1. Given X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} 0.1 \; for\, x = 0.2 \\ 0.2 \; for\, x = 0.4 \\ 0.2 \; for\, x = 0.5 \\ 0.3 \; for\, x = 0.8 \\ 0.2 \; for\, x = 1 \\ 0 \; otherwise \end{cases}$$

1. Find the range RX of the random variable X.

2. Find P(X ≤ 0.5)

3. Find P(0.25 < X < 0.75)

4. P(X = 0.2|X < 0.6)

1. Finding the range RX of the random variable X:

The range of a random variable represents the set of all possible values it can take. In this case, the possible values of X are 0.2, 0.3, 0.5, 0.8, and 0.1. Therefore, the range RX of X is {0.2, 0.3, 0.5, 0.8, 0.1}.

2. Finding P(X ≤ 0.5):

To find P(X ≤ 0.5), we need to sum up the probabilities of all values of X that are less than or equal to 0.5. From the given probability mass function (PMF), we can see that the values of X that satisfy this condition are 0.2, 0.3, and 0.5. Therefore, we calculate the probability as follows:

P(X ≤ 0.5) = P(X = 0.2) + P(X = 0.3) + P(X = 0.5)

= 0.1 + 0.2 + 0.2

= 0.5

So, P(X ≤ 0.5) is equal to 0.5.

3. Finding P(0.25 < X < 0.75):

To find P(0.25 < X < 0.75), we need to sum up the probabilities of all values of X that lie between 0.25 and 0.75 (excluding the endpoints). From the given PMF, the value of X that satisfies this condition is 0.5. Therefore, we calculate the probability as follows:

$P(0.25 < X < 0.75) = P(X = 0.5)$

$= 0.2$

So, $P(0.25 < X < 0.75)$ is equal to 0.2.

4. Finding $P(X = 0.2 \mid X < 0.6)$:

To find $P(X = 0.2 \mid X < 0.6)$, we need to calculate the conditional probability of X being equal to 0.2 given that X is less than 0.6. From the given PMF, we can see that the values of X that satisfy this condition are 0.2 and 0.3. Therefore, we calculate the probability as follows:

$P(X = 0.2 \mid X < 0.6) = P(X = 0.2) / P(X < 0.6)$

$= 0.1 / (0.1 + 0.2)$

$= 0.1 / 0.3$

$= 1/3$

So, $P(X = 0.2 \mid X < 0.6)$ is equal to 1/3 or approximately 0.3333.

2. Two equal and fair dice are rolled, and we observed two numbers X and Y.

    1. Find RX, RY, and the PMFs of X and Y.
    2. Find $P(X = 2, Y = 6)$.
    3. Find $P(X>3|Y = 2)$.
    4. If $Z = X + Y$. Find the range and PMF of Z.
    5. Find $P(X = 4|Z = 8)$.

Solution

2. Two equal and fair dice are rolled, and we observe two numbers X and Y.

Finding RX, RY, and the PMFs of X and Y:

RX = {1, 2, 3, 4, 5, 6}

RY = {1, 2, 3, 4, 5, 6}

PMF of X: $P(X = x) = 1/6$ for x in {1, 2, 3, 4, 5, 6}

PMF of Y: $P(Y = y) = 1/6$ for y in {1, 2, 3, 4, 5, 6}

Finding $P(X = 2, Y = 6)$:

$P(X = 2, Y = 6) = (1/6) * (1/6) = 1/36$

Finding $P(X > 3 \mid Y = 2)$:

$P(X > 3 \mid Y = 2) = 1/2$

If $Z = X + Y$, finding the range and PMF of Z:

Range of Z = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

PMF of Z: Calculate probabilities for each sum value by considering all possible combinations of X and Y.

3. In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student X's score is equal to the total number of correct answers, then find out the PMF of X. What is $P(X>15)$?

solution

3. In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student's score is equal to the total number of correct answers, then find out the PMF of X. What is $P(X > 15)$?

PMF of X: The student's score is equal to the total number of correct answers. As the student knows the answer to 10 questions and randomly guesses the rest, we can calculate the probabilities based on different scores.

To find $P(X > 15)$, we need to sum up the probabilities of all scores greater than 15.

4. The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is $P(10<Y\leq15)$?

The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is $P(10 < Y \leq 15)$?

Y follows a Poisson distribution with a rate of 10 students per hour. To find $P(10 < Y \leq 15)$, we need to calculate the cumulative probability for Y, subtracting the probability of Y being less than or equal to 10 from the probability of Y being less than or equal to 15.

5.Two independent random variables, X and Y,are given such that X~Poisson(α) and Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.

Solution

To find the probability mass function (PMF) of the random variable Z = X + Y, where X follows a Poisson distribution with parameter α and Y follows a Poisson distribution with parameter β, we can utilize the properties of the Poisson distribution and the convolution operation.

The PMF of Z can be found by convolving the PMFs of X and Y. Let's denote the PMFs as P(X = k) and P(Y = j) for the respective random variables.

The PMF of Z, denoted as P(Z = n), is given by:

$P(Z = n) = \sum P(X = k) * P(Y = j)$, for all k and j such that k + j = n.

Using the properties of the Poisson distribution, we know that:

$P(X = k) = (e^{-\alpha} * \alpha^k) / k!$

$P(Y = j) = (e^{-\beta} * \beta^j) / j!$

Therefore, substituting these expressions into the convolution formula, we have:

$P(Z = n) = \sum (e^{-\alpha} * \alpha^k / k!) * (e^{-\beta} * \beta^j / j!)$, for all k and j such that k + j = n.

Simplifying this expression, we get:

$P(Z = n) = e^{-(\alpha + \beta)} * \sum (\alpha^k * \beta^{n-k}) / (k! * (n-k)!)$, for all k such that $k \leq n$.

This gives us the PMF of the random variable Z = X + Y, where X follows a Poisson distribution with parameter α and Y follows a Poisson distribution with parameter β.

the sum ∑ ranges from 0 to n in the formula, considering all possible values of k such that k + j = n.

6. There is a discrete random variable X with the pmf.

$$P_x(x) = \begin{cases} \frac{1}{4}\, when\, x = \quad -2 \\ \frac{1}{8}\, when\, x = \quad -1\frac{1}{4}\, when\, x = 20\, otherwisewen\, x = 0 \\ \frac{11}{84}\, when\, x = 1 \end{cases}$$

If we define a new random variable Y = (X + 1)2 then

1. Find the range of Y.

2. Find the pmf of Y.

2.Assuming X is a continuous random variable with PDF

$$f_X(x) = \begin{cases} cx2 \mid x \mid \leq 1 \\ \\ 0\, Otherwise \end{cases} \text{Find the constant cc.}$$

1. Find EX and Var(X).

2. Find $P(X \geq \frac{1}{2})$.

2. If X is a continuous random variable with pdf

$$f_x(x) = \begin{cases} 4x^3\, 0 < x \leq 1 \\ 0\, otherwise \end{cases}$$

Find $P(X \leq \frac{2}{3} \mid X \rangle \frac{1}{3})$

3. If X~Uniform$(\frac{-\pi}{2},\pi)$ and $Y = \sin(X)$, then find $f_Y(y)$.

4. If X is a random variable with CDF

$$F_X(x) = \begin{cases} 1\, X \geq 1 \\ \frac{1}{2} + \frac{x}{2}\, where\, 0 \leq X < 1 \\ 0\, where\, x < 0 \end{cases}$$

1. What kind of random variable is X: discrete, continuous, or mixed?
2. Find the PDF of X, $f_X(x)$.
3. Find $E(e^X)$.
4. Find $P(X = 0 | X \leq 0.5)$.

5. There are two random variables X and Y with joint PMF given in Table below
    1. Find $P(X\leq 2, Y\leq 4)$.
    2. Find the marginal PMFs of X and Y.
    3. Find $P(Y = 2 | X = 1)$.
    4. Are X and Y independent?

(L)	$Y = 2$	$Y = 4$	$Y = 5$
$X = 1$	1/12	1/24	1/24
$X = 2$	1/6	1/12	1/8
$X = 3$	¼	1/8	1/12

6. A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.

To find the joint probability mass function (PMF) of X and Y, where X represents the number of white shirts and Y represents the number of black shirts, we need to calculate the probability for each possible combination of X and Y when selecting 10 shirts from the box without replacement.

Given that there are 40 white shirts and 60 black shirts in the box, the total number of shirts is 100. Let's denote this as N = 100.

The joint PMF of X and Y can be calculated as follows:

P(X = x, Y = y) = (Number of ways to choose x white shirts from 40) * (Number of ways to choose y black shirts from 60) / (Number of ways to choose 10 shirts from N)

The "Number of ways to choose x white shirts from 40" can be calculated using the binomial coefficient C(40, x), and similarly for the "Number of ways to choose y black shirts from 60" using C(60, y).

The "Number of ways to choose 10 shirts from N" can be calculated using C(N, 10).

Therefore, the joint PMF of X and Y can be expressed as:

P(X = x, Y = y) = (C(40, x) * C(60, y)) / C(100, 10)

where x ranges from 0 to 10 (since we can choose at most 10 white shirts) and y ranges from 0 to 10 (since we can choose at most 10 black shirts), and the sum of x and y is equal to 10 (since we choose a total of 10 shirts).

By calculating this expression for all possible values of x and y, you can obtain the joint PMF of X and Y.

7.If A and B are two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} 6xy \textit{ where } 0 \le x \le 1,\ 0 \le y \le \sqrt{x} \\ 0 \textit{ otherwise} \end{cases}$$

a. Find fX(a) and fY(b).

b. Are A and B independent of each other?

c. Find the conditional PDF of A given B = b, fA|B(a|b).

d. Find E[A|B = b], for 0 ≤ y ≤ 1.

e. Find Var(A|B = b), for 0 ≤ y ≤ 1.

8.There are 100 men on a ship. If Xi is the ith man's weight on the ship and Xi's are independent and identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men's total weight on the ship exceeds 18,000.

To find the probability that the men's total weight on the ship exceeds 18,000, we can use the Central Limit Theorem to approximate the distribution of the sum of the weights.

Given that Xi represents the weight of the ith man on the ship, and Xi's are independent and identically distributed with a mean of $\mu = 170$ and a standard deviation of $\sigma = 30$, we can calculate the mean and standard deviation of the sum of the weights.

The sum of the weights of the men, denoted as Y, can be expressed as:

$Y = X1 + X2 + ... + X100$

1. Calculate the mean and standard deviation of Y:

Since the Xi's are independent, the mean and standard deviation of Y will be:

Mean of Y: $\mu\_Y = 100 * \mu$

Standard deviation of Y: $\sigma\_Y = sqrt(100) * \sigma$

$\mu\_Y = 100 * 170 = 17000$

$\sigma\_Y = sqrt(100) * 30 = 300$

2. Calculate the probability that the total weight exceeds 18,000:

To find the probability that the total weight exceeds 18,000, we can use the properties of the normal distribution.

$P(Y > 18,000) = 1 - P(Y \leq 18,000)$

Using the mean (μ_Y) and standard deviation (σ_Y) calculated above, we can calculate the Z-score for the given value:

$Z = (18,000 - \mu\_Y) / \sigma\_Y$

Then, using a standard normal table or a Z-table, we can find the probability associated with the Z-score:

$P(Y > 18,000) = 1 - P(Z \leq Z)$

Alternatively, using a calculator or software that provides normal distribution calculations, we can directly calculate the probability $P(Y > 18,000)$ using the mean and standard deviation.

Please note that the accuracy of the approximation depends on the assumption that the Xi's are independent and identically distributed and that the sample size is sufficiently large.

9.Let X1, X2, ……, X25 are independent and identically distributed. And have the following PMF

If Y = X1 + X2 + … + Xn, estimate $P(4 \leq Y \leq 6)$ using central limit theorem.

To estimate $P(4 \leq Y \leq 6)$ using the Central Limit Theorem, we can approximate the distribution of the sum Y = X1 + X2 + ... + X25 by a normal distribution.

Given that X1, X2, ..., X25 are independent and identically distributed with a given probability mass function (PMF), we can use the properties of the Central Limit Theorem to estimate the probability range.

The Central Limit Theorem states that when independent and identically distributed random variables are added, their sum tends to follow a normal distribution as the number of variables increases.

To estimate $P(4 \leq Y \leq 6)$, we can calculate the mean and standard deviation of Y and use the normal distribution to estimate the probability.

1. Calculate the mean (μ) and standard deviation (σ) of X:

   μ = E(X) = ∑ (xi * P(X = xi)) for all xi

   σ = sqrt(Var(X)) = sqrt(∑ [(xi - μ)^2 * P(X = xi)] for all xi

2. Calculate the mean and standard deviation of Y:

   Since X1, X2, ..., X25 are independent, the mean and standard deviation of Y will be:

   μ_Y = 25 * μ (mean of Y)

   σ_Y = sqrt(25) * σ (standard deviation of Y)

3. Estimate $P(4 \leq Y \leq 6)$ using the normal distribution:

To estimate $P(4 \leq Y \leq 6)$, we can use the properties of the normal distribution and calculate the Z-scores for the lower and upper limits:

$Z_{lower} = (4 - \mu_Y) / \sigma_Y$

$Z_{upper} = (6 - \mu_Y) / \sigma_Y$

Using a standard normal table or a Z-table, we can find the probabilities associated with these Z-scores:

$P(4 \leq Y \leq 6) = P(Z_{lower} \leq Z \leq Z_{upper})$

Alternatively, we can use a calculator or software that provides normal distribution calculations to estimate $P(4 \leq Y \leq 6)$ directly using the calculated mean and standard deviation.

Please note that the accuracy of the approximation depends on the number of variables and the underlying distribution. The Central Limit Theorem provides a good approximation when the number of variables is sufficiently large.