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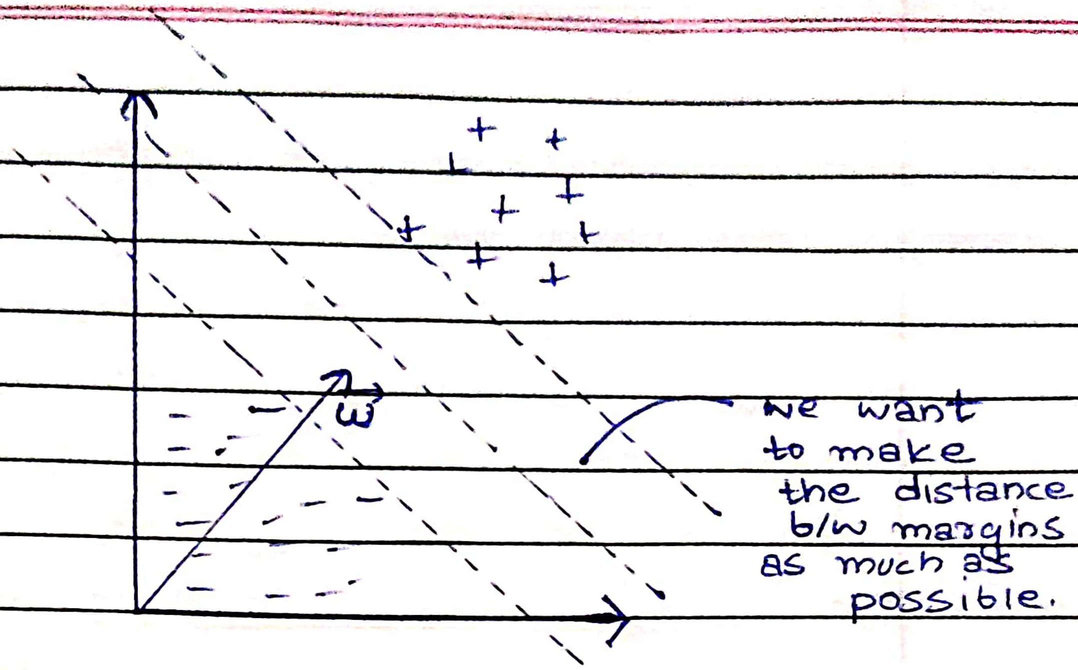


The equations for the margins are:

$$\vec{w} \cdot \vec{x} + b = 1$$

$$\vec{w} \cdot \vec{x} + b = -1$$

For positive samples

$$\vec{w} \cdot \vec{x_+} + b \geq 1$$

For negative samples

$$\vec{w} \cdot \vec{x_-} + b \leq -1$$

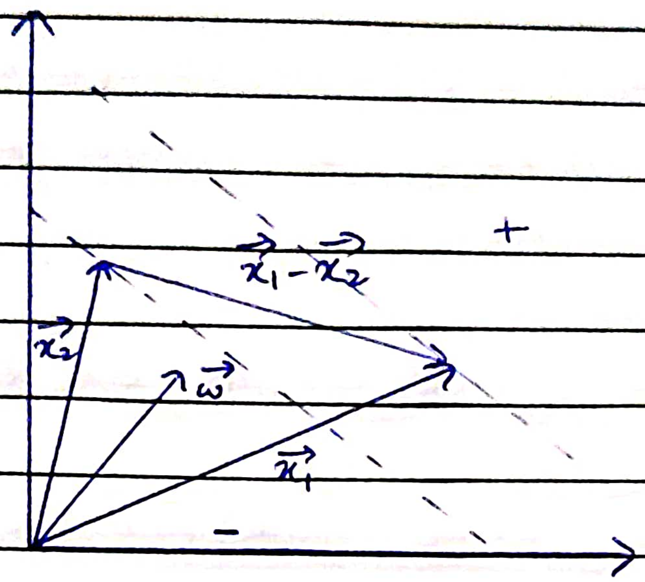
For +ve samples , $y_i = +1$

For -ve samples , $y_i = -1$

So $\quad y_i(\vec{w} \cdot \vec{x_i} + b) \geq 1$

$$\Rightarrow \quad y_i(\vec{w} \cdot \vec{x_i} + b) - 1 \geq 0$$

$y_i(\vec{w} \cdot \vec{x_i} + b) - 1 = 0$ for x_i on margins.



Let x_1 and x_2 be 2 points of different classes both lying on the margin.

Then width of the line =

$$\frac{(\vec{x_1} - \vec{x_2}) \cdot \vec{w}}{\|w\|}$$

x_1 is of +ve class

so $y_1 = +1 \Rightarrow 1 \cdot (\vec{w} \cdot \vec{x_1} + b) - 1 = 0$

$\Rightarrow \vec{x_1} \cdot \vec{w} = 1 - b$.

similarly, $\vec{x_2} \cdot \vec{w} = -1 - b$

So width $= \dfrac{(\vec{x_1} - \vec{x_2}) \cdot \vec{w}}{\|w\|} = \dfrac{(1-b) - (-1-b)}{\|w\|}$

$$= \frac{2}{\|w\|}$$

We have to maximize the width $= \frac{2}{||w||}$

we can equivalently minimize $||w||$

or minimize $\frac{1}{2}||w||^2$.

So we minimize $\frac{1}{2}||w||^2$.

with condition $y_i(\vec{w} \cdot \vec{x_i} + b) \geq 1 \quad \forall i$