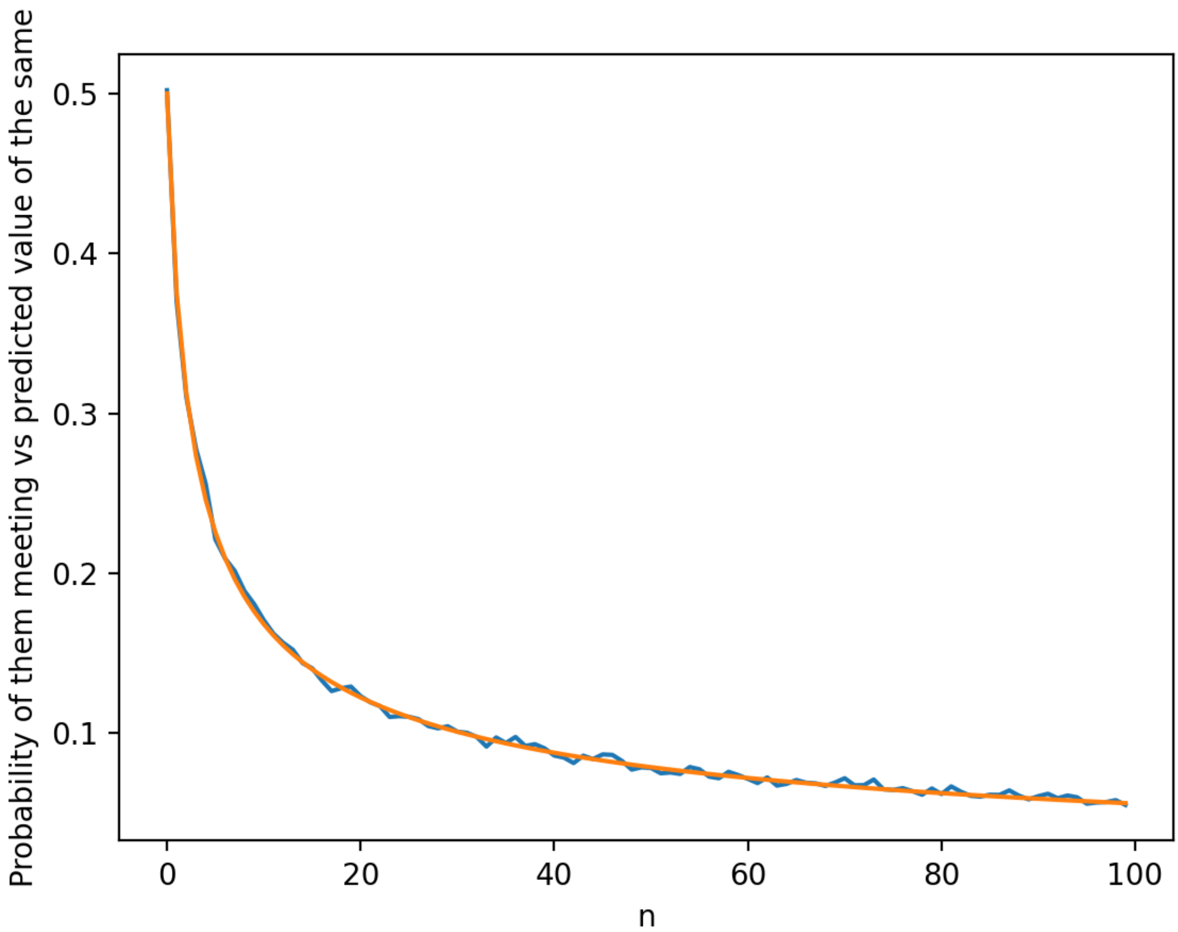
In the below simulation, we ran 100 trials for each value of `n`, and used the results of these trials to find the probability that they will end at the same spot after n steps.

Blue represents the estimated value, while orange is the actual mathematical probability



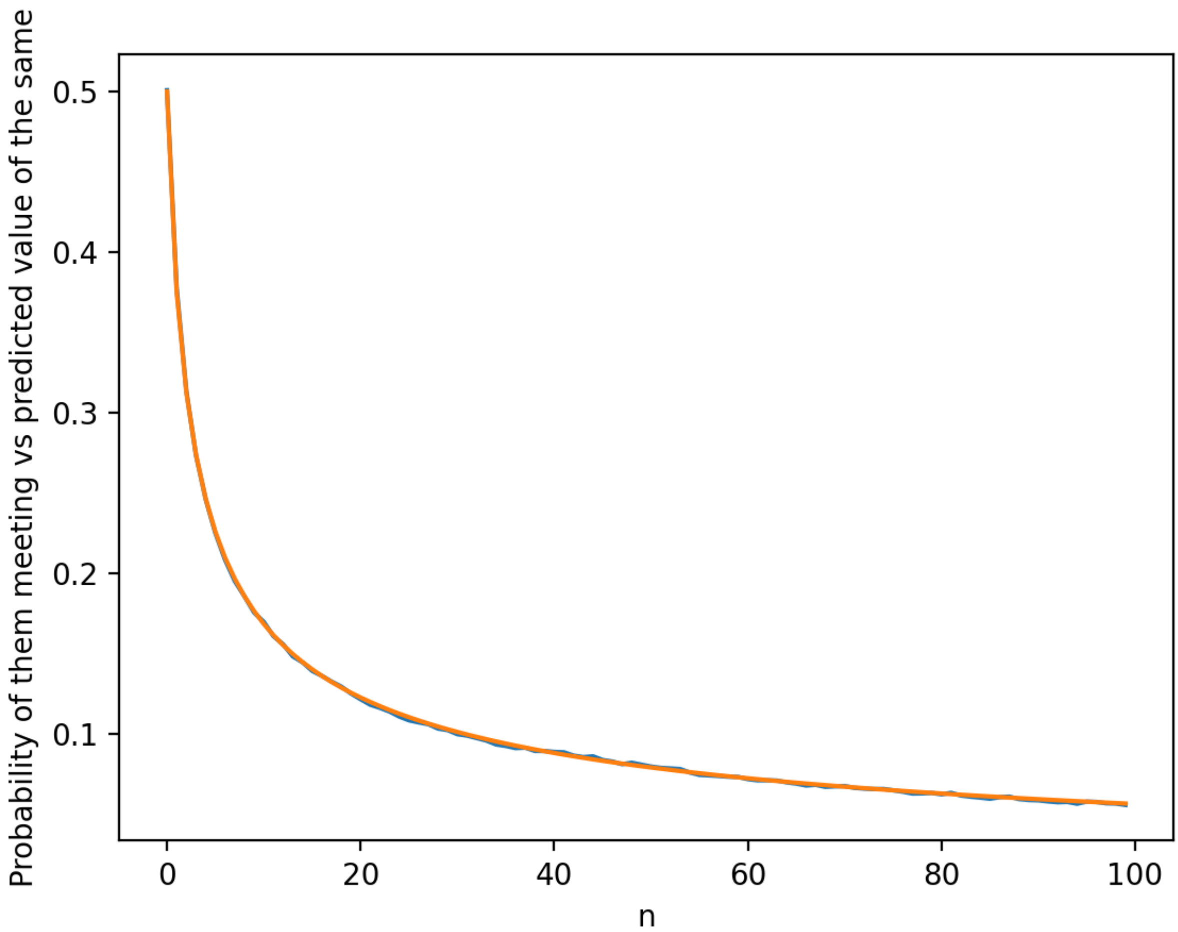
Here, we ran 10000 trials.

Blue represents the estimated value, while orange is the actual mathematical probability



Here, we ran 100000 trials.

Blue represents the estimated value, while orange is the actual mathematical probability

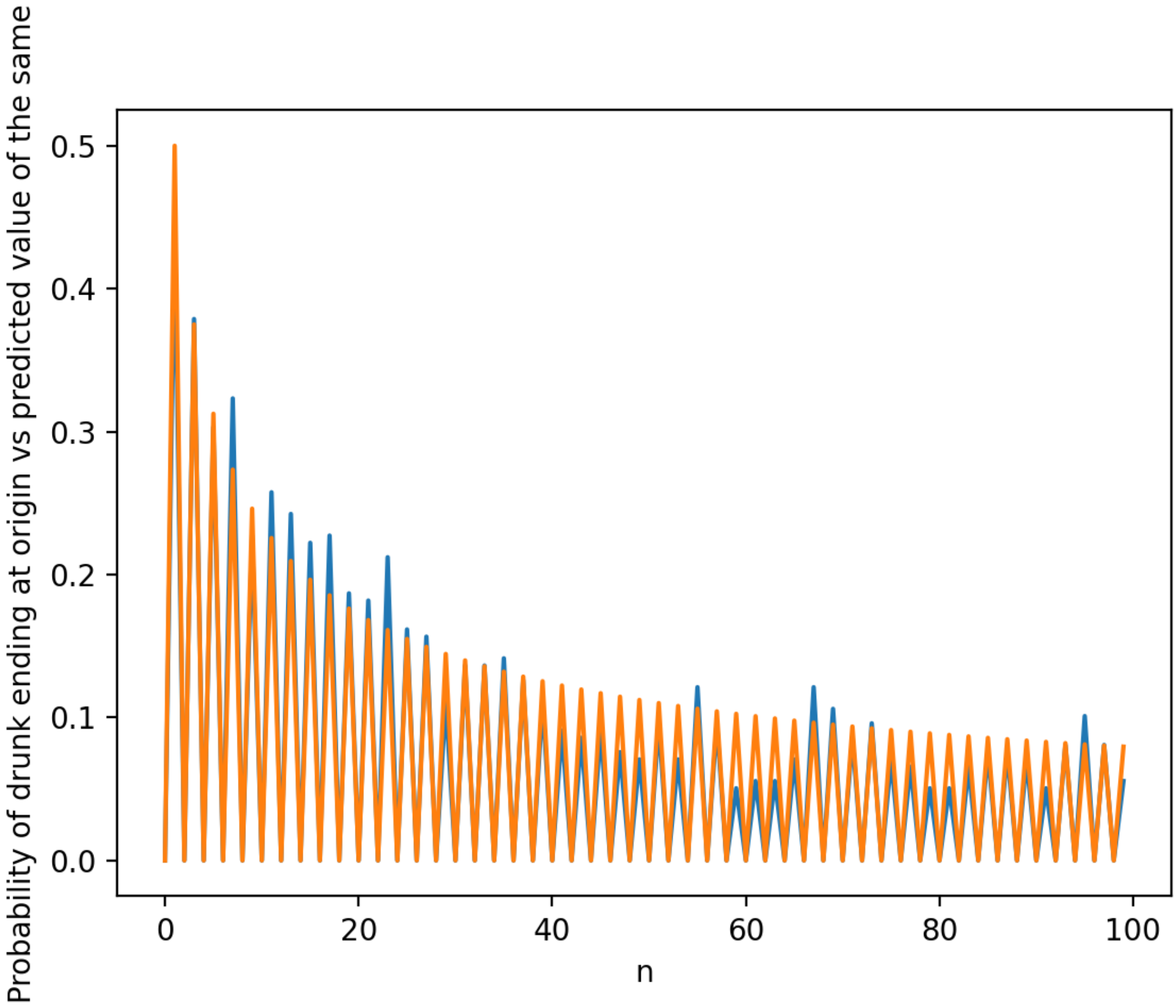


as you can see, when we run more trials, our estimate gets closer to the actual value

probability that they end at the origin

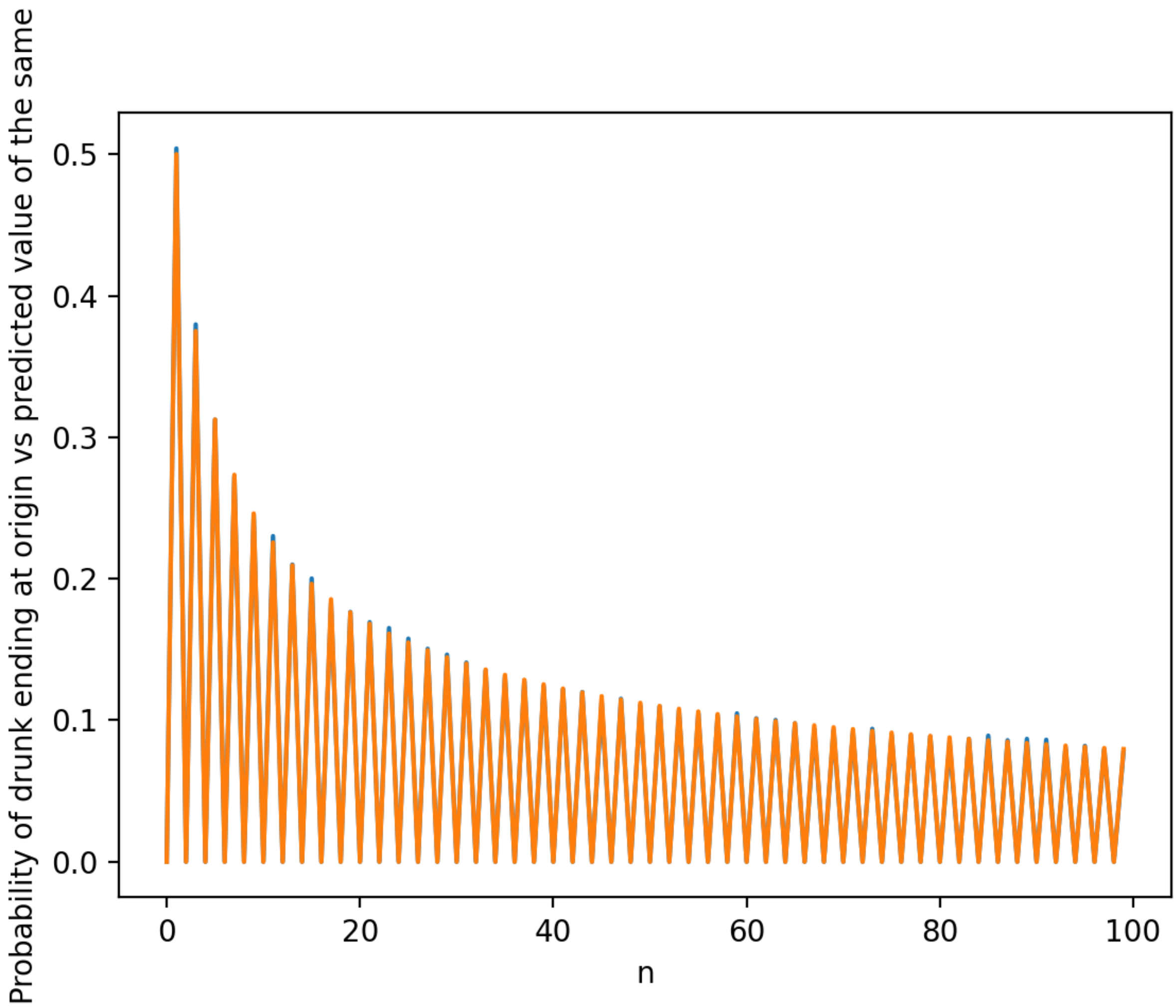
In the below simulation, we ran 200 trials for each value of `n`, and used the results of these trials to find the probability that they will end at the origin.

Blue represents the estimated value, while orange is the actual mathematical probability



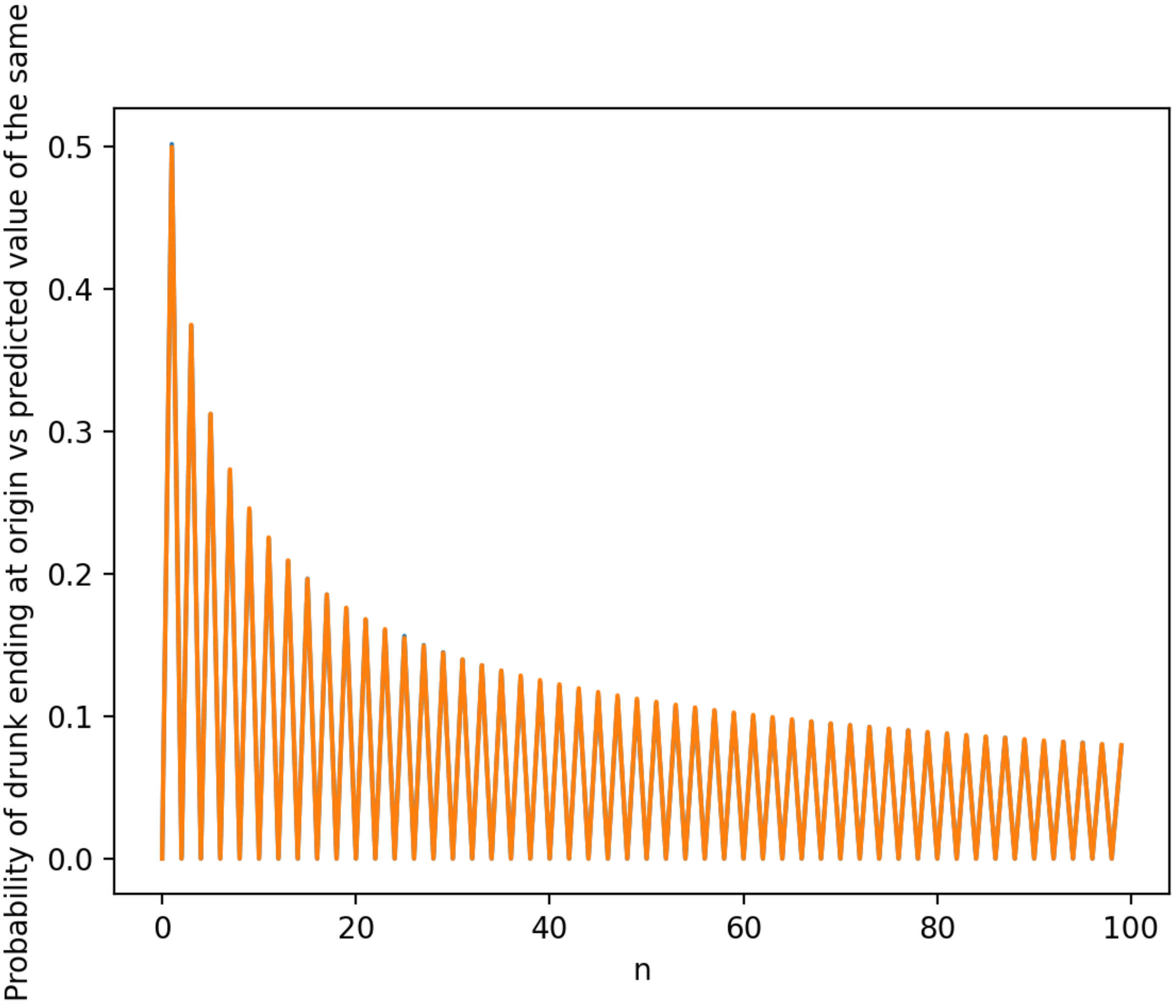
Here, we ran 20000 trials.

Blue represents the estimated value, while orange is the actual mathematical probability



Here, we ran 200000 trials.

Blue represents the estimated value, while orange is the actual mathematical probability

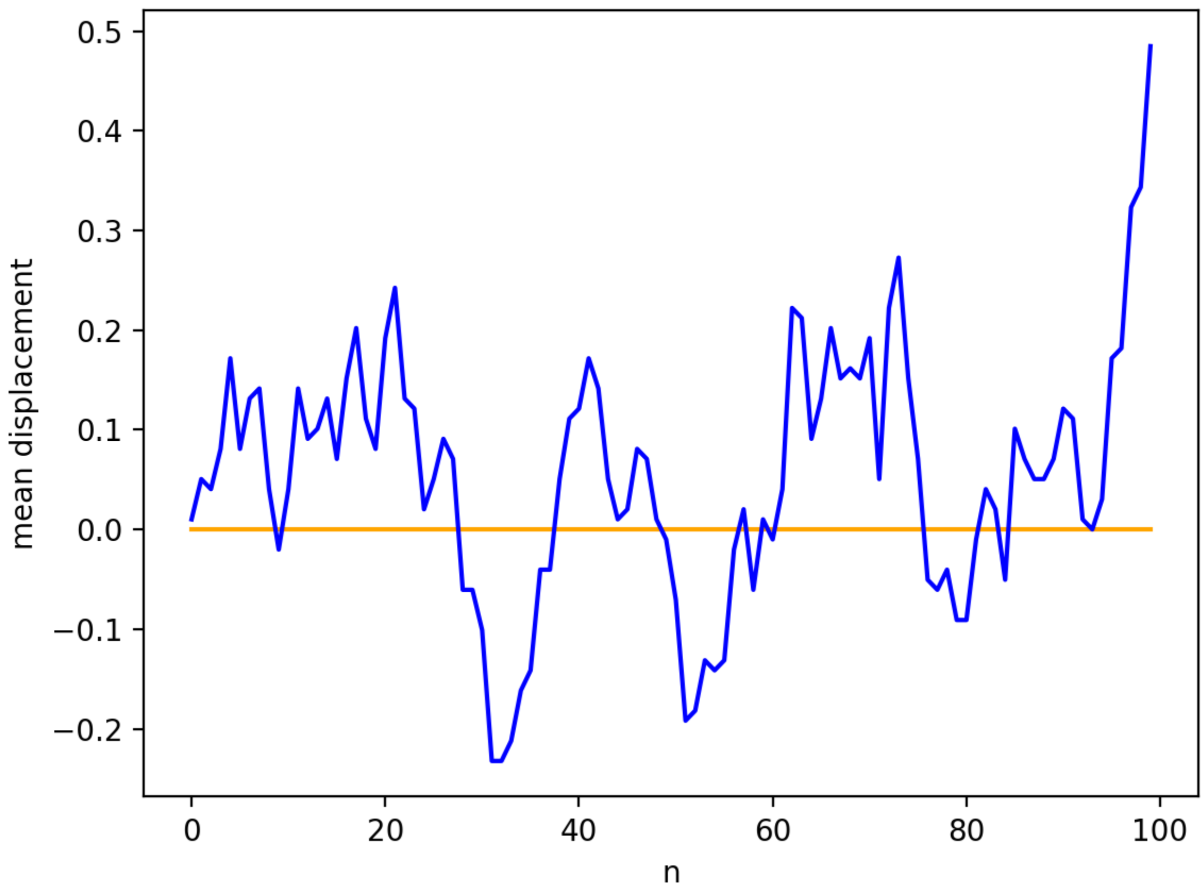


As you can see, when we run more trials, our estimate gets closer to the actual value

Mean Displacement

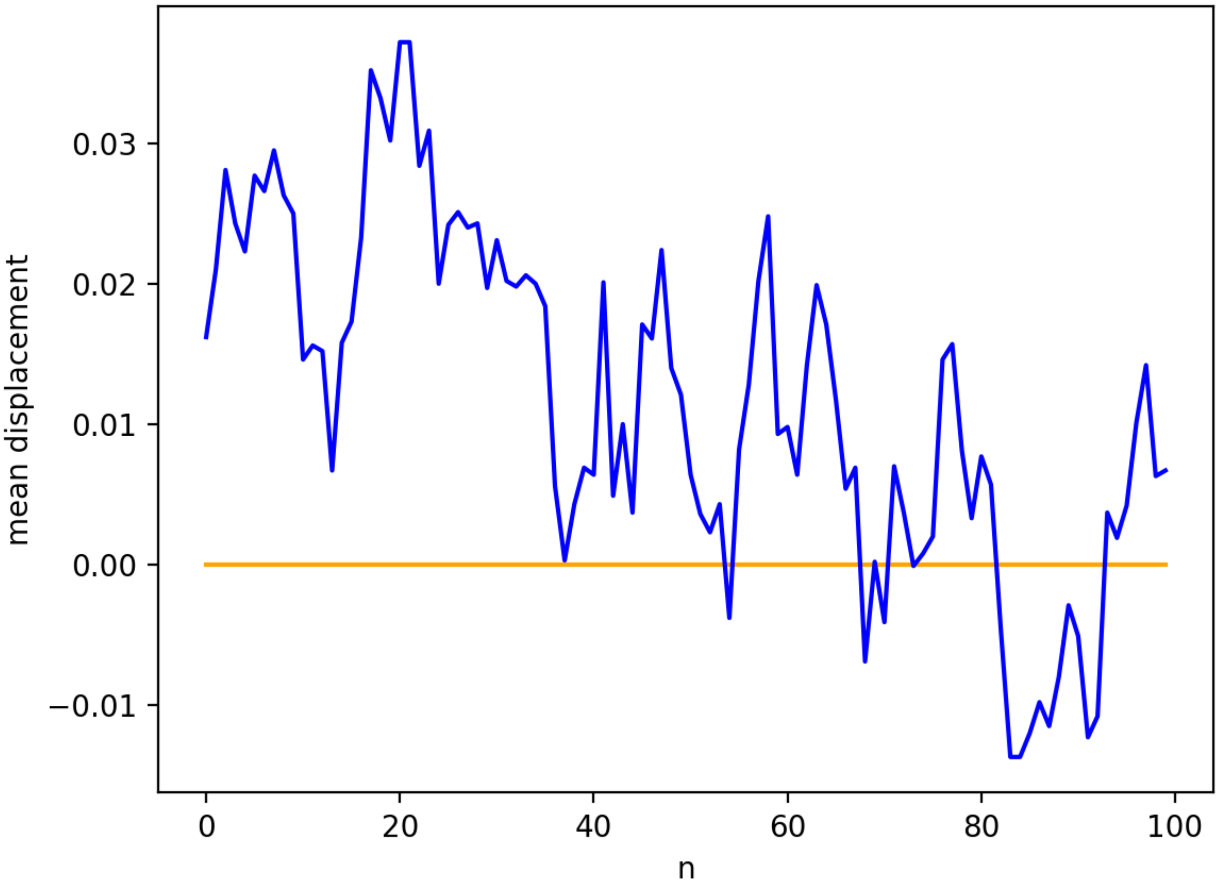
In the below simulation, we ran 100 trials for each value of `n`, and used the results of these trials to find the mean displacement

Blue represents the estimated value, while orange is the actual mathematical probability



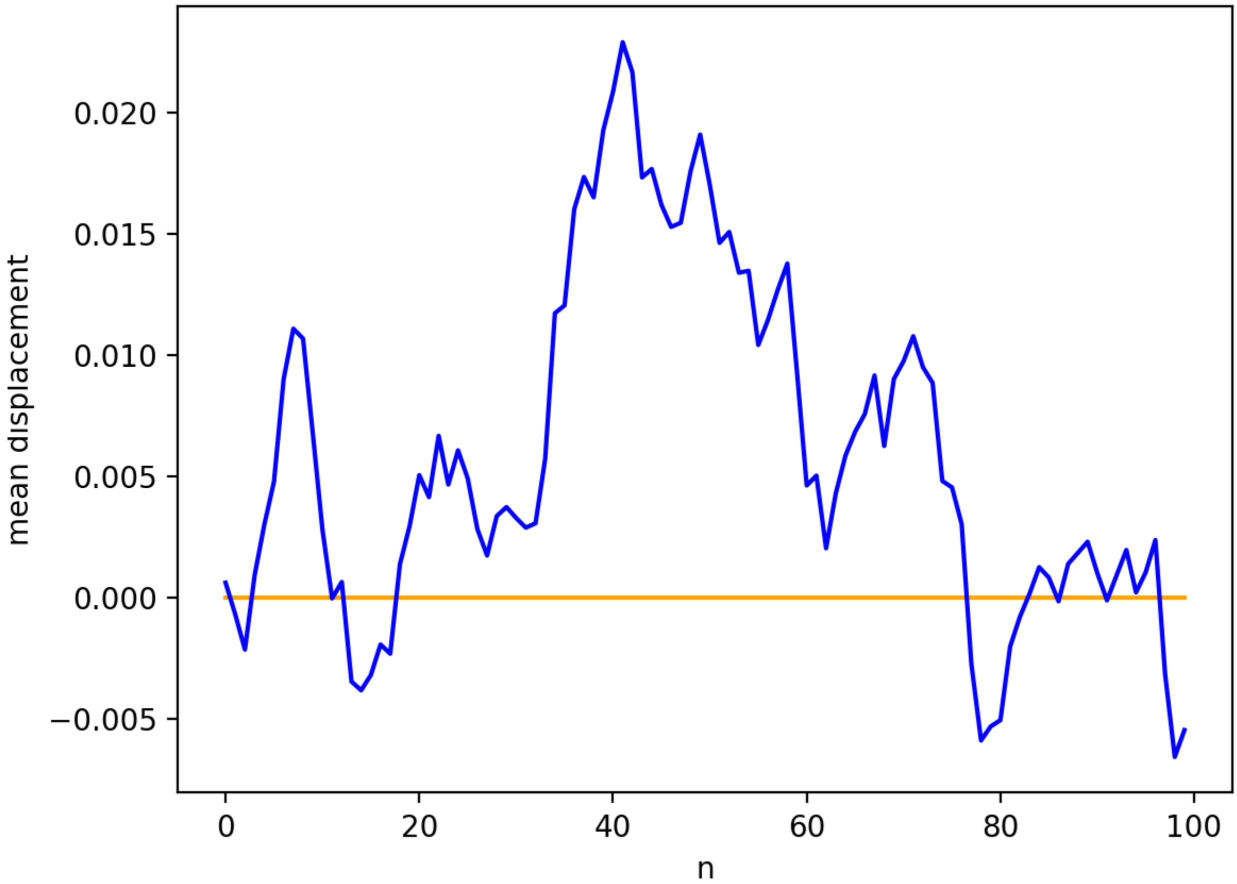
Here, we ran 10000 trials.

Blue represents the estimated value, while orange is the actual mathematical probability



Here, we ran 100000 trials.

Blue represents the estimated value, while orange is the actual mathematical probability

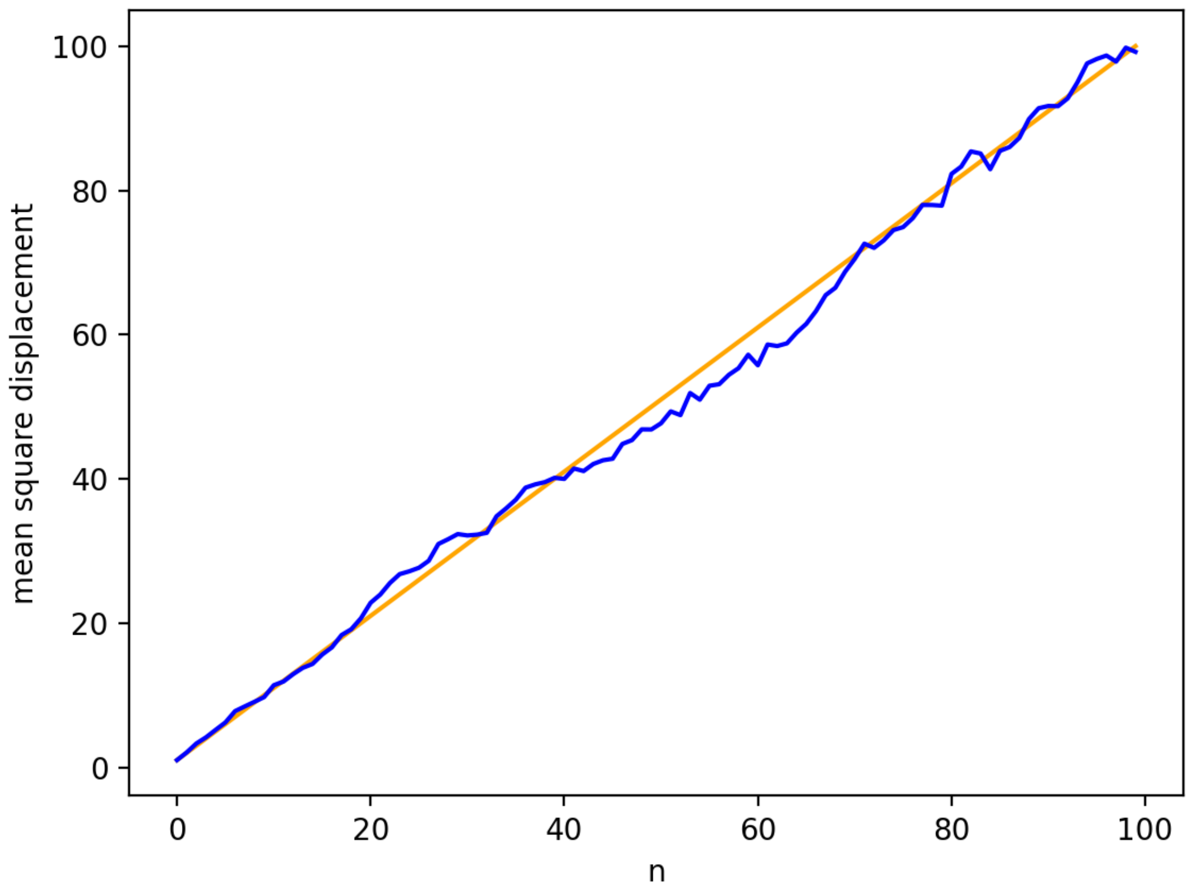


This gives random values, because of the nature of the problem. More trials wont necessarily give a more accurate value.

Mean Squared Displacement

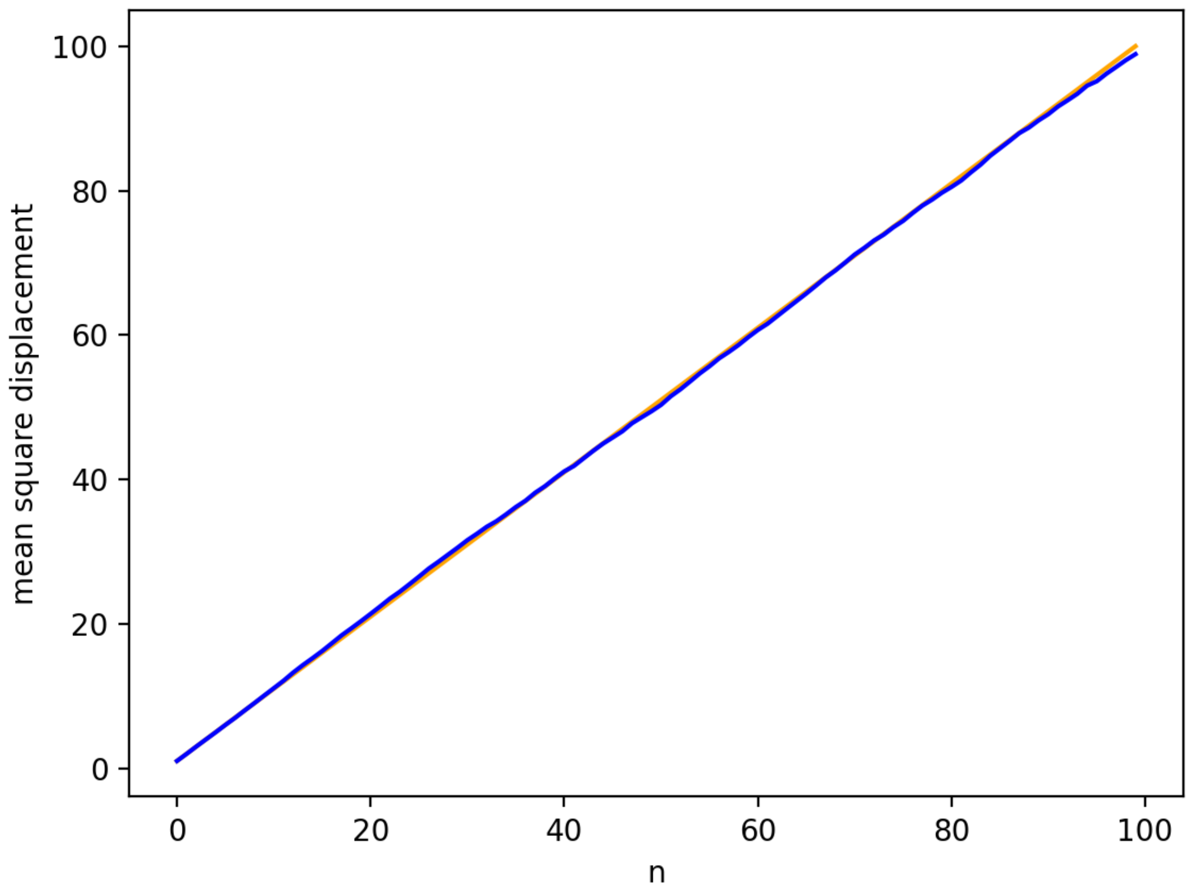
In the below simulation, we ran 100 trials for each value of `n`, and used the results of these trials to find the average squared displacement

Blue represents the estimated value, while orange is the actual mathematical probability



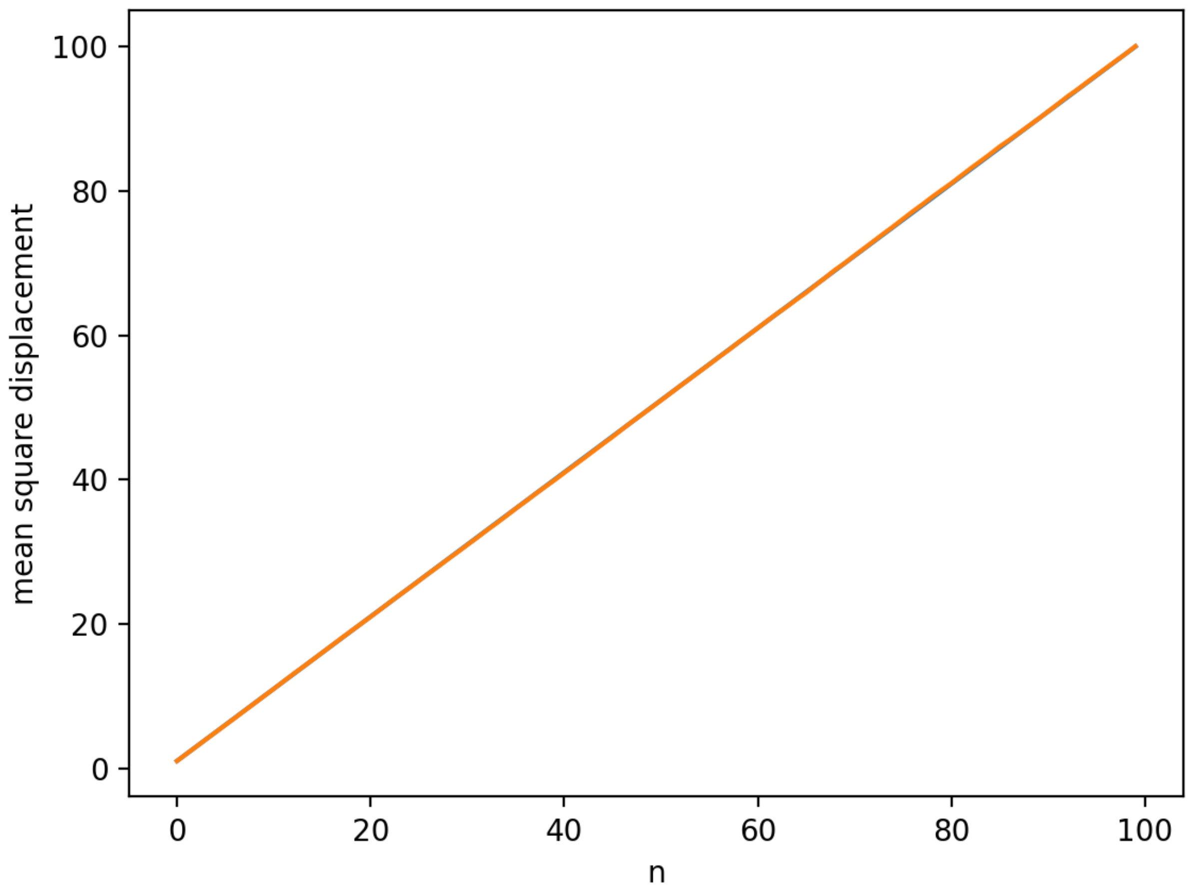
Here, we ran 10000 trials.

Blue represents the estimated value, while orange is the actual mathematical probability



Here, we ran 100000 trials.

Blue represents the estimated value, while orange is the actual mathematical probability



As you can see, when we run more trials, our estimate gets closer to the actual value