Eigen Values and Eigen Vector Summary

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1 Summary

The linear transformations are from the complex vector space Cn to itself. Observe that in this case, the matrix of the linear transformation is annnmatrix. All the matrices are square matrices and a vector x means x= $(x_1, x_2, ..., x_n)^t$ for some positive integer n

Definition 6.1.2 (Characteristic Polynomial, Characteristic Equation) Let A be a square matrix of order n.The polynomial det($A\lambda$I) is called the characteristic polynomial of A and is denoted by pA(λ) (in short, p(λ), if the matrix A is clear from the context). The equation p(λ) = 0 is called the characteristic equation of A. If λF is a solution of the characteristic equation p(λ) = 0,then λ is called a characteristic value of A.

Theorem 6.1.3 : Let A$\in M_n$ (F). Suppose $\lambda = \lambda_0 \in$ F is a root of the characteristic equation. Then there exists a non-zero v$\in$ F_n such that Av=λ_0v.

Definition 6.1.5 (Eigenvalue and Eigenvector) : Let A$\in M_n$(F)and let the linear system Ax=x has a non-zero solution x$\in F^n$ for some $\in$F. Then
1. $\lambda\in$F is called an eigenvalue of A,
2. x$\in F^n$ is called an eigenvector corresponding to the eigenvalue λ of A, and
3. The tuple(λ,x) is called an eigen-pair.

Remark 6.1.6 :To understand the difference between a characteristic value and an eigenvalue, we give the following example.
Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Then pA(λ) =λ2+ 1. Also, define the linear operator T_A:$F^2 \to F^2$ by T_A(x) =Ax for every x$\in F^2$.

1. Suppose F=C, i.e., A$\in M_2$(C). Then the roots of p(λ) = 0 in C are i . So, A has (i,(1,i)^t) and (i,(i, 1)^t) as eigen-pairs.
2. If A$\in M_2$(R), then p(λ) = 0 has no solution in R. Therefore, if F=R, then A has no eigen value but it has i as characteristic values