

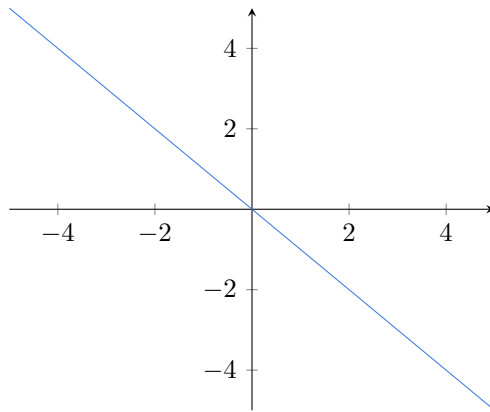
An Investigation in Cartesian Planes

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We can make equations that represent a line on the Cartesian plane.

$$x + y = 0 \tag{1}$$



(2)

(3)

These are also called linear equations.

Let's say we do it the other way around and make an equation, whose solution is what we choose.

We want an equation, whose solution is

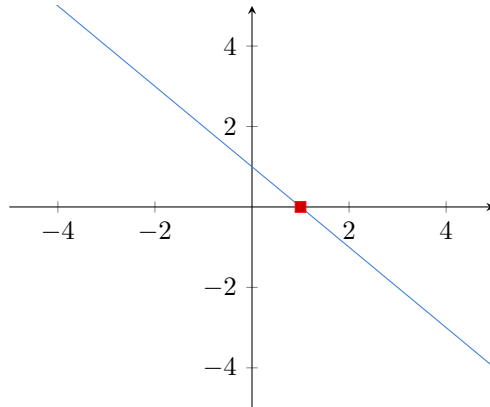
$$x = 1, y = 0 \tag{4}$$

Let's try adding the equations, so,

$$x + y = 1 + 0 \tag{5}$$

$$x + y = 1 \tag{6}$$

Let's verify it graphically,



(7)

Yay! We did it!

Now what about 2 solutions equations?

Umm, I think these equation are also called Quadratic Equations.
Anyway, Let's say the required points are

$$x_1 = 1 \quad (8)$$

$$y_1 = 0 \quad (9)$$

$$x_2 = 2 \quad (10)$$

$$y_2 = 0 \quad (11)$$

Now to create the equation. Let's first make a equation for each point first.

$$x + y = 1 + 0 \quad (12)$$

$$x + y = 1 \quad (13)$$

$$x + y = 2 + 0 \quad (14)$$

$$x + y = 2 \quad (15)$$

Upon further rearrangement,

$$x + y - 1 = 0 \quad (16)$$

$$x + y - 2 = 0 \quad (17)$$

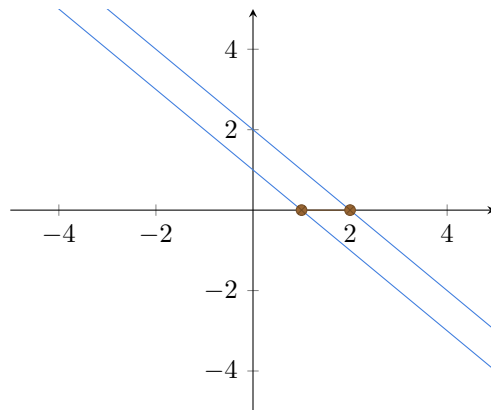
Let's use the factor theorem to make a equation

$$(x + y - 1)(x + y - 2) = 0 \quad (18)$$

$$x^2 + xy - 2x + xy + y^2 - 2y - x - y + 2 = 0 \quad (19)$$

$$x^2 + y^2 + 2xy - 3(x + y) + 2 = 0 \quad (20)$$

We have a equation now! Let's verfiy using a graph.



(21)

Damn, that's satisfying! Do you know what's even more satisfying? There are infinite solutions to our question.

Let's try finding the other solutions.

This is the factored form of the equation we had obtained.

$$(x + y - 1)(x + y - 2) = 0 \quad (22)$$

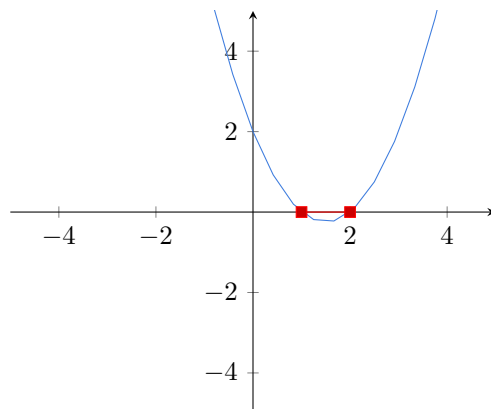
If we manipulate them and take y as the function we get,

$$f(x) = (1 - x)(2 - x) \quad (23)$$

$$f(x) = 2 - x - 2x + x^2 \quad (24)$$

$$f(x) = x^2 - 3x + 2 \quad (25)$$

Let's graph this one and try again.



(26)

We found another equation, I am sure that there are many more! But I haven't found another one yet. But if you find one email me at tusharmaharana44674@gmail.com.