An Investigation in Cartesian Planes

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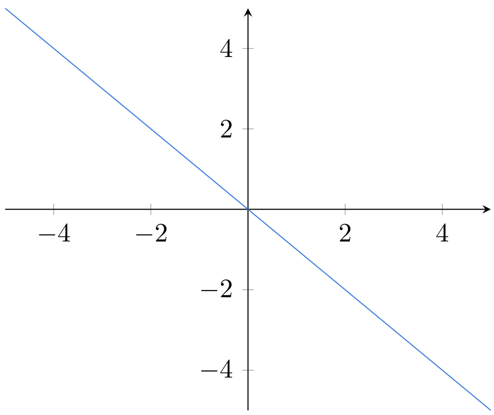
We can make equations that represent a line on the Cartesian plane.

$$x + y = 0 \tag{1}$$



(2)

These are also called linear equations.
Let's say we do it the other way around and make a equation, who's solution is what we choose.
We want a equation, whose solution is

$$x = 1, y = 0 \tag{3}$$

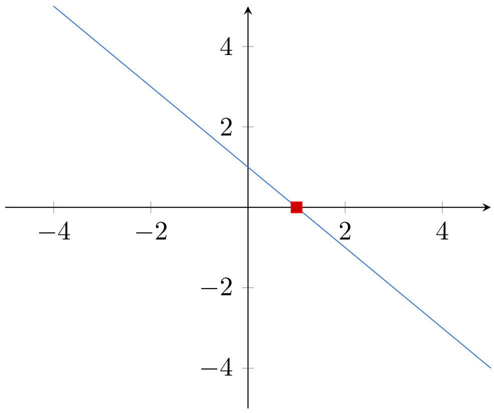
Let's try adding the equations, so,

$$x + y = 1 + 0 \tag{4}$$

$$x + y = 1 \tag{5}$$

Let's verify it graphically,

[*]Existence#6673 (Discord user)



$$(6)$$

Yay! We did it!

Now what about 2 solutions equations?

Umm, I think these equation are also called Quadratic Equations.
Anyway, Let's say the required points are

$$x_1 = 1 \tag{7}$$
$$y_1 = 0 \tag{8}$$
$$x_2 = 2 \tag{9}$$
$$y_2 = 0 \tag{10}$$

Now to create the equation. Let's first make a equation for each point first.

$$x + y = 1 + 0 \tag{11}$$
$$x + y = 1 \tag{12}$$

$$x + y = 2 + 0 \tag{13}$$
$$x + y = 2 \tag{14}$$

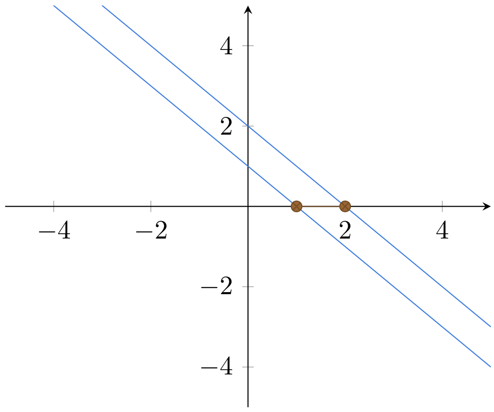
Upon further rearrangement,

$$x + y - 1 = 0 \tag{15}$$
$$x + y - 2 = 0 \tag{16}$$

Let's use the factor theorem to make a equation

$$(x + y - 1)(x + y - 2) = 0 \tag{17}$$
$$x^2 + xy - 2x + xy + y^2 - 2y - x - y + 2 = 0 \tag{18}$$
$$x^2 + y^2 + 2xy - 3(x + y) + 2 = 0 \tag{19}$$

We have a equation now! Let's verfiy using a graph.



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Damn, that's satisfying! Do you know what's even more satisfying? There are infinite solutions to our question.
Let's try finding the other solutions.

This is the factored form of the equation we had obtained.

$$(x + y - 1)(x + y - 2) = 0 \tag{21}$$

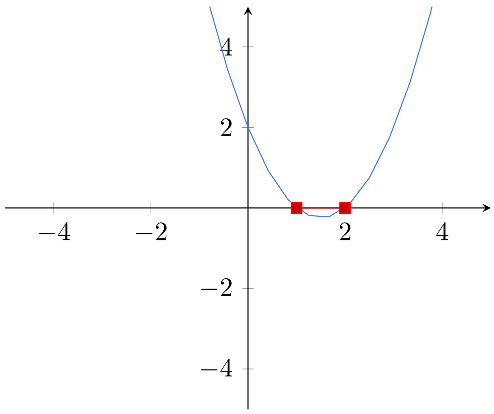
If we manipulate them and take y as the function we get,

$$f(x) = (1 - x)(2 - x) \tag{22}$$

$$f(x) = 2 - x - 2x + x^2 \tag{23}$$

$$f(x) = x^2 - 3x + 2 \tag{24}$$

Let's graph this one and try again.



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We found another equation, I am sure that there are many more! But I haven't found another one yet. But if you find one email me at tusharmaharana44676@gmail.com.

1 Existence#6673

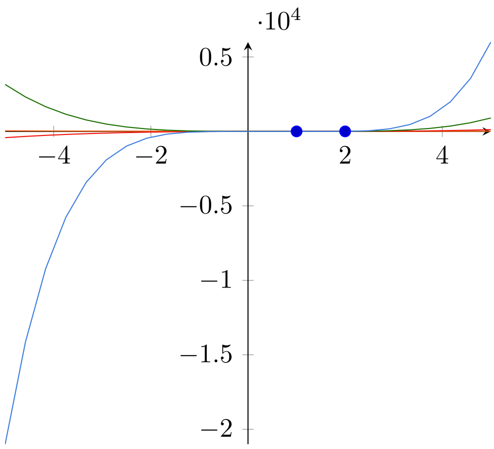
Existence told me about the general form of these equations.

Any equation having the solutions 1 and 2 ($y = 0$) is in the form of,

$$f(x) = ax^n(x-1)(x-2) \tag{26}$$

We just got a infinite supply of these equations!
Let's create four of them and graph them!

$$f(x) = (x-1)(x-2), a = 1, n = 0 \tag{27}$$
$$f(x) = 2x(x-1)(x-2), a = 2, n = 1 \tag{28}$$
$$f(x) = 3(x^2)(x-1)(x-1), a = 3, n = 2 \tag{29}$$
$$f(x) = 4(x^3)(x-1)(x), a = 4, n = 3 \tag{30}$$



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