Practical :- 1

Topic :- Limits and Continuity

Ex 1].
$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$\lim_{x\to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$=\frac{1}{3}\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$=\frac{1}{3}\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$=\frac{1}{3}\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$=\frac{1}{3}\frac{2\sqrt{a}+2\sqrt{a}}{2\sqrt{3a}}$$

$$=\frac{1}{3}\frac{\cancel{4}^{2}\cancel{\sqrt{a}}}{\cancel{2}\sqrt{3}\sqrt{\cancel{a}}}$$

$$=\frac{2}{3\sqrt{3}}$$

2].
$$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

$$\lim_{y\to 0}\frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$$

$$\lim_{y\to 0}\frac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{a+y}-\sqrt{a})}$$

$$=\frac{1}{\sqrt{a+0}(\sqrt{a+0}+\sqrt{a})}$$

$$=\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$$

$$=\frac{1}{\sqrt{a}(2\sqrt{a})}$$

$$=\frac{1}{2a}$$

3].
$$\lim_{x\to\frac{\pi}{6}}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$$

By substituting $x-\frac{\pi}{6}=h$

$x=h+\pi/6$

where $h\to 0$

$$\lim_{h\to 0}\frac{\cos(h+\pi/6)-\sqrt{3}\sin(h+\pi/6)}{\pi-6(h+\pi/6)}$$

$$\lim_{h\to 0}\frac{\cos h.\cos\pi/6-\sin h.\sin\pi/6-\sqrt{3}\sin h.\cos\pi/6+\cos h.\sin\pi/6}{\pi-\cancel{6}\left(\frac{6h+\pi}{\cancel{6}}\right)}$$

$\cos\pi/6=\cos 30°=\sqrt{3}/2$

$\sin\pi/6=\sin 30°=1/\sqrt{2}$

$$\lim_{h\to 0} \frac{\cosh \frac{\sqrt3}{2} - \sinh \frac{1}{\sqrt2} - \sqrt3\left(\sinh\cdot\sqrt3/2 + \cosh\cdot\frac{1}{\sqrt2}\right)}{\pi - 6h + \pi}$$

$$\lim_{h\to 0} \frac{\cancel{\cos\frac{\sqrt3}{2}h} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cancel{\cos\frac{\sqrt3}{2}h}}{\cancel{-6x} - 6h}$$

Ex

$$\lim_{h\to 0} \frac{\cancel{-}\sin\frac{4h}{2}}{\cancel{-}6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{\frac{12}{3}h}$$

$$\frac{1}{3}\lim_{h\to 0}\frac{\sin h}{h}$$

$$= \frac{1}{3}\times 1 = \frac{1}{3}$$

4]. $\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$

By rationalizing numerator & Denominator both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}-\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\left[\frac{\overset{4}{\cancel{8}}(\sqrt{x^2+3}+\sqrt{x^2+1})}{\cancel{2}(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$4\lim_{x\to\infty}\left[\frac{\sqrt{x^2(1+\frac{3}{x^2})} + \sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+5/x^2)} + \left(\sqrt{x^2(1-3/x^2)}\right)}\right]$$

After applying limit we get, $= 4$

5) ~~i) if (x)~~

5) i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$ } at $x = \pi/2$

$= \frac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f. at $x = \pi/2$ define.

as $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \frac{\cos x}{\pi - 2x}$

By substituting method

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$

where $h\to 0$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{\pi - \cancel{2}\left(\frac{2h+\pi}{\cancel{2}}\right)}$$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{-2h}$$ using $\cos(A+B) = \cos A\cdot\cos B -$

$$\lim_{h\to 0}\frac{\cosh\cdot\cos\pi/2 - \sinh\cdot\sin\pi/2}{-2h}$$

$$\lim_{h\to 0}\frac{\cosh\cdot 0 - \sinh}{-2h}$$

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$$\lim_{h\to 0} \frac{\sin h}{2h}$$

Ex.

$$\frac{1}{2}\lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2}$$

b) $\lim_{x\to \pi/2} f(x) = \lim_{x\to \pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ using $\sin 2x = 2\sin x \cdot \cos x$

$$\lim_{x\to \pi/2} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x\to \pi/2} \frac{2\cancel{\sin x}\cdot \cos x}{\sqrt{2}\,\cancel{\sin x}}$$

$$\lim_{x\to \pi/2} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x\to \pi/2} \cos x$$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore f$ is not continuous at $x = \pi/2$.

5) ii) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x \le 6$

$= \frac{x^2+9}{x+3}$ $6 \le x < 9$ } at $x = 3$ & $x = 6$

at $x = 3$

i) $f(x) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ L.H.L = R.H.L

f is continuous at $x=3$

for $x = 6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{\cancel{27}^{3}}{\cancel{9}_{1}} = 3$

2) $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} (x+3) = 3+6 = 9$

$\therefore$ R L.H.S $\neq$ R.H.S

function is not continuous

6] i) $f(x) = \frac{1-\cos 4x}{x^2}$ $x < 0$

$= k$ $x = 0$ } at $x = 0$

Sol^n :- f is continuous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

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$$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

Ex:

$$2(2)^2 = k$$

$$\therefore k = 8$$

iii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$; $= k$ $\quad x = 0$ $\Big\}$ at $x = 0$

Sol^n: $f(x) = (\sec^2 x)^{\cot^2 x}$

using $\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

$\cot 2x = \frac{1}{\tan 2x}$

$$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$$

We know that

$$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$$

$$\therefore k = e$$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$; $= k$ $\quad x = \pi/3$ $\Big\}$ at $x = \pi/3$

$$x - \pi/3 = h$$

$$x = h + \pi/3$$

where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$ using $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$\lim_{h \to 0} \frac{\sqrt{3} - \frac{\tan \pi/3 + \tanh}{1 - \tan \pi/3 \cdot \tanh}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\frac{\sqrt{3}(1 - \tan \pi/3 \cdot \tanh) - (\tan \pi/3 + \tanh)}{1 - \tan \pi/3 \cdot \tanh}}{-3h}$$

$$\lim_{h \to 0} \frac{\frac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \tanh) - (\sqrt{3} + \tanh)}{1 - \sqrt{3} \tanh}}{-3h}$$

$$\lim_{h \to 0} \frac{\frac{(\cancel{\sqrt{3}} - 3\tanh - \cancel{\sqrt{3}} - \tanh)}{1 - \sqrt{3} \tanh}}{-3h}$$

$$\lim_{h \to 0} \frac{\cancel{-} 4\tanh}{\cancel{-}3h(1 - \sqrt{3}\tanh)}$$

$$\lim_{h \to 0} \frac{4\tanh}{3h(1 - \sqrt{3}\tanh)}$$

$$\frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \lim_{h \to 0} \frac{1}{(1 - \sqrt{3}\tanh)}$$ $\quad \frac{\tanh}{h}$

$$= \frac{4}{3} \frac{1}{(1 - \sqrt{3}(0))}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$$

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7] is $f(x) = \dfrac{1-\cos 3x}{x \tan x}$ $x \neq 0$
$= 9$ $x = 0$ } at $x = 0$

Ex 1

$f(x) = \dfrac{1-\cos 3x}{x \cdot \tan x}$

$\lim_{x \to 0} = \dfrac{2\sin^2 3/2\, x}{x \cdot \tan x}$

$\lim_{x \to 0} = \dfrac{\dfrac{2\sin^2 3/2\, x}{x^2} \times x^2}{x \cdot \dfrac{\tan x}{x^2} \times x^2}$

$= \cancel{x}\, \cancel{\lim} \; 2 \cdot \lim_{x \to 0} \dfrac{(3/2)^2}{1}$

$= \dfrac{2 \cdot (9/4)}{1}$

$= \dfrac{9}{2}$

$\lim_{x \to 0} f(x) = \dfrac{9}{2}$ $9 = f(0)$

f is not continuous at $x = 0$

Redefine function

$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x \cdot \tan x} & x \neq 0 \\ 9/2 & x = 0 \end{cases}$

Now $\lim_{x \to 0} f(x) = f(0)$

f has ~~retionable~~ removable discontinuous at $x = 0$

ii> $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$ $x \neq 0$
$= \pi/6$ $x = 0$ } at $x = 0$

$\lim_{x \to 0} \dfrac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$\lim_{x \to 0} \dfrac{e^{3x}-1}{x} \; \lim_{x \to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$\lim_{x \to 0} \dfrac{3 \cdot e^{3x}-1}{3x} \; \lim_{x \to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$\cancel{3} \log e \cdot \dfrac{\pi}{\cancel{180}\,60} = \pi/60 = f(0)$

f is continuous at $x = 0$

8]. $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continuous at $x = 0$

∴ When, f is continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \dfrac{e^{x^2} - \cos x}{x^2} = f(0)$

$\lim_{x \to 0} \dfrac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$\lim_{x \to 0} \dfrac{(e^{x^2}-1) + (1-\cos x)}{x^2}$

$\lim_{x \to 0} \dfrac{e^{x^2}-1}{x^2} + \lim_{x \to 0} \dfrac{1-\cos x}{x^2}$

$\log e + \lim_{x \to 0} \dfrac{2\sin^2 x/2}{x^2}$

$\therefore \log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$

Multiply with 2 in Num & Denominator

$= 1 + 2' \times \frac{1}{4 \cdot 2} = \frac{3}{2} = f(0)$

Ex.1] 9]. $f(x) = \frac{\sqrt{2} - \sqrt{1 - \sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1-\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \left(\frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}\right)$$

$$\lim_{x \to \pi/2} \frac{1 + \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)(\cancel{1+\sin x})(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

Practical :- 2

Topic :- Derivative

1] Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

is $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a) \tan x \cdot \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(\cancel{a}+h-\cancel{a}) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$$

Formula :- $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A - B)(1 + \tan A \cdot \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a - a - h) \cdot (1 + \tan a \cdot \tan(a+h))}{h \times \tan(a+h) \tan a}$$

$= \lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$

Ex 1

$= -1 \times \frac{1+\tan 2a}{1+\tan 2a}$

$= \frac{-\sec^2 a}{\tan 2a}$

$= \frac{-1}{\cos 2a} \times \frac{\cos 2a}{\sin 2a}$

$= -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\cos 2a$

$\therefore f$ is differentiable ~~at~~ $\forall\ a \in R$

ii) cosec x

$f(x) = \operatorname{cosec}(x)$

$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$

$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$

$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \cdot \sin x}$

Put ~~x=a~~ $x-a=h$

$x = a+h$

$x\to a,\ h\to 0$

$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$

Formula:- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\cdot \sin\left(\frac{C-D}{2}\right)$

$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$

$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos(2a+h/2)}{\sin a \sin(a+h)}$

$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$

$= \frac{-\cos a}{\sin 2a} = -\cot a \cdot \operatorname{cosec} a$

iii) sec x

$f(x) = \sec x$

$Df(x) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$

$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$

$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cdot \cos x}$

Put ~~x=a~~ $x-a=h$

$x = a+h$

as $x\to a,\ h\to 0$

$Df(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$

Formula:- $2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$

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$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin -\frac{h}{2} \times -\frac{1}{2}}{\cos a \cos(a+h) \times -h/2}$$

$$= \frac{-\frac{1}{2} \times -2\sin\left(\frac{2a+0}{2}\right)}{\cos \cdot \cos(a+0)}$$

$$= -\frac{1}{2} \times -2 \frac{\sin a}{\cos a \times \cos 0}$$

Ex. 1

$= \tan a \cdot \sec a$

Q.2]. If $f(x) = 4x+1 \quad x \leq 2$
$= x^2+5 \quad x > 0$, at $x=2$, then
Find function is differentiable or not

Sol^n:- L.H.D:-

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(a)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{x-2}$$

$Df(2^-) = 4$

R.H.D:-

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 2+2$

$Df(2^+) = 4$ R.H.D = L.H.D $\therefore f$ is differentiable at $x=2$

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3]. If $f(x) = 4x+7, \quad x < 3$
$= x^2+3x+1, \quad x \geq 3$ at $x=3$, then
find f is differentiable or not?

Sol^n:-
R.H.D:-

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$= 3+6$

$Df(3^+) = 9$

L.H.D:-

$$Df(3^-) = \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$$

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$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\frac{-h}{2} \times -\frac{1}{2}}{\cos a \cos(a+h) \times -h/2}$$

$$= -\frac{1}{2} \times -2 \frac{\sin\left(\frac{2a+0}{2}\right)}{\cos \cdot \cos(a+0)}$$

$$= -\frac{1}{2} \times -2 \frac{\sin a}{\cos a \times \cos 0}$$

Ex $= \tan a \cdot \sec a$

Q.2]. If $f(x) = 4x+1 \quad x \le 2$
$= x^2+5 \quad x > 0$, at $x=2$, then
Find function is differentiable or not

Solⁿ :- L.H.D :-

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(a)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{x-2}$$

$Df(2^-) = 4$

R.H.D :-

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 2+2$

$Df(2^+) = 4$ R.H.D = L.H.D $\therefore f$ is differentiable at $x=2$

Q.3]. If $f(x) = 4x+7 \quad , x < 3$
$= x^2+3x+1, \; x \ge 3$ at $x=3$, then
find f is differentiable or not?

Solⁿ :-

R.H.D :-

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$= 3+6$

$Df(3^+) = 9$

L.H.D :-

$$Df(3^-) = \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$Df(3^-) = 4$

$\therefore R.H.D \neq L.H.D$

$\therefore f$ is not differentiable at $x=3$

Q.4) If $f(x) = 8x-5$, $x \leq 2$
$= 3x^2-4x+7$, $x > 2$ at $x=2$, then find f is differentiable or not?

Solⁿ:-

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

R.H.D:-

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{(x-2)}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 3 \times 2 + 2$

$Df(2^+) = 8$

L.H.D:-

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$$

$= 8$

$Df(2^-) = 8$

$\therefore L.H.D = R.H.D$

$\therefore f$ is differentiable ~~function~~ at $x=3$.

06/02/2021

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Practical No.:- 3

Topic: Application of Derivative

① Find the intervals in which function is increasing or decreasing

Ex

1> $f(x) = x^3 - 5x + 1$

2> $f(x) = x^2 - 4x$

3> $f(x) = 2x^3 + x^2 - 20x + 4$

4> $f(x) = x^3 - 27x + 5$

5> $f(x) = 69 - 24x - 9x^2 + 2x^3$

② Find the intervals in which function is concave upwards or concave downwards.

1> $y = 3x^2 - 2x^3$

2> $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

3> $y = x^3 - 27x + 5$

4> $y = 69 - 24x - 9x^2 + 2x^3$

5> $y = 2x^3 + x^2 - 20x + 4$

i> To Find increasing

$f'(x) > 0$

$f'(x) = 3x^2 - 5 > 0$

$= 3x^2 > 5$

$\therefore x = \pm\sqrt{5/3}$

$+ \quad - \quad +$

$-\infty \quad -\sqrt{5/3} \quad +\sqrt{5/3} \quad \infty$

$x \in (-\infty, -\sqrt{5/3}) \cup (+\sqrt{5/3}, +\infty)$

To Find decreasing $f'(x) < 0$

$f'(x) < 0$

$= 3x^2 - 5 < 0$

$3x^2 < 5$

$x = \pm\sqrt{5/3}$

$x \in (-\sqrt{5/3}, +\sqrt{5/3})$

ii> $f(x) = x^2 - 4x$

To Find increasing

$f'(x) > 0$

$f'(x) = 2x - 4 > 0$

$= \cancel{2}(x-2) > 0$

$x - 2 > 0$

$x = 2$

$x \in (2, \infty)$

To Find decreasing

$f'(x) < 0$

$f'(x) = 2x - 4 < 0$

$x - 2 < 0$

$\therefore x = 2$

$x \in (-2, -\infty)$

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iii) $f(x) = 2x^3 - x^2 - 20x + 4$

$f(x) > 0$ for increasing

$f'(x) = 6x^2 - 2x - 20 > 0$

$= 6x^2 - 12x + 10x - 20 > 0$

$= 6x(x-2) + 10(x-2) > 0$

$= 6x + 10 > 0 \quad , x - 2 > 0$

$x = -5/3 \quad , x = 2$

$x \in (-5/3, -\infty) \cup (2, +\infty)$

$f'(x) < 0$ for decreasing

$f'(x) = 6x^2 - 2x - 20 < 0$

$= 6x(x-2) + 10(x-2) < 0$

$= 6x + 10 < 0 \quad , x - 2 < 0$

$\therefore x = -5/3 \quad , x = 2$

$x \in (-5/3, 2)$

4) $f(x) = x^3 - 27x + 5$

$f'(x) > 0$ for increasing

$f'(x) = 3x^2 - 27 > 0$

$= x^2 - 9 > 0$

$= (x-3)(x+3) = 0$

$x = \pm 3$

$x \in (-3, -\infty) \cup (+3, +\infty)$

$f'(x) < 0$ for decreasing

$f'(x) = 3x^2 - 27 = 0$

$= x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x = \pm 3$

$x \in (-3, +3)$

5) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) > 0$ for increasing

$f'(x) = 6x^2 - 18x - 24 > 0$

$= x^2 - 3x - 4 > 0$

$= x^2 - 4x + x - 4 > 0$

$= x(x-4) + 1(x-4) > 0$

$(x+1) > 0 , (x-4) > 0$

$x \in (-1, -\infty) \cup (4, +\infty)$

$f'(x) < 0$ for decreasing

$f'(x) = 6x^2 - 18x - 24 < 0$

$= x^2 - 3x - 4 < 0$

$= x^2 - 4x + x - 4 < 0$

$= x(x-4) + 1(x-4) < 0$

$x + 1 < 0 \quad x - 4 < 0$

$x \in (-1, 4)$

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Q.2.]

is $y = 3x^2 - 2x^3$

Ex

case 1 : $f(x)$ is concave upwards iff $f''(y) > 0$

$f'(y) = 6x - 6x^2$

$f''(y) = 6 - 12x$

$f''(y) > 0$

6

$6 - 12x > 0$

$x = \frac{1}{2}$

+ − +

$-\infty \quad \frac{1}{2} \quad +\infty$

$x \in (-\infty, \frac{1}{2})$

case: f^x is concave downward iff $f''(y) < 0$

$f''(y) < 0$

$6 - 12x < 0$

$x = \frac{1}{2}$

+ − +

$+\infty \quad \frac{1}{2} \quad -\infty$

$x \in (\frac{1}{2}, +\infty)$

2> $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

caseI : f^x is concave upwards iff $f''(y) > 0$

$f'(y) = 4x^3 - 18x^2 + 24x + 5$

$f''(y) = 12x^2 - 36x + 24$

$\therefore f''(y) > 0$

$12x^2 - 36x + 24 > 0$

$x^2 - 3x + 2 > 0$

$x(x-2) - 1(x-2) > 0$

$(x-1)\ (x-2) > 0$

$x = 1,\ x = 2$

$x \in (1, -\infty) \cup (2, +\infty)$

Case II :- $f''(y) < 0$, then it is concave downwards

$f''(y) < 0$

$= 12x^2 - 36x + 24 < 0$

$x^2 - 3x + 2 < 0$

$x^2 - 2x - x + 2 < 0$

$x(x-2) - 1(x-2) < 0$

$(x-1)\ (x-2) < 0$

$x = 1,\ x = 2$

$\therefore x \in (1, 2)$

3> $y = x^3 - 27x + 5$

$f''(y) > 0$ for concave upward

$f'(y) = 3x^2 - 27$

$f''(y) = 6x$

$f''(y) > 0$

$6x > 0$

$\therefore x = 0$

− +

$-\infty \quad 0 \quad +\infty$

$x \in (0, +\infty)$

$f''(y) < 0$ for concave downward

$f''(y) = 6x < 0$

$\therefore x = 0$

$\therefore x \in (0, -\infty)$

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Ex

Q

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f''(y) > 0$ for concave upwards

$f'(y) = -24x - 18x + 6x^2$

$f''(y) = 12x - 18$

$f''(y) > 0$

$12x - 18 > 0$

$12x > 18$

$x = 18/12$

$x = 3/2$

$x \in (\frac{3}{2}, +\infty)$

$f''(y) < 0$ for concave downwards

$f''(y) < 0$

$12x - 18 < 0$

$12x < \cancel{F0}\ 18$

$x = 3/2$

$\therefore x \in (3/2, -\infty)$

5) $y = 2x^3 + x^2 - 20x + 6$

$f''(y) > 0$ for concave upwards

$f'(y) = 6x^2 - 2x - 20$

$f''(y) = 12x + 2$

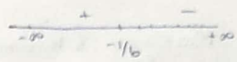
$\therefore f''(y) > 0$

$\therefore 12x + 2 > 0$

$12x = -2$

$x = -2/12$

$x = -\frac{1}{6}$



$\therefore x \in (-\frac{1}{6}, +\infty)$

$f''(y) < 0$ for concave downward

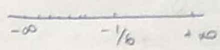
$f''(y) < 0$

$12x + 2 < 0$

$12x = -2$

$x = -2/12$

$x = -1/6$



$x \in (-\infty, -\frac{1}{6})$

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Practical No:- 4

Topic :- Application of Derivative & Newton's Method

Q.1] Find Maximum & minimum value of following functions

1) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6 = 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$, and $x = -2$

2) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider

$f'(x) = 0$

~~$15x^2 + 1$~~ $-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$\therefore f''(1) = -30 + 60$

$= 30 > 0$

$\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore f$ has the maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

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3) $f(x) = x^3 - 3x^2 + 1$
$\therefore f'(x) = 3x^2 - 6x$

Consider,
$f'(x) = 0$
$\therefore 3x^2 - 6x = 0$
$3x(x-2) = 0$
$3x = 0$ or $x - 2 = 0$
$x = 0$ or $x = 2$
$f''(x) = 6x - 6$
$f''(0) = 6(0) - 6$
$= -6 < 0$
$\therefore f$ has ~~min~~ maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$
$= 1$
$f''(2) = 6(2) - 6$
$= 12 - 6$
$= 6 > 0$
$\therefore f$ has minimum value at $x = 2$
$f(2) = (2)^3 - 3(2)^2 + 1$
$= 8 - 3(4) + 1$
$= 9 - 12$
$= -3$
$\therefore f$ has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$

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$f(x) = 2x^3 - 3x^2 - 12x + 1$
$\therefore f'(x) = 6x^2 - 6x - 12$
Consider
$f'(x) = 0$
$\therefore 6x^2 - 6x - 12 = 0$
$6(x^2 - x - 2) = 0$
$x^2 - x - 2 = 0$
$x^2 + x - 2x - 2 = 0$
$x(x+1) - 2(x+1) = 0$
$(x-2)(x+1) = 0$
$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$
$\therefore f''(2) = 12(2) - 6$
$= 24 - 6$
$= 18 > 0$
$\therefore f$ has minimum value at $x = 2$
$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$
$= 2(8) - 3(4) - 24 + 1$
$= 16 - 12 - 24 + 1$
$= -19$
$f''(-1) = 12(-1) - 6$
$= -12 - 6$
$= -18 < 0$
$\therefore f$ has maximum value at $x = -1$
$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1$
$= -2 - 3 + 12 + 1$
$= 8$

$\therefore f$ has maximum value 8 at $x = -1$ and f has minimum value -19 at $x = 2$

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Q.2] Find the ~~root of~~ root of following equ by Newton's method (Take 4 iteration only) correct upto 4 decimal

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$

$\therefore f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$$x_1 = 0.1727$$

$$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$$
$$= 0.0051 - 0.0895 - 9.4985 + 9.5$$
$$= -0.0829$$

$$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$
$$= 0.0895 - 1.0362 - 55$$
$$= -55.9467$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 0.1727 - \frac{0.0829}{55.9467}$$
$$= 0.1712$$

$$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$
$$= 0.0050 - 0.0879 - 9.416 + 9.5$$
$$= 0.0011$$

$$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$$
$$= 0.0879 - 1.0272 - 55$$
$$= -55.9393$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 0.1712 + \frac{0.0011}{55.9393}$$
$$= 0.1712$$

$\therefore$ The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$ [2,3]

$f'(x) = 3x^2 - 4$

$$f(2) = 2^3 - 4(2) - 9$$
$$= 8 - 8 - 9$$
$$= -9$$

$$f(3) = 3^3 - 4(3) - 9$$
$$= 27 - 12 - 9$$
$$= 6$$

Let $x_0 = 3$ be the initial approximation,

$\therefore$ By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

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$= 3 - \frac{6}{23}$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.7392 - \frac{0.596}{18.5096}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$

$= 19.8386 - 10.8284 - 9$

$= 0.0102$

$f'(x) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9$

$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$

$= 21.8943 - 4$

$= 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$= 2.7015 + 0.0050$

$= 2.7065$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ [1, 2]

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 + 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17$

$= -2.2$

Let $x_0 = -2$ be initial approximation

By Newton's Method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_2)}{f'(x_2)}$

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$= 2 - \frac{2.2}{5.2}$

$= 2 - 0.4230$

$= 1.577$

Ex

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.92193 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$\therefore x_2 = x_1 - \frac{(f x_1)}{f'(x_1)}$

$= 1577 + \cancel{0.65}\ \frac{0.6755}{8.2164}$

$= 1.577 + 0.0822$

$\cancel{= 1.6592} = 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.6592 + \frac{0.0204}{7.7143}$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= -7.6977$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 1.6618 + \frac{0.0004}{7.6977}$

$= 1.6618$

$\therefore$ ~~To~~ The root of equation is 1.6618

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Practical No: 5

Topic :- Integration

Ex

Q.1. Solve the following Integration

1) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$$I = \int \frac{1}{\sqrt{x^2+2x-3}}\,dx$$

$$= \int \frac{1}{\sqrt{x^2+2x+1-4}}\,dx$$

$$= \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx = \int \frac{1}{\sqrt{(x+1)^2-(2)^2}}\,d$$

$\because a^2+2ab+b^2 = (a+b)^2$

Substitute

$x+1 = t$

$dx = \frac{1}{t} x\, dt$ where $t = 1$, $t = x+1$

$$\int \frac{1}{\sqrt{t^2-4}}\,dt$$

$= \log(|t + \sqrt{t^2-4}|)$

$\left[\because \int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log|x+\sqrt{x^2 - a^2}|\right]$

$t = x+1$

$= \log(|x+1 + \sqrt{(x+1)^2-4}|)$

$= \log[|x+1 + \sqrt{(x^2+2x-3)}|] + c$

2) $\int (4e^{3x}+1)\,dx$

$$I = \int (4e^{3x}+1)\,dx$$

$$= \int 4e^{3x}\,dx + \int 1\,dx$$

$$= 4\int e^{3x}\,dx + \int 1\,dx$$

$$= \frac{4e^{3x}}{3} + x \qquad \left[\because \int e^{ax}\,dx = \frac{1}{a}e^{ax}\right]$$

$$= \frac{4e^{3x}}{3} + x + c$$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$$I = \int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$$

$$= \int 2x^2 - 3\sin x + 5x^{1/2}\,dx$$

$$= \int 2x^2\,dx - 3\int \sin x\,dx + 5\int x^{1/2}\,dx$$

$$= \frac{2x^3}{3} + 3\cos x + \frac{10\,{}^3\sqrt{x}}{3} + c$$

$\left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c\right.$

$\int \sin x\,dx = -\cos x + c$

$$= \frac{2x^3 + 10\,{}^3\sqrt{x}}{3} + 3\cos x + c$$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$I = \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right)dx$

$= \int \left(\frac{x^3}{x^{1/2}}\,dx + \int \frac{3x}{x^{1/2}}\,dx + \frac{4x}{x^{1/2}}\,dx\right.$

$= \int x^{5/2}\,dx + 3\int x^{1/2}\,dx + 4\int x^{-1/3}\,dx$

$= \frac{x^{5/2+1}}{5/2+1} + 3\frac{x^{1/2+1}}{1/2+1} + \frac{x^{-1/2+1}}{-1/2+1}$

$= \frac{x^{7/2}}{7/2} + 3\left(\frac{x^{3/2}}{3/2}\right) + \frac{4^{1/2}}{1/2}$

$= \frac{2x^{7/2}}{7} + 2x^{3/2} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\,dt$

Put $u = 2t^4$

$du = 2\times 4t^3$

$= \int t^7 * \sin(2t^4) + \frac{1}{2\times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du$

$= \frac{t^4 \times \sin(2t^4)}{8}\,du$

Substitute t^4 with $\frac{u}{2}$

$= \int \frac{\frac{u}{2} \times \sin(2^{u/2})}{8}\,\frac{}{2}\,du$

$= \int \frac{\frac{u}{2} \times \sin(u)}{\frac{2}{8}}\,du$

$= \int \frac{u \times \sin(u)\,du}{16}$

$= \frac{1}{16}\int u + \sin(u)\,du$

$= \frac{1}{16}\left(u + (-\cos(u)) - \int -\cos(u)\,du\right.$

[$\because \int u\,dv = uv - \int v\,du$
where $u = v$
$dv = \sin(u)\times du$
$du = 1\,du$ ^ $v = -\cos(u)$]

$= \frac{1}{16} \times \left(u \times (-\cos v)\right) + \int \cos(u)\,du$

$= \frac{1}{16}\left(4 \times (-\cos(u)) + \sin(u)\right)$

[$\because \int \cos x\,dx = \sin x\,dx$]

Resubstituting $u = 2t^4$

$= \frac{1}{16}\left(2t^4 \times (-\cos(2t^4)) + \sin(2t^4)\right)$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

6> $\int \sqrt{x}\,(x^2-1)\,dx$

$I = \int \sqrt{x}\,(x^2-1)\,dx$

$= \int x^{1/2}\,(x^2-1)\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= \frac{x^{5/2+1}}{5/2+1} - \frac{x^{1/2+1}}{1/2+1}$

$= \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2}$

$= \frac{2x^{7/2}}{7} - \frac{2x^3}{3} + C$

7> $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$I = \int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \frac{\cos x}{\sin x^{3/2}}\,dx$

Put $t = \sin x$

$dt = \cos x\,dx$

$\frac{1}{t^{2/3}}\,dt$

$(t)^{-2/3}\,dt$

$= \frac{t^{-2/3+1}}{-2/3+1}$

$= \frac{t^{1/3}}{1/3}$

$= 3\sqrt[3]{t} + C$

$= 3\sqrt[3]{\sin x} + C$

9> $\int e^{\cos^2 x}\,\sin 2x\,dx$

$I = \int e^{\cos^2 x}\,\sin 2x\,dx$

Put $\cos^2 x = t$

$2\cos x\,(-\sin x)\,dx = dt$

$-\sin 2x\,dx = dt$

$\sin 2x = -dt$

$\therefore \int e^t\,(-dt)$

$-\int e^t\,dt$

$= -e^t + C$ $\quad [\because \int e^x\,dx = e^x + c]$

$= -e^{\cos^2 x} + C$

10> $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

Put $x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x)\,dx = dt$

$3(x^2 - 2x)\,dx = dt$

$(x^2 - 2x)\,dx = \frac{dt}{3}$

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$$\therefore \int \left(\frac{1}{t}\right) \frac{dt}{3}$$

$$\frac{1}{3}\int \left(\frac{1}{t}\right) dt$$

$$= \frac{1}{3} \log|t| + c \quad \left[\because \int \left(\frac{1}{x}\right)dx = \log|x| + c\right]$$

Resubstituting $x^3 - 3x^2 + 1 = t$

$$\therefore \frac{1}{3} \log|x^3 - 3x^2 + 1| + c$$

AK 02/01/2020

Practical No: 06

Topic: Application of integration & Numerical Integration

Q.1) $x = t - \sin t, \quad y = 1 - \cos t, \quad t \in [0, 2\pi]$

$$= \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \cancel{\sqrt{2-2\cos}}\ \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2\left|\sin \frac{t}{2}\right| dt$$

$$= \int_0^{2\pi} 2 \sin \frac{t}{2}\, dt$$

$$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi}$$

$$= (-4\cos\pi) - (-4\cos 0) = 4 + 4 = 8$$

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2) $\cancel{y = \sqrt{y-x^2}}$ $y = \sqrt{4-x^2}$ $x \in [-2,2]$

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$

$= \frac{-x}{\sqrt{4-x^2}}$

$= \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_{-2}^{2} \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$

$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\, dx$

$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$

$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$

$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$

$= 2\pi$

3) $y = x^{3/2}$ $x \in [0,4]$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$= \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_0^4 \sqrt{1 + \left(\frac{9x}{4}\right)}\, dx$

$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$

$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]$

$= \frac{1}{27}\left[(4+9x)^{3/2}\right]$

$= -\frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$

$= \frac{1}{27}\left[40^{3/8} - 8\right)$ unit

4) $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

$\frac{dx}{dt}$ $3\cos t$ $\frac{dy}{dx} = -3\sin t$

$= \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 + 9\cos^2} + dt$

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$$= \int_0^{2x} 3\sqrt{1}\, dt$$

$$= 3\int_0^{2x} dt$$

$$= 3[x]_0^{2x}$$

$$= 3(2x-0)$$

$$= 6x \text{ units}$$

5) $x = \frac{1}{6}y^3 + \frac{1}{2y} \quad y \in [1,2]$

$$\therefore \frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$$

$$\frac{dx}{dy} = \frac{y^2-1}{4y^2}$$

$$= \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1} + \left[\left(\frac{y^3-1}{4y^2}\right)^2\right] dy$$

$$= \cancel{\int_1^2 \sqrt{\frac{(y^6-1)+4xy^3 \times 1\, dy}{4y}}}$$

$$= \int_1^2 \sqrt{\frac{(y^4+1}{(2y)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2 dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{4}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{1}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$= \frac{17}{12} \text{ units}$$

Q2) 17) $\int_0^2 e^x\, dx$ with ~~x=4~~ $n=4$

$$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.284	2.71	9.58	54.59
	y	y_1	y_2	y_3	y_4

$$= \frac{0.5}{3}\left[\cancel{(1+(54.54)} + 5(1.28+9.68) + 2\times 2.7883)\right]$$

$$= \frac{0.5}{3}\left[55.982 + 43.868 + 5.426\right]$$

$$= \int_0^2 ex^2\, dx = 17.3535$$

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2) $\int_0^4 x^2\,dx$ $n = 4$ $h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^1 x^2\,dx = \frac{h}{3}\left[(y_0+y_0) + 4(y_1+y_3) + 2 y_2\right]$$

$$= \frac{1}{3}\left(16 + 4(10) + 8\right)$$

$$= \frac{64}{3}$$

$$= \int_0^3 x^2\,dx = 21.333$$

3) $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ $n = 6$

$$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	5π
0	0.416	0.584	0.70	0.875	0
y_0	y_1	y_2	y_3	y_4	y_5

$$\int_0^{\pi/3} \sqrt{\sin x}\,dx = \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{\pi/18}{3}\left[0.416 + 0.93 + 4(0.4167 + 0.707 + 0.9752) + 2(0.5848 + 0.807)\right]$$

$$= \frac{\pi}{54}\left[1.3473 + 7.996 + 2.773\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$= \int_0^{\pi/2} \sqrt{\sin x}\,dx = 0.7049$$

Practical No: 7

Topic: Differential Equations

1) Solve the following differential Equations.

i) $x \frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\quad Q(x) = e^x/x$

$IF = e^{\int 1/x \, dx}$

$= e^{\int dx}$

$= e^x$

$y(I.F) = \int Q(x) (I.F) \, dx + c$

$xy = \int \frac{e^x}{x} x * x \, dx + c$

$= \int e^x \, dx + c$

$xy = e^x + c$

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2) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \qquad Q(x) = e^x$

$I.F = e^{\int P(x) dx}$

$= e^{\int 2 dx}$

$= e^{2x}$

$y\,(I.F.) = \int Q(x)\,(I.F)\,dx + c$

$y \cdot e^{2x} = \int e^{-x} \cdot x\, e^{2x/x} + c$

$= \int e^{-x+2x}\, dx + c$

$= \int e^x\, dx + c$

$y \cdot e^{2x} = e^x + c$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2}{x}y = \frac{\cos x}{x^2}$

$P(x) = 2/x \qquad Q(x) = \cos x/x^2$

$I.F = \int l(x)\, dx$

$= e^{\int 2/x}\, dx$

$= e^{2/x}$ x

$= e^{x}$ x^2

$I.F = x^2$

$y\,(I.F.) = \int Q(x)\,(I.F.)\,dx + c$

$= \int \frac{\cos^2 x}{x^2}\, x^2\, dx + c$

$= \int \cos x + c$

$x^2 \cdot y = \sin x + c$

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4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3}{x} y = \frac{\sin x}{x^2}$

$P(x) = 3/x \qquad Q(x) = \sin x / x^2$

$(I.F) = e^{\int P(x) dx}$

$= e^{\int 3/x \, dx}$

$= e^{\int 3x}$

$= e^{x n^3}$

$IF = x^3$

$y(I.F.) = \int Q(x) (IF) dx + c$

$x^3 y = \int \frac{\sin x}{x^3} \times x^3 dx + c$

$= \int \sin x \, dx + c$

$x^3 y = -\cos x + c$

5) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \qquad Q(x) = 2xe^{-2x}$

$I.F = e^{\int 8(x) dx}$

$= e^{\int 2x} dx$

$= e^{2x}$

$y(I.F) = \int Q(x) (I.F) dx + c$

$ye^{2x} = \int 2xe^{-2x} \times e^{2x} dx + c$

$ye^{2x} = \int 2x \, dx + c$

$ye^{2x} = \cancel{x} x^2 + c$

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6) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$\frac{\sec^2 x \, dx}{\tan x} = \frac{-\sec^2 y \, dy}{\tan y}$

$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$

$\log|\tan x| - \log|\tan y| + 1$

$\log|\tan x - \tan y| = c$

$\tan x \cdot \tan y = e$

7) $\frac{dy}{dx} = \sin^2(x-y+1) - x$

Differentiating both sides

$x-y+1 = v$

$\cancel{1 - \frac{dy}{dx} = \frac{dv}{dx}}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dy}{dx} = \cos^2 x$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + c$

$\tan(x+y+1) = x + c$

8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x+3y = v$

$2 + \frac{3dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2} = \frac{3v+3}{v+2}$

$\int \frac{v+1}{v+1} dv + \int \frac{1}{v+1} dv = 3x + c$

$v + \log(v+1) = 3x + c$

$2x + 3y + \log|2x+3y+1| = 3x + c$

$3y = x - \log|2x+3y+1| + c$

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Practical No:- 8

Using Euler's Method Find the foll...

① $\frac{dy}{dx} = y + e^x - 2$, $y(u) = 2$, $h = 0.5$ Find $y(2)$

Sol^n:-

$f(x) = y + e^x - 2$, $x_0 = 0$.

$y(0) = 2$ $h = 0.5$

$y(0.2) = ?$

n	x_n	$y(0)$	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.14787	3.5743
2	1	3.5443	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215

$y(2) = 9.8215$

② $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$ find $y(1)$

$y_2 = 0$; $y_0 = 0$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ Find $y(1)$

$y(0) = 1$, $x_0 = 0$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0394
2	0.4	1.6894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3813	0.7694	1.5061
5	1	1.5051		

$y(1) = 1.5051$

4) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$

For $h = 0.5$ & $h = 0.25$

$h = 0.5$ & $h = 0.25$ $y_0 = 2$ $x_0 = 1$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	2	4
1	1.5	1.5	4	7.875
2	2	2	7.875	

$y(2) = 7.875$

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5) $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ find (1.2)

with $h = 0.2$ $y(0) = 1$ $x(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1) = 3.6$

Practical No :- 9

Topic :- Limits & Partial Order Derivative

Q1] Evaluate the following limits :-

i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

$\frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{(-4)(-1) + 5}$

$\frac{64 + 3 + 1 - 1}{4 + 5}$

$\therefore \frac{67}{9}$

ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

$\frac{(0+1)(2 + 0^2 - 4(2))}{2 + 3(0)}$

$\frac{1(4 + 0 - 8)}{8}$

$-\frac{4}{2}$

$= -2$

3) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

$\lim_{(x,y,z)\to(1,1,1)} \frac{(x)^2-(yz)^2}{x^2(x-y^2)}$

$\lim_{(x,y,z)\to(1,1,1)} \frac{(x+yz)(x-yz)}{x^2(x-y^2)}$ $[\because a^2-b^2 = (a+b)(a-b)]$

$\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x}$

$\therefore \frac{1+(1)(1)}{1^2} = 1$

Q.2] Find f_x, f_y for the following

1) $f(x,y) = xye^{x^2+y^2}$

$f(x) = \frac{\partial f}{\partial x}$

$= \frac{\partial(xye^{x^2+y^2})}{\partial x}$

$= y\frac{\partial(xe^{x^2+y^2})}{\partial x}$

$= y[x\cdot\frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}\frac{d}{dx}(x)]$

$= y[x\cdot e^{x^2+y^2}\cdot 2x + e^{x^2+y^2}(1)]$

$= y\cdot e^{x^2+y^2}[2x+1]$

Now,

$f(y) = \frac{\partial f}{\partial y}$

$= \frac{\partial(xye^{x^2+y^2})}{\partial y}$

$= x\cdot\frac{\partial}{\partial y}(y\cdot e^{x^2+y^2})$

$= x[y\frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}\cdot\frac{d}{dy}(y)]$

$\because \frac{d}{dx}(uv) = uv' + vu'$

$= x[2y^2\cdot e^{x^2+y^2} + e^{x^2+y^2}]$

$= x\cdot e^{x^2+y^2}[2y^2+1]$

2) $f(x,y) = e^x\cos y$

$\therefore f(x) = e^x\cos y$

$f(y) = e^x\frac{d}{dy}(\cos y)$

$= e^x(-\sin y)$

$= -e^x\sin y$

3) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3y^2 - 3x^2y + y^3+1)}{\partial x}$

$= 3x^2y^2 - 3(2x)y$

$= 3x^2y^2 - 6xy$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3y^2 - 3x^2y + y^3+1)}{\partial y}$

$= x^3(2y) - 3(1)x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

Q3.] Using defination, find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

Sol^n:- $f_x(a,h) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$(a,b) = (0,0)$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$\lim_{h\to 0} \frac{2h-0}{2} = 2$

Similarly, $f_y(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0-0}{h} = 0 \quad \therefore f_x = 2 \quad f_y = 0$

Q4.] Find all second order partial derivatives of f. Also find whether $f_{xy} = f_{yx}$

i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(y^2-xy)}{x^2}$

$= \frac{x^2 \cdot \frac{d}{dx}(y^2-xy) - (y^2-xy)\frac{d}{dx}(x^2)}{(x^2)^2}$

$\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$

$= \frac{x^2(-y) - (y^2-xy)(2x)}{x^4}$

$= \frac{x(xy - 2y^2)}{x^4}$

$f_x = \frac{xy - y^2}{x^3}$

$f_y = \frac{\partial f}{\partial y} = \partial\left(\frac{y^2 - xy}{x^2}\right) = \frac{\partial\left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right)}{\partial y}$

$= \partial\left(\frac{y^2}{x^2} - \frac{y}{x}\right)/\partial y$

$= \frac{1}{x^2} 2y - \frac{1}{x}$

$\therefore f_y = \frac{2y - x}{x^2}$

$$f(x,x) = \frac{\partial\left(\frac{xy-2y^2}{x^3}\right)}{\partial x}$$

$$= \frac{x^3\frac{d}{dx}(xy-2y^2)-(xy-2y^2)\frac{d}{dx}(\ldots}{(x^3)^2}$$

$$= \frac{x^3(y)-(xy)y^2(3x^2)}{6}$$

$$= \frac{x^3y-3x^3y+6x^2y^2}{6}$$

$$= \frac{6x^2y^2-2x^3y}{x^6}$$

$$= \frac{x^2(6y^2-2xy)}{x^6}$$

$$= \frac{6y^2-2xy}{x^4}$$

$$f(yy) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial y}$$

$$= \frac{\frac{\partial}{x^2}(2y-x)}{\partial y} = \frac{1}{x}(2) = \frac{2}{x^2}$$

$$f(xy) = \frac{\partial\left(\frac{xy-2y^2}{x^3}\right)}{\partial y} = \frac{\partial\left(\frac{xy}{x^3}-\frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{\partial\left(\frac{y}{x^2}-\frac{2y^2}{x^3}\right)}{\partial y}$$

$$\frac{1}{x^2} - \frac{1}{x^3}\cdot x(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3-4yx^2}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$f(yx) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial x}$$

$$= \frac{\partial\left(\frac{2y}{x^2}-\frac{x}{x^2}\right)}{\partial x} = \frac{\partial\left(\frac{2y}{x^2}-\frac{1}{x}\right)}{\partial x}$$

$$= 2y\left(\frac{-2}{x^3}\right)-\left(-\frac{1}{x^2}\right)$$

$$= -\frac{4y}{x^3}+\frac{1}{x^2}$$

$$= \frac{-4yx^2+x^3}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$\therefore f(xy) = f(yx) = \frac{x-4y}{x^4}$$

Hence verified.

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2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial((x^3+3x^2y^2) - (\log(x^2+1)))}{\partial x}$

$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$

$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$= 0 + 3(2y)(x^2) + 0$

$f(y) = 6x^2y$

$f(xx) = \frac{\partial}{\partial x} fx = \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 6x + 6y^2(1) - 2\left[\frac{x^2+1(1) - x(2x)}{(x^2+1)^2}\right.$

$\left[\frac{d}{dx}\frac{u}{v} = \frac{v \cdot u' - u \cdot v'}{v^2}\right.$

$= 6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)\Big]$

$= 6x + 6y^2 - 2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$

$f(yy) = \frac{\partial f y}{\partial y} = \frac{\partial(6x^2y)}{\partial y}$

$= 6x^2(1) = 6x^2$

$f(xy) = \frac{\partial\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)}{\partial y}$

$= 0 + 6x(2y)$

$= 12xy$

$f(xy) = f(yx) = 12xy$

Hence verified

3) $f(x,y) = \sin(xy) + e^{x+y}$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial x}$

$= \cos(xy)(y) + e^{x+y}(1)$

$= y\cos xy + e^{x+y}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$

$= \cos(xy)(x) + e^{x+y}(1)$

$= x\cos xy + e^{x+y}$

$f(xx) = \frac{\partial}{\partial x} fx$

$= \frac{\partial(y\cos xy + e^{x+y})}{\partial x}$

$= y\cos xy(y) + e^{x+y}(1)$

$= y^2\cos xy + e^{x+y}$

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$$f(yy) = \frac{\partial f_y}{\partial y} = \frac{\partial\left(x\cos xy + e^{x+y}\right)}{\partial y}$$

$$= x\cos xy(x) + e^{x+y}(1)$$

$$= x^2\cos xy + e^{x+y}$$

$$f(xy) = \frac{\partial f_x}{\partial y} = \frac{\partial\left(y\cos xy + e^{x+y}\right)}{\partial y}$$

$$= y[-\sin(xy)(x) + \cos xy(1)] + e^{x+y}(1)$$

$$\left[\because \frac{d}{dx} uv = u.v' + \ldots\right.$$

$$= -xy\sin(xy) + \cos xy + e^{x+y}$$

$$f(yx) = \frac{\partial f_y}{\partial x} = \frac{\partial\left(x\cos xy + e^{x+y}\right)}{\partial x}$$

$$= \cos xy(1) + x(-\sin(xy))(y) + e^{x+y}$$

$$= -xy\sin(xy) + \cos(xy) + e^{x+y}$$

$$f(xy) = f(yx) = -xy\sin(xy) + \cos(xy) + e^{x+y}$$

Q5) Find the linerization of $f(x,y)$ at given pt.

1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$$f(1,1) = \sqrt{(1)^2+(1)^2}$$

$$= \sqrt{2}$$

$$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$$

$$f_x(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$f_y(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \frac{2 + x - 1 + (y-1)}{\sqrt{2}}$$

$$= \frac{2 + x - 1 + (y-1)}{\sqrt{2}}$$

$$= \frac{2 + x + y - 2}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$$

2) $f(x,y) = 1-x+y\sin x$ at $\left(\frac{\pi}{2},0\right)$

$f\left(\frac{\pi}{2},0\right) = 1-\frac{\pi}{2}+0 \quad \left(\sin\left(\frac{\pi}{2}\right)\right)$

$= 1-\frac{\pi}{2}$

$f(x) = -1+y\cos x$

$f(y) = 1$

$fx\left(\frac{\pi}{2},0\right) = -1+0\cdot\cos\left(\frac{\pi}{2}\right)$

$= -1$

$fy\left(\frac{\pi}{2},0\right) = 1$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$

$= 1-\frac{\pi}{2} + (-1)\left(x-\frac{\pi}{2}\right) + 1(y-0)$

$= 1-\frac{\pi}{2} - x + \frac{\pi}{2} + y$

$= y-x+1$

3) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1)$

$= 0$

$f(x)\ fx(1,1) = 1$

$f(y) = \frac{1}{y}$

$fy(1,1) = 1$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a-b)(y-b)$

$= 0 + 1(x-1) + (y-1)$

$= x-1+y-1$

$= x+y-2$

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Practical No:- 10

Q.1] Find the directional derivative of the following fn. at given points & in the directn of given vector.

Soln :- $f(x,y) = x+2y-3$ $a=(1,-1)$ $u=3i-j$

Here, $u=3i-j$ not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = 1+2(-1)-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1, + \frac{-h}{\sqrt{10}}\right)$

$= 1+\frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$

$= \lim_{h\to 0} \frac{-4+h/\sqrt{10}+4}{h} = \frac{1}{\sqrt{10}}$

2] $f(x) = y^2 - 4x+1$ $a(3,4)$ $u=i+5j$

Here $u=i+5j$ is not unit vector

$|\bar{u}| = \sqrt{1^2+5^2} = \sqrt{26}$

u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3)+1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3+\frac{h}{\sqrt{26}}, 4+\frac{5h}{\sqrt{26}}\right)$

$f(a+hu) = \left(4+\frac{5h}{\sqrt{26}}\right)^2 + 4\left(3+\frac{h}{\sqrt{26}}\right)+1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + 40h - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h\to 0} \frac{\frac{25}{h^2} + \frac{36h}{\sqrt{26}} + 5 - 5}{h} = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

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1) $2x+3y$ $\quad a=(1,2)$ $\quad u=(3i+4j)$

$u = 3i+4j$ is not a unit vector

$|u| = \sqrt{3^2+4^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1)+3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$

$f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h \to 0} \frac{18h}{5} + 8 - 8/h$

$= \frac{18h}{5}$

Q2] Find ~~generation vector~~ gradient vector for the following:

1) $f(x,y) = x^y + y^x \quad a(1,1)$

$f_x = yx^{y-1} + y^x \log y$

$f_y = x^y \log x + xy^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y,\ x^y \log x + xy^{x-1})$

$\nabla f(1,1) = (1+0, 1+0)$

$= (1,1)$

2) $f(x,y) = (\tan^{-1} x) \cdot y^2 \quad a(1,-1)$

$f(x) = \frac{1}{1+x^2} \cdot y^2$

$f(y) = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$\nabla f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \frac{1}{2}, \frac{\pi}{4}(-2)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

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3) $f(x,y,z) = xyz - e^{x+y+z}$ $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$f(x,y,z) = f_x, f_y, f_z$

$= f_z - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(-1,-1,0) = ((-1)(0) - e^{(1+(-1)+0)}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0})$

$= (0 - e^0, 0 - e^0, -1, -e^0)$

$= (-1, -1, -2)$

Q.3.] Find the equ^n of tangent & normal to each of the following.

1) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y\, 2x + e^{ex} + y$

$f_y = x^2(-\sin y) + e^{xy}.x$

$(x_0, y_0) = (1,0)$ $\therefore x_0 = 1$, $y_0 = 0$

Eq^n of tangent $:- f_x(x-x_0) + f_y(y-y_0) = 0$

$f_x(x_0,y_0) = \cos 0 \; 2(0) + e.0$

$= 1(2) + 0$

$= 2$

$f_y(x_0,y_0) = (1)^2 (-\sin 0) + e^0.1$

$= 0 + 1.1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

$\therefore$ It is the required eq^n of tangent.

Equation of Normal : $ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$

$1 + 2(0) + d = 0$ at $(1,0)$

$d + 1 = 0$

$\therefore d = -1$

2) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2$; $y_0 = -2$

$f_x(x_0,y_0) = 2(2) - 2 = 2$

$f_y(x_0,y_0) = 2(-2) + 3 = -1$

Equation of tangent.

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + (1(y+2)) = 0$

$\cancel{2x+} 2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$

$\therefore$ The equ. of tangent is found.

Equation of Normal.

$-ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 - 4 + d = 0$ $-6 + d = 0$

$\therefore d = 6$

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Q4] Find the equ. of tangent & normal to each of the following.

i) $x^2 - 2y^2 + 3y + x^2 = 7$ at $(2, 1, 0)$

$f_x = 2x - 0 + 0 + 2$

$f_x = 2x + 2$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0)$ $\therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

Equation of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$= 4x + 3y - 11 = 0$

$\therefore$ Equation of tangent is found.

Equation of Normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x - 2}{4} = \frac{y - 1}{3} = \frac{z + 11}{0}$$

ii) $3xy^2 - x - y + z = -4$ at $(1, -1, 2)$

$3xy^2 - x - y + z + 4 = 0$

$f_x = 3yz - 1 - 0 + 0 + 0$ at $(1, -1, 2)$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_x(x_0, y_0, z_0) = 3(1)(2) - 1 = 5$

$f_x(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

Equation of tangent $-7(x - 1) + 5(y + 1) - 2(z - 2) = 0$

$= -7x + 7 + 5y + 5 - 2z + 4 = 0$

$= -7x + 5y - 2z + 16 = 0$

This is required Eq^n. of tangent

Equation of Normal $= (-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x - 1}{-7} = \frac{y + 1}{5} = \frac{z - 2}{-2}$$

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Q.5)
1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$
$f_x = 6x + 0 - 3y + 6 = 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$
$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ —(1)

$f_y = 0$
$2y - 3x - 4 = 0$
$2y - 3x = 4$ —(2)
Multiply eq^n (1) by (2)
$4x - 2y = -4$
$y - 3x = 4$
$x = 0$
Subs. value of x in eq^n (1)
$2(0) - y = -2$
$y = 2$
Critical points are $(0,2)$
$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here $r > 0$
$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$.
f has maximum at $(0,2)$
$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$
$= 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$
$= 0 + 4 - 0 + 0 + 8$
$= -4$

2) $f(x,y) = 2x^4 + 3x^2y - y^2$
$f_x = 8x^3 + 6xy$
$f_y = 3x^2 - 2y$

$f_x = 6$
$8x^3 + 6xy = 0$
$2x(4x^2 + 3y) = 0$
$4x^2 + 3y = 0$ —(1)

$f_y = 0$
$3x^2 - 2y = 0$ —(2)

Solving eq^n. (1) & (2)
$12x^2 - 9y = 0$
$-12x - 9y = 0$
$y = 0$

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$4x^2 + 3(0) = 0$
$4x^2 = 0$
$x = 0$

critical points in (0,0)
$r = f_{xx} = 24x^2 + 6x$
$t = f_{yy} = 0 - 2 = -2$
$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at (0,0)
$= 24(0) + 6(0) = 0$
$\therefore r = 0$

f(x,y) at 0,0
$2(0)^4 + 3(0)^2$ (0)
$0 + 0$
$= 0$

$rt - s^2 = 0(-2) - (0)^2$
$= 0 - 0$
$= 0$

$\therefore$ Nothing to say $\because r = 0$ & $rt - s^2 = 0$

3) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$
$f_y = -2y + 8$
$f_x = 0$
$2x + 2 = 0$
$x = -1$
$f_y = 0$
$-2y + 8 = 0$
$y = +4$

Critical points are (1,-4)

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$r = f_{xx} = 2$
$t = f_{yy} = -2$
$s = f_{xy} = 0$
$r > 0$
$rt - s^2 = 2(-2) - 0^2$
$= -4 < 0$

f(x,y) at (-1, +4)
$= (-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$
$= 47 - 70$
$= -23$

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