Q 1)

1) Find P of them ending at same point after n step.

we can model this as a system where one of the drunks is stuck at the origin, and the other drunk takes 2n steps.

Now for them to end up at the same point, the moving drunk has to move n steps right and n steps left in any order.

the number of such orders is

$^{2n}C_n$.

Hence, the probability of this is

$^{2n}C_n \times \left(\frac{1}{2}\right)^{2n}$

~~(as going either right or left has a probability of $\frac{1}{2}$)~~

(as total number of possibilities is 2^{2n})

2) P of drunk ending up at origin

for this to happen, he has to move right exactly $\frac{n}{2}$ times, and move left exactly $\frac{n}{2}$ times. hence, if n is even, then the number of ways he can do this is,

${}^{n}C_{\frac{n}{2}}$

So $P = \begin{cases} {}^{n}C_{\frac{n}{2}} \times \left(\frac{1}{2}\right)^{n} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$

3) Mean displacement.

let us define X_i as a random variable, where if the i^{th} step is right, $X_i = 1$, and if it is left the $X_i = -1$.

Clearly

Mean displacement $= E(X_1 + X_2 + X_3 \dots X_n)$

as X_i are independent of each other,

$E(x_1 + x_2 + x_3 \dots x_n) = \sum_{i=1}^{n} E(x_i)$

now, $E(x) = \frac{1}{2} \cdot (-1) + \frac{1}{2} \cdot (1)$

$= 0$

So $\sum_{i=1} E(x_i) = 0$

So mean displacement $= 0$.

4) Average Squared displacement

We use the same x_i as in the the previous part.

mean squared displacement $= E\left(\left(\sum_{i=1} x_i\right)^2\right)$

$= E\left(\left(x_1^2 + x_2^2 + x_3^2 \ldots\right) + 2\left(x_1 x_2 + x_2 x_3 + x_3 x_1 \ldots\right)\right)$

$= E(x_1^2) + E(x_2^2) \ldots + 2\left(E(x_1 x_2) + E(x_2 x_3) \ldots\right)$

$E(x_i^2) = 1$ as x_i = either 1 or -1

So $x_i^2 = 1$

So $E(x_i^2) = 1$

$E(x_i x_j) = 0$ as $x_i x_j = 1$ with $P = \frac{1}{2}$

-1 with $P = \frac{1}{2}$

So $E(x_i x_j) = 1 \cdot \frac{1}{2} + -1 \cdot \frac{1}{2}$

$= 0$

So ..

mean squared displacement = $\frac{(1+1+1 \ldots \text{n times}) + (0+0+0 \ldots)}{n}$