4) Given

$$\frac{\partial P(x,y,t)}{\partial t} = D_x \frac{\partial^2 P(x,y,t)}{\partial x^2} + D_y \frac{\partial^2 P(x,y,t)}{\partial y^2}$$

$$-L \le y \le L \; ; \; -L \le x \le L \; ; \; 0 \le t \le T$$

Initial conditions: $P(x,y,0) = 1$ for $x=0$ and $y=0$

$= 0$ otherwise

Boundary conditions: $P(x,y,t) = 0$ $\quad x^2+y^2 = L^2$ all t

$$\cancel{P(L,y,t) = 0 \text{ all } y,t}$$

$$\cancel{P(x,-L,t) = 0 \text{ all } x,t}$$

$$\cancel{P(x,L,t) = 0 \text{ all } x,t}$$

Forward euler scheme:

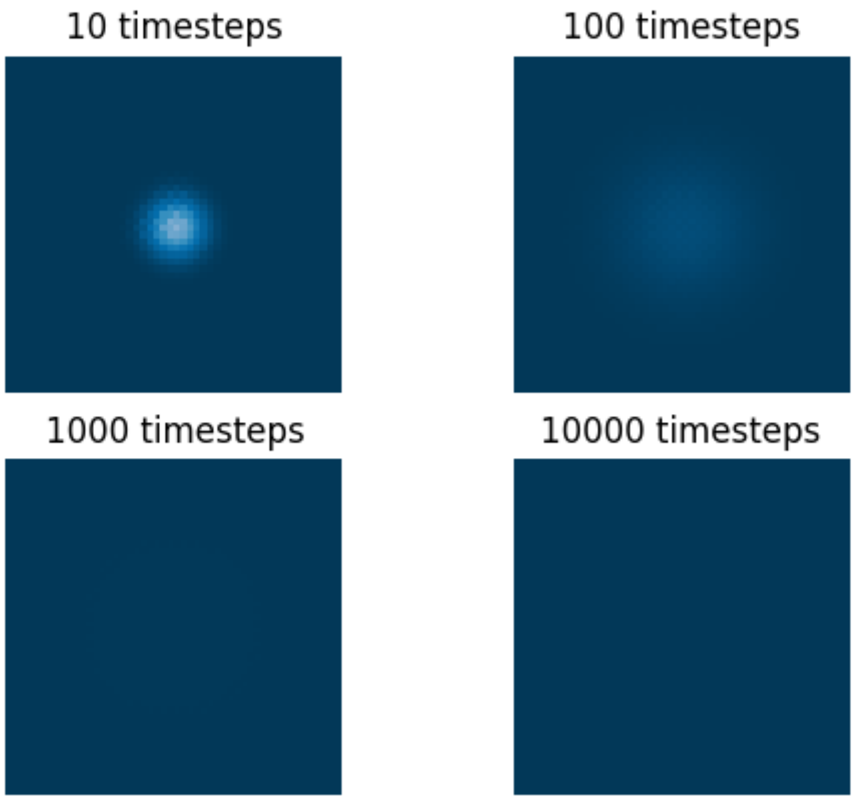
$t = n\Delta t, \quad n \in 0,1,2 \ldots N_t$

$$P(i,j,n+1) = P(i,j,n) + \frac{D_x \Delta t}{(\Delta x)^2}\Big[P(i+1,j,n) - 2P(i,j,n) + P(i-1,j,n)\Big] + \frac{D_y \Delta t}{(\Delta y)^2}\Big[P(i,j+1,n) - 2P(i,j,n) + P(i,j-1,n)\Big]$$

The plots are as shown

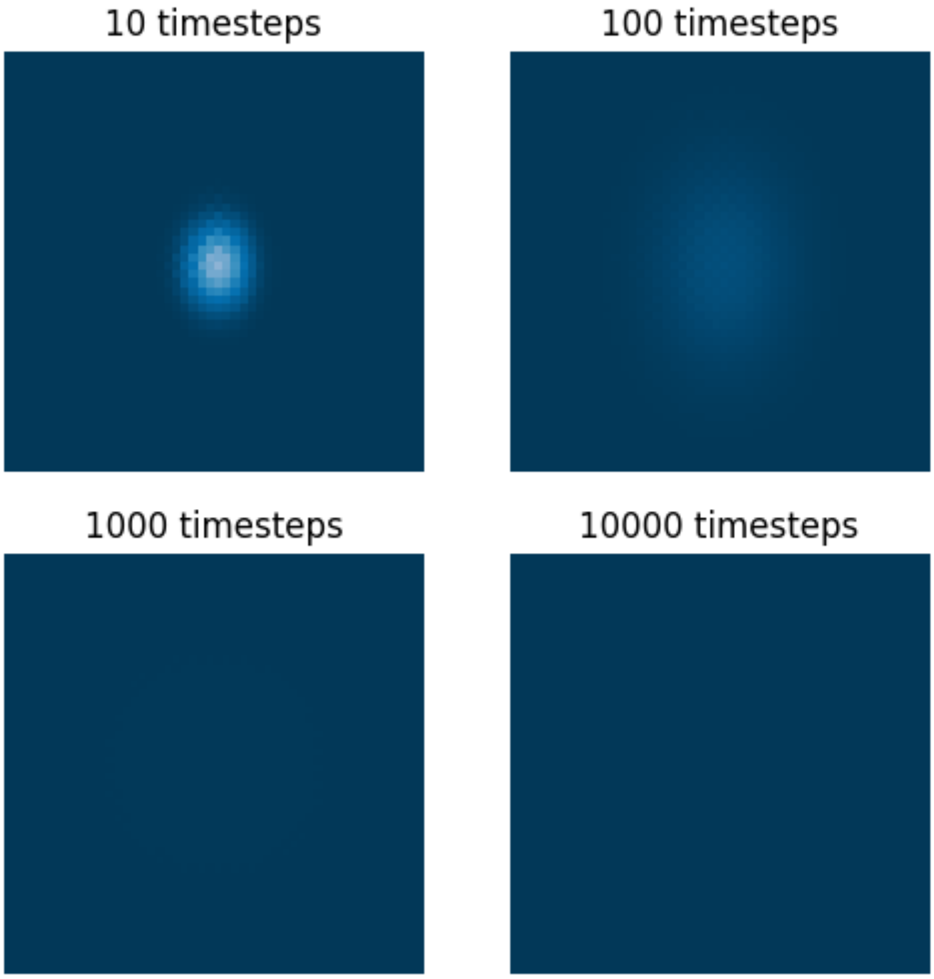
Dx = Dy

Dx = 4, Dy =4



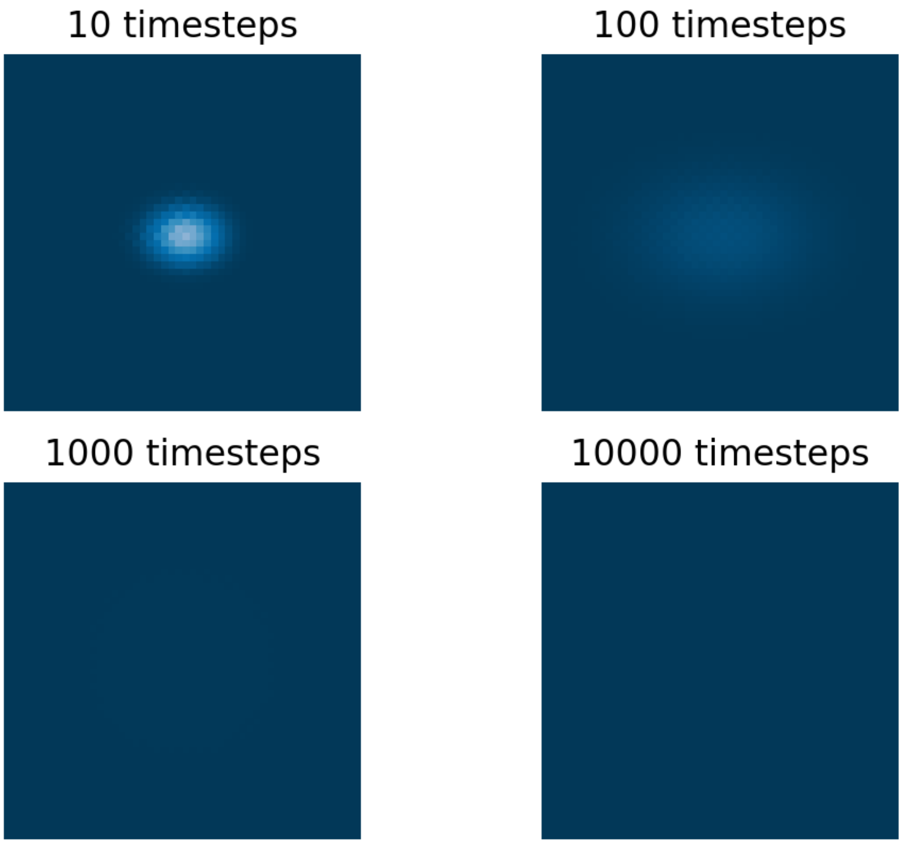
Dx > Dy

Dx = 9, Dy = 4


10 timesteps
100 timesteps
1000 timesteps
10000 timesteps

Dx < Dy

Dx = 4, Dy = 9


10 timesteps
100 timesteps
1000 timesteps
10000 timesteps