

Physics

(02-02-2020)

Charge q unit — C (Coulomb's), e.s.u.

Coulomb's law :- A law stating that like charges repel and opposite charges attract, with a force proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$F = C \frac{q_1 q_2}{r^2}$$

if $r = 1 \text{ cm}$, $F = 1 \text{ dyne}$, $C = 1$
 $q_1 = q_2 = q$ $\therefore q = \pm 1 \text{ e.s.u}$

or if $r = 1 \text{ m}$ $F = 1 \text{ N}$

$$q_1 = q_2 = q = 1 \text{ C}$$

$$1 \text{ C} = 9 \times 10^9 \text{ e.s.u}$$

$$\vec{F} = C \frac{q_1 q_2}{r^2} \cdot \vec{r}$$

Waves:-

Hooke's law: Hooke's law is a principle of physics that states, the force needed to extend or compress a spring by some distance is proportional to that distance.

$$F \propto -x$$

$$\Rightarrow F = -kx$$

k = proportional constant in case of spring it is called spring constant or stiffness constant.

"Within elastic limit the strain (deformation) of a elastic body is proportional to the stress applied to it."

Oscillatory motion (oscillation)

Oscillatory motion can be defined as the repeated motion in which an object repeats the same movement and passes a point again and again.

$$T \propto \frac{1}{f}$$

Vector magnetic potential \vec{A}

$$\vec{\nabla} \cdot \vec{B} = ? \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

• $\vec{A} \times \vec{B} \rightarrow \vec{B} \times \vec{A}$

We know from Biot-Savart law

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^3} \quad \text{--- (i)}$$

$$\text{and } \vec{\nabla} \left(\frac{1}{r} \right) = - \frac{\vec{r}}{r^3} \quad \left(\begin{array}{l} \text{vector} \\ \text{Algebra} \end{array} \right) \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

From (i) and (ii) we have

$$\vec{B} = - \frac{\mu_0 I}{4\pi} \int d\vec{l} \times \vec{\nabla} \left(\frac{1}{r} \right)$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \vec{\nabla} \left(\frac{1}{r} \right) \times d\vec{l}$$

Again using the identity

$$\vec{\nabla} \cdot (s \times \vec{A}) = \vec{\nabla} \times (s \vec{A}) - s (\vec{\nabla} \times \vec{A})$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \left[\vec{\nabla} \times \frac{d\vec{l}}{r} - \frac{1}{r} (\vec{\nabla} \times d\vec{l}) \right]$$

$$= \frac{\mu_0 I}{4\pi} \int \vec{\nabla} \times \frac{d\vec{l}}{r}$$

Simple harmonic motion:

If an oscillatory particle moves back and forth about an equilibrium position such as oscillator is called simple harmonic motion-oscillator and its motion is called simple harmonic motion (S.H.M)

The S.H.M has the following characteristics

- (i) The motion is oscillatory and is repeated after equal interval at time.
- (ii) The restoring force is proportional to displacement
- (iii) The acceleration is always directed to mean position

Differential equation of simple harmonic oscillator:

Suppose the force F is applied on a body of mass ' m ' displacement is x . Then from Hooke's law $F \propto -x$ or $F = -Kx$... (i)

From Newton's 2nd law of motion $F = ma$
 $= m \cdot \frac{dv}{dt} = m \cdot \frac{d}{dt} \cdot \frac{dx}{dt} = m \cdot \frac{d^2x}{dt^2}$... (ii)

From (i) and (ii) $m \cdot \frac{d^2x}{dt^2} = -kx$ or $m \frac{d^2x}{dt^2} + kx = 0$.

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m} x = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m} x = 0 \quad \left| \quad \sqrt{\frac{k}{m}} = \omega \quad \therefore \omega = \sqrt{\frac{k}{m}} \right.$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \dots (iii) \text{ This is the}$$

equation of motion of S.H.O

Multiplying both sides of (iii) by $2 \cdot \frac{dx}{dt}$ we

obtain :

$$2 \cdot \frac{dx}{dt} \cdot \frac{d^2x}{dt^2} + \omega^2 x \cdot 2 \cdot \frac{dx}{dt} = 0$$

$$\text{or } \frac{d}{dt} \left(\frac{dx}{dt} \right)^2 + \omega^2 \frac{d}{dt} (x^2) = 0 \quad \text{Integrating w.r.t } t$$

$$\Rightarrow \left(\frac{dx}{dt} \right)^2 + \omega^2 x^2 = C \dots (iv)$$

If $x = A$, maximum displacement that is constant

$$\text{then } \frac{dx}{dt} = 0$$

$$\therefore C = \omega^2 A^2 \dots (v) \quad \text{putting into (iv)}$$

$$\left(\frac{dx}{dt} \right)^2 + \omega^2 x^2 = \omega^2 A^2 \quad \text{or } \left(\frac{dx}{dt} \right)^2 = \omega^2 A^2 - \omega^2 x^2$$

$$\text{or } \left(\frac{dx}{dt} \right)^2 = \omega^2 (A^2 - x^2)$$

or $\frac{dx}{dt} = \pm \omega \sqrt{A^2 - x^2}$ considering the +ve value

only $\frac{dx}{dt} = \omega \sqrt{A^2 - x^2}$

$\Rightarrow \frac{dx}{\sqrt{A^2 - x^2}} = \omega dt \dots (vi)$ Integrating we get

$$\sin^{-1} \frac{x}{A} = \omega t + \phi$$

or $\frac{x}{A} = \sin(\omega t + \phi)$

$\therefore x = A \sin(\omega t + \phi) \dots$

This is the differential equation at SH.O

For initial phase $\phi = 0$

$x = A \sin(\omega t)$
 or $\frac{x}{A} = \sin \omega t \dots (i)$

$v = \frac{dx}{dt} = A \omega \cos(\omega t)$

or $v = A \omega \sqrt{1 - \sin^2 \omega t} = A \omega \sqrt{1 - \frac{x^2}{A^2}}$

$\Rightarrow v = \frac{A \cdot 2\pi}{T \cdot A} \sqrt{A^2 - x^2} = \omega \sqrt{A^2 - x^2}$

$\therefore v = \omega \sqrt{A^2 - x^2}$

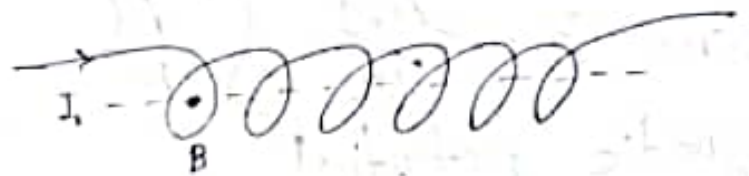
At $x=0$, $v = \frac{2\pi A}{T}$

At $x=A$, $v = \frac{2\pi}{T} (0-0) = 0$

Magnetic field inside a magnetic material :-

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I_0 \dots \textcircled{i}$$

$$B = \mu_0 n I_0 \dots \textcircled{ii}$$



where I_0 is the current flowing through the solenoid & turns n per unit length.

When the solenoid is filled with a magnetic material (iron) the magnetic field

$$B = B_0 + B_m \dots \textcircled{iii}$$

B_0 is the magnetic field absence of magnetic material, B_m is the induced magnetisation

But we know $B = \mu H$

From (3)

$$B = \mu_0 H + \mu_0 M \quad \left| \begin{array}{l} B_0 = \mu_0 H \\ B_m = \mu_0 M \end{array} \right.$$

$$= \mu_0 H \left(1 + \frac{M}{H} \right) \textcircled{5}$$

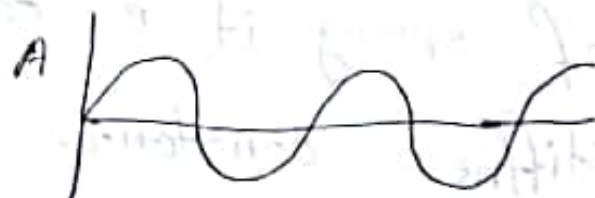
$$\underline{E \quad D \quad P}$$

Simple harmonic motion (SHM) :-

If an oscillatory particle moves back and forth about an equilibrium position such as oscillator is called simple harmonic oscillator and its motion is called SHM.

Characteristics of SHM :-

- (i) The motion is oscillatory and is repeated after equal interval of time.
- (ii) The restoring force is proportional to displacement.
- (iii) acceleration is always directed towards the mean position.



Characteristic parameters of SHM :-

(i) time period (निम्नतम अवधि)

(ii) $\omega T = 2\pi$ $\therefore \omega = \frac{2\pi}{T}$

$$T = \frac{2\pi}{\omega}$$

From Fig :-

$$x = a \tan \theta$$

$$dx = a \sec^2 \theta d\theta$$

$$E_p = \frac{1}{4\pi\epsilon_0} \int_{-\pi/2}^{+\pi/2} \frac{a \sec^2 \theta d\theta}{(a^2 + x^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \int_{-\pi/2}^{+\pi/2} \frac{\sec^2 \theta d\theta}{a^3 (1 + \tan^2 \theta)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \int_{-\pi/2}^{+\pi/2} \frac{\sec^2 \theta d\theta}{a^3 \sec^3 \theta}$$

$$= \frac{1}{4\pi\epsilon_0 \cdot a} \int_{-\pi/2}^{+\pi/2} \cos \theta d\theta$$

$$= \frac{1}{4\pi\epsilon_0 \cdot a} [\sin \theta]_{-\pi/2}^{+\pi/2}$$

$$\therefore E_p = \frac{1}{2\pi\epsilon_0 a}$$

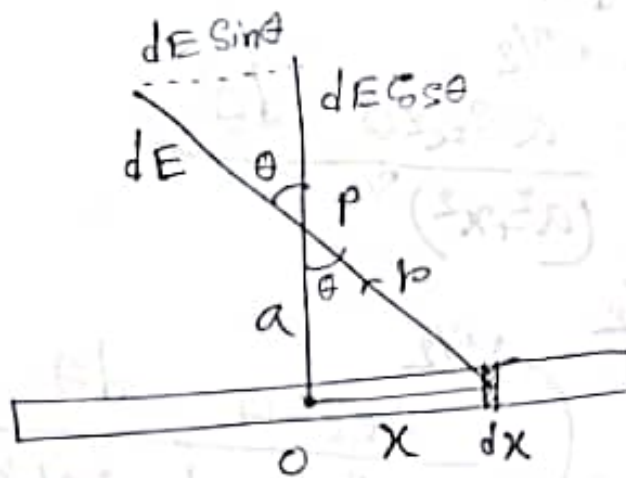
$$r^2 = a^2 + x^2$$

$$x = +a \quad \theta = +\pi/2$$

$$x = -a \quad \theta = -\pi/2$$

V.V.I
* Electric field due to a straight charge wire:

Let



Electric field at P for $1 \cdot dx$ charge

$$\therefore dE = \frac{1}{4\pi\epsilon_0} \times \frac{1 dx}{r^2}$$

For the wire :-

$$\begin{aligned} dE \cos \theta &= \frac{1}{4\pi\epsilon_0} \cdot \frac{1 dx}{r^2} \cdot \cos \theta \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{1 dx}{r^2} \cdot \frac{a}{r} \end{aligned}$$

The total

$$E_P = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{1 dx \cdot a}{r^3}$$

The net flux ϕ

$$\oint d\phi = \frac{\mu_0 I}{2\pi} \int \frac{dr}{r}$$

$$\phi = \frac{\mu_0 I}{2\pi} \ln \frac{b}{a}$$

We know the e.m.f

$$\mathcal{E} = \frac{d\phi}{dt}$$

$$= \frac{d}{dt} \left\{ \frac{\mu_0 I}{2\pi} \ln \left(\frac{b}{a} \right) \right\}$$

$$\mathcal{E} = \frac{\mu_0}{2\pi} \ln \frac{b}{a} \cdot \frac{dI}{dt} \dots \textcircled{i}$$

$$\text{and } \mathcal{E} = L \cdot \frac{dI}{dt} \dots \textcircled{ii}$$

comparing (i) and (ii)

$$L = \frac{\mu_0}{2\pi} \ln \frac{b}{a}$$

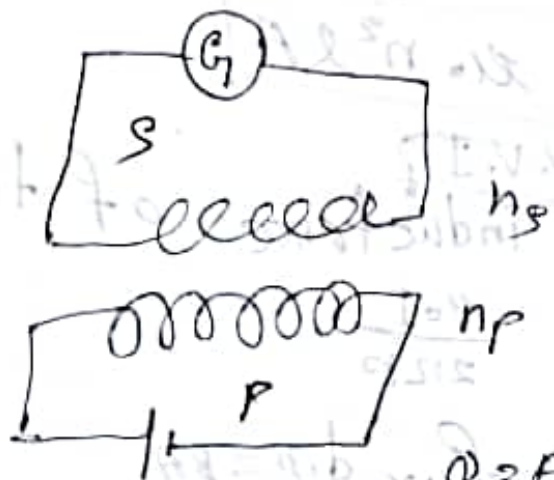
Mutual induction:

$$n_s \phi \propto I$$

$$n_s \phi = M I$$

where M is the mutual inductance

$$\mathcal{E} = M \cdot \frac{dI}{dt}$$



$$\phi_P = \mu_0 n_p A_p I$$

Self induction :-

$$N\phi \propto I$$

$$N\phi = LI$$

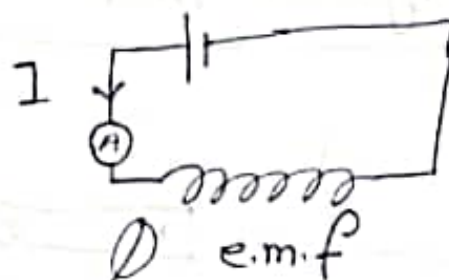
Where L is the self inductance

Again we know

$$\mathcal{E} = \frac{d}{dt}(N\phi) = \frac{d}{dt}(LI)$$

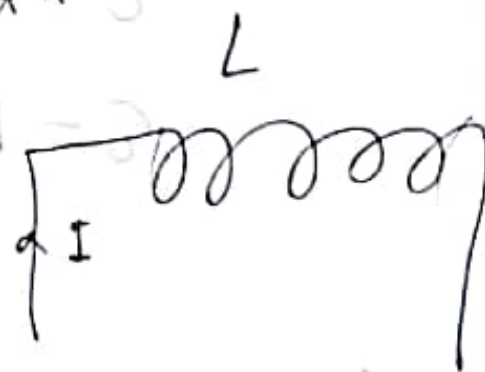
$$\therefore \mathcal{E} = L \cdot \frac{dI}{dt}$$

$$\therefore L = \frac{\mathcal{E}}{dI/dt}$$

Self inductance of a solenoid: *** V.V.I

$$B = \mu_0 n I \quad n = \frac{N}{l}$$

where n is the number of turns per unit length and I is the current flowing through the solenoid. The flux induced in the solenoid



$$\phi = BA = \mu_0 n I A$$

If N is the total number of turns in the solenoid

Biot-Savart Law :-

Class - 07

23-02-20

$$dB \propto \frac{Idl \sin \theta}{r^2}$$

$$dB = K \frac{Idl \sin \theta}{r^2}$$

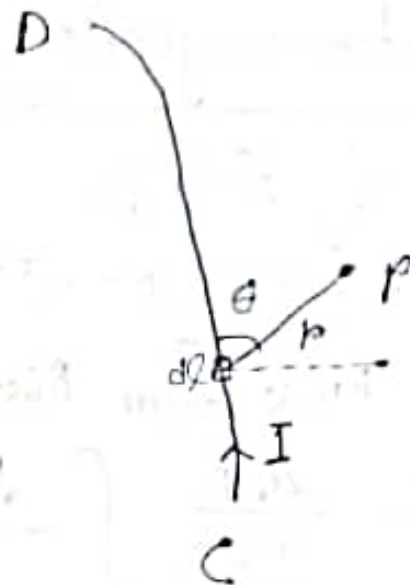
$$K = \frac{\mu_0}{4\pi}$$

$$B = \frac{\mu_0}{4\pi} \int \frac{Idl \sin \theta}{r^2}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin \theta}{r^2} \frac{\vec{r}}{r}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{dl r \sin \theta}{r^3}$$



(i) $B = \frac{\mu_0 I}{2\pi a}$

(ii) $B = \frac{\mu_0 I}{2r}$

Magnetic Vector potential :-

$$\vec{E} = -\vec{\nabla} V$$

Electric potential

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

$$\vec{A} = A? \quad \frac{\mu_0}{4\pi} \int \frac{J dV}{r}$$

$$\text{curl} = 0$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{B} = \vec{\nabla} ?$$

magnetic potential

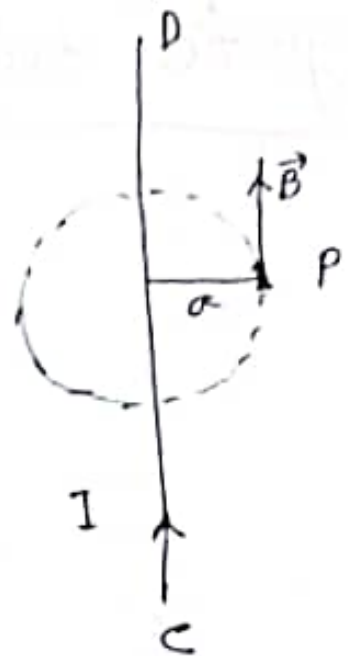
$$\therefore \vec{B} = \vec{\nabla} \times \vec{A}$$

From ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\Rightarrow B \oint dl = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi a}$$



(ii) magnetic field for solenoid:-

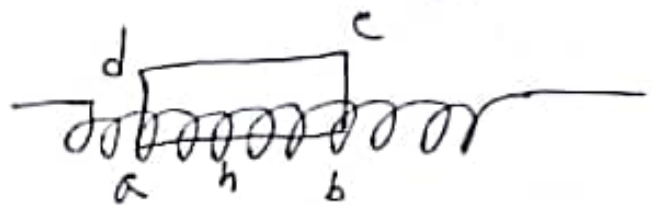
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\Rightarrow \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\Rightarrow Bh + 0 + 0 + 0 = \mu_0 I$$

$$\Rightarrow Bh = \mu_0 h n I$$

$$\Rightarrow B = \mu_0 n I$$



num. of turns
per unit
length

$$n = \frac{N}{l}$$

Relative magnetic permeability / dielectric cons. $K = \frac{F_e}{F_m} = \frac{\epsilon_m}{\epsilon_0}$

$$K_m = \frac{\mu_m}{\mu_0} \dots \dots \textcircled{6}$$

and

$$B = \mu_m H \dots \dots \textcircled{7}$$

$$\Rightarrow B = K_m \mu_0 H \dots \dots \textcircled{7}$$

Using equation (6) and (7)

$$K_m \mu_0 H = \mu_0 H \left(1 + \frac{M}{H} \right)$$

$$\Rightarrow K_m = 1 + \frac{M}{H}$$

$$\Rightarrow M = (K_m - 1) H$$

$$\Rightarrow M = X H$$

$X = (K_m - 1)$ is the magnetic susceptibility.

$$B = \mu H, \quad M = X H$$

M, H - Both are parallel

(iii) magnetic field for toroid :-



$$I_{enc} = N I$$

$$I_{enc} = N I$$

$$\frac{I_{enc}}{2\pi r} = \frac{N I}{2\pi r}$$

direction of field is tangential



$$I_{enc} = N I$$

$$I_{enc} = N I$$

$$I_{enc} = N I$$

$$I_{enc} = N I$$

$$I_{enc} = N I$$

from
length
of wire
area
of cross
section

$$B = \frac{\mu_0 N I}{2\pi r}$$

** Ampere's Law :-

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \oint \vec{J} \cdot d\vec{S}$$

$$\Rightarrow \oint (\vec{\nabla} \times \vec{B}) \cdot d\vec{S} = \mu_0 \oint \vec{J} \cdot d\vec{S}$$

$$\Rightarrow \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Proof
Then

$$B \propto \frac{I}{r}$$

$$B = K \frac{I}{r}$$

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$$

$$B(2\pi r) = \mu_0 I$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\frac{I}{A} = \vec{J} \rightarrow \text{Vector}$$

$$\vec{J} \propto I$$

Application of Ampere's law
(i) Magnetic field for a current carrying long wire :-

(ii) Sphere

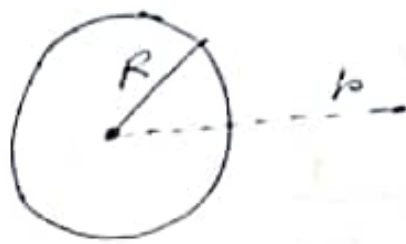
$$\alpha = \frac{q}{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$\oint E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{4\pi r^2 \epsilon_0}$$

$$\therefore E = \frac{\sigma}{\epsilon_0}$$



Add:- ***

Volume density charge

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$\Rightarrow \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q = \int \rho \, dv$$

$$\Rightarrow \epsilon_0 \int \vec{\nabla} \cdot \vec{E} \, dv = \int \rho \, dv$$

$$\Rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Differential form of Gauss law

it's also Gauss law

$$\rho = \frac{dq}{dv} = \frac{q}{V}$$

$$\Rightarrow \int dq = \int \rho \, dv$$

[Divergence theorem]

$$\frac{1}{V} \int \rho \, dv = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

$$\rho = \frac{q}{V}$$

Gauss Law:-

Class-03

16-02-2020

Electric flux $Q_E = \frac{q}{\epsilon_0}$

The component of E along the direction of $d\vec{A}$ is $E \cos \theta$

$$E_n = E \cos \theta$$

$$E_n dA = E \cos \theta dA$$

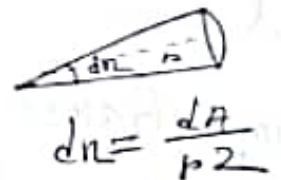
$$E_n dA = \frac{q}{4\pi\epsilon_0} \times \frac{\cos \theta dA}{r^2}$$

$$\Rightarrow \oint E_n dA = \frac{q}{4\pi\epsilon_0} \oint \frac{\cos \theta dA}{r^2}$$

$$\Rightarrow \oint E \cos \theta dA = \frac{q}{4\pi\epsilon_0} \times \oint dn$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q}{4\pi\epsilon_0} \times 4\pi$$

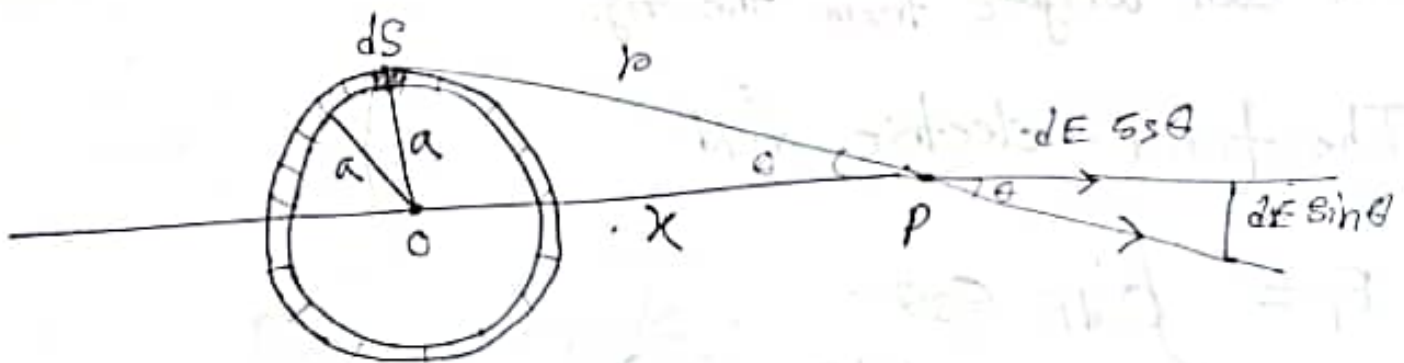
$$\Rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$



E_n = Number of flux

Electric field at a point on the axis of charged ring

concept of electricity and magnetism
Refikur DU
electricity and magnetism
KK TEWARI



The amount of charge in dS is $= \frac{q dS}{2\pi a}$

The electric field at point P for $\frac{q dS}{2\pi a}$ charge

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{q dS}{2\pi a r^2}$$

$$\vec{B} = \vec{\nabla} \times \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}}{r}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

where $\vec{A} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}}{r}$ is the vector magnetic potential

We know, current density

$$J = \frac{I}{dS}$$

$$I = J dS = J \cdot \frac{dV}{dl}$$

$$\therefore \vec{A} = \frac{\mu_0}{4\pi} \int \frac{I dl}{r} \hat{n}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{J dV}{r} \hat{n}$$

Magnetisation :-

dipole moment = m_i

$$\sum_{i=1}^N m_i = m$$

$$q(2l) = p$$

$$q(2l) = m$$

$$\text{magnetisation vector } M = \frac{m}{V} = \frac{1}{V} \sum_{i=1}^N m_i$$

then the net flux

$$\Phi_B = \mu_0 n I A N$$

we know the e.m.f induced

$$\mathcal{E} = \frac{d\Phi_B}{dt}$$

$$\mathcal{E} = \frac{d}{dt} (\mu_0 n I A N)$$

$$\mathcal{E} = \mu_0 n A N \cdot \frac{dI}{dt} \dots \textcircled{i} \text{ and } \mathcal{E} = L \cdot \frac{dI}{dt} \dots \textcircled{ii}$$

Comparing \textcircled{i} and \textcircled{ii}

$$\boxed{L = \mu_0 n A N} \quad \text{Proved}$$

$$L = \mu_0 \cdot \frac{N}{l} \cdot A \cdot N$$

$$= \mu_0 \frac{N^2}{l} \cdot A$$

$$\boxed{L = \mu_0 n^2 l A}$$

Self ^{V.V.I} inductance of two co-axial cylinder:

$$\beta = \frac{\mu_0 I}{2\pi r}$$

$$\text{Then flux } d\Phi = \beta A$$

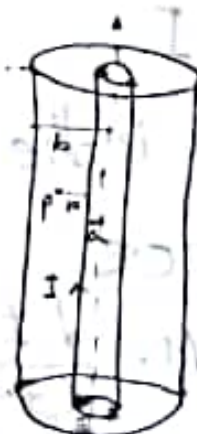
$$= \beta dr$$

$$\therefore d\Phi = \frac{\mu_0 I}{2\pi r} \cdot dr$$

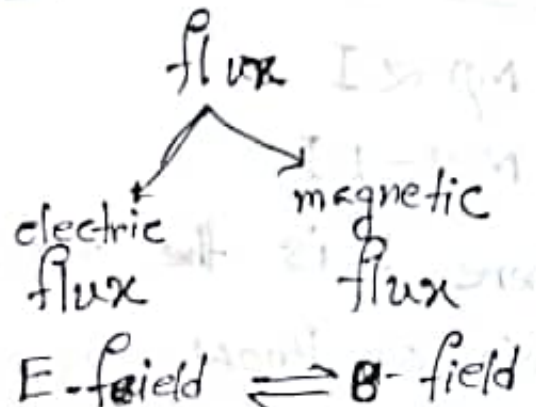
$$A = dr \cdot l$$

$$= dr$$

$$l = 1\text{m}$$



Electromagnetic Induction:



Faraday's law :-

1st law :-

2nd law :-

$$\epsilon \propto \frac{d\phi}{dt}$$

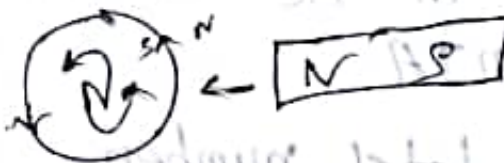
$$\epsilon = k \cdot \frac{d\phi}{dt}$$

$$\text{or } \epsilon = - \frac{d\phi}{dt}$$

If N is the number of turns

$$\epsilon = N \cdot \frac{d\phi}{dt}$$

Lenz's law :-



The result

$$E_0 // E_p$$

$$E = \sqrt{E_\theta^2 + E_r^2 + 2E_\theta E_r \cos 90^\circ}$$

$$= \frac{P}{4\pi\epsilon_0 r^3} \sqrt{4 \cos^2 \theta + \sin^2 \theta}$$

$$= \frac{P}{4\pi\epsilon_0 r^3} \sqrt{1 + 3 \cos^2 \theta}$$

Torque (τ)

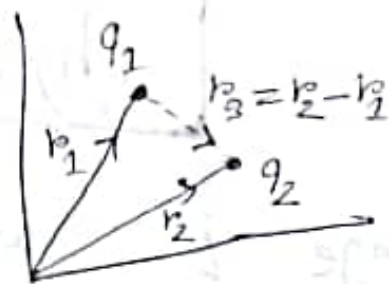
$$\tau = F \cdot 2l \sin \theta$$

$$\tau = Eq(2l) \sin \theta$$

$$\tau = PE \sin \theta$$

$$\tau = \vec{p} \times \vec{E}$$

$$\vec{F}_2 = C \cdot \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$$



For discrete charge distribution charge density

$$\vec{F}_1 = \sum_{i \neq j}^{j=N} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|^3} (\vec{r}_i - \vec{r}_j)$$

$$\lambda = \frac{q}{l}$$

$$\sigma = \frac{q}{A}$$

$$\rho = \frac{q}{V}$$

$$\vec{F}_q = C q \int \frac{\rho(\vec{r}') dV}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') + C q \oint_S \frac{\sigma(\vec{r}') dA}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

Electric field :

$$\vec{E} = \frac{\vec{F}}{q}$$

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2}$$

Frequency: $f = \frac{1}{T}$

Amplitude:

phase:-

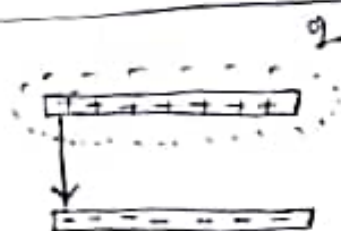
Initial phase is called epoch.

~~***~~

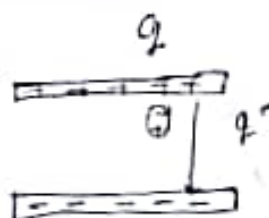
N.V. I. Sure

Class-05

Gauss's law for dielectric medium:-



(a)



(b)

From Fig (a)

$$\epsilon \cdot \oint \vec{E}_0 \cdot d\vec{A} = q$$

$$\Rightarrow E_0 = \frac{q}{\epsilon \cdot A}$$

— (i)

13-02-20

(i) if $x \gg a$

$$E_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2}$$

(ii) $x \ll a$ then $E_p = 0$

(iii) $x = a$

$$E_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{qa}{2^{3/2} \cdot a^3}$$

Electric dipole

$$P = q \times 2l$$

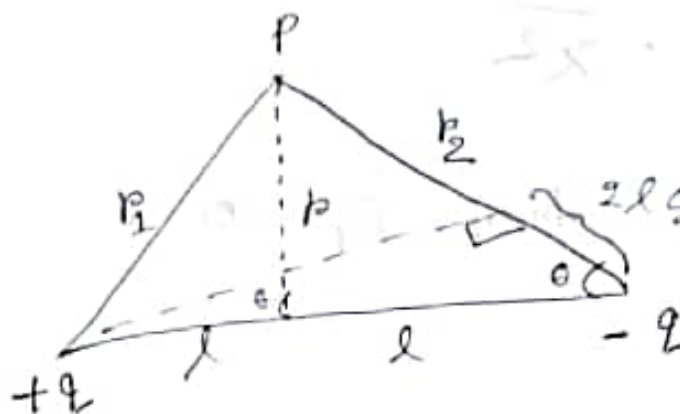
$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

$$V = Ed$$

$$E = - \frac{dV}{dr}$$

E and V for electric dipoles :- $\frac{1}{4\pi\epsilon_0} \cdot \frac{P}{r^3} = V$

E and V for electric dipole :-



The potential at point P for dipole

$$V = \frac{1}{4\pi\epsilon_0} \cdot \left(\frac{q}{r_1} - \frac{q}{r_2} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \cdot \frac{r_2 - r_1}{r_1 \cdot r_2}$$

$$= \frac{q}{4\pi\epsilon_0} \cdot \frac{2l \cos \theta}{r^2} \quad [r_1 \approx r_2 \approx r]$$

$$\therefore V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \cos \theta}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot \vec{r}}{r^3}$$

With variation r , we have

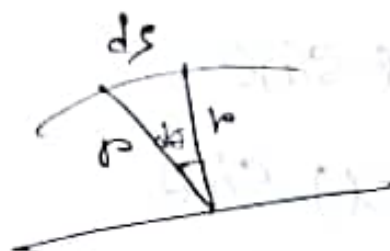
$$E_r = - \frac{dV}{dr}$$

$$= - \frac{d}{dr} \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{p \cos\theta}{r^2} \right)$$

$$= - \frac{p \cos\theta}{4\pi\epsilon_0} \times - \frac{2}{r^3}$$

$$\therefore E_r = \frac{2p \cos\theta}{4\pi\epsilon_0 \cdot r^3}$$

Radial component



$$s = r\theta$$

$$ds = r d\theta$$

With variation θ

$$E_\theta = - \frac{dV}{r d\theta}$$

$$= - \frac{d}{r d\theta} \cdot \left(\frac{p \cos\theta}{4\pi\epsilon_0 \cdot r^2} \right)$$

$$= - \frac{1}{r^3} \cdot \left(- \frac{p \sin\theta}{4\pi\epsilon_0} \right) = \frac{p \sin\theta}{4\pi\epsilon_0 \cdot r^3}$$

$$E_\theta = \frac{p \sin\theta}{4\pi\epsilon_0 \cdot r^3}$$

Again,

$$V = \frac{q}{l}$$

$$C = \frac{1}{V}$$

$$\Rightarrow C = \frac{1}{\frac{1}{2\pi\epsilon_0} \cdot \ln \frac{b}{a}}$$

$$\Rightarrow C = \frac{2\pi\epsilon_0}{\ln \frac{b}{a}}$$

For l Length

$$C = \frac{2\pi\epsilon_0 l}{\ln \frac{b}{a}}$$

Capacitance for sphere

$$C = 4\pi\epsilon_0 r$$

We can write from the fig.

The total electric field

$$E_p = \oint dE \cos \theta$$

$$= \oint \frac{1}{4\pi\epsilon_0} \times \frac{q ds}{2\pi a \cdot r^2} \times \frac{x}{r}$$

$$= \frac{qx}{4\pi\epsilon_0} \times \frac{1}{2\pi a} \cdot \frac{x}{r^3} \int ds$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{2\pi a} \cdot \frac{x}{(a^2 + x^2)^{3/2}} \cdot 2\pi a$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{qx}{(a^2 + x^2)^{3/2}}$$



From Fig (b) :-

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = (q - q')$$

$$\Rightarrow E = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \quad \text{--- (ii)}$$

Dielectric cons. $K = \frac{E_0}{E}$

$$\Rightarrow E = \frac{E_0}{K}$$

From (ii) $\frac{E_0}{K} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A}$

$$\Rightarrow \frac{q}{\epsilon_0 A K} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A}$$

$$\Rightarrow \frac{q}{K} = q - q'$$

$$\Rightarrow q' = q - \frac{q}{K}$$

Now

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q - q'$$

$$\Rightarrow \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q - q + \frac{q}{K}$$

$$\Rightarrow \epsilon_0 K \oint \vec{E} \cdot d\vec{A} = q$$

But differentiate

$$\Rightarrow \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

$$\Rightarrow C = \frac{\epsilon_0 A}{d}$$

other medium $C = \frac{\epsilon_0 k A}{d}$

Capacitance of a co-axial cylinder :-
अक्षीय चिह्न

$$\oint \vec{E} \cdot d\vec{A} = q$$

$$\Rightarrow \epsilon_0 E \cdot 2\pi r l = q$$

$$\Rightarrow E = \frac{1}{2\pi \epsilon_0 \cdot r}$$

$$\frac{q}{l} = 1$$



$$V = V_a - V_b$$

$$= \int_a^b \vec{E} \cdot d\vec{r}$$

$$= \int_a^b E dr$$

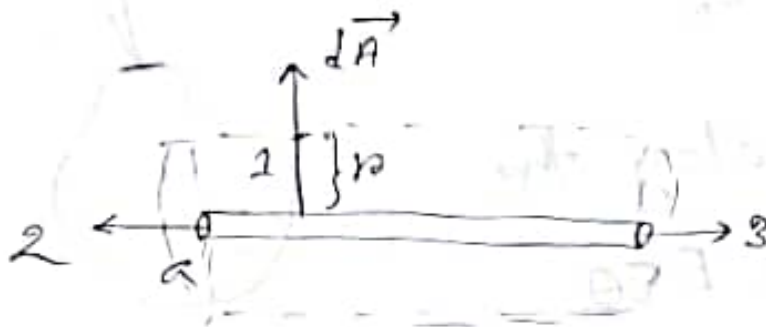
$$\Rightarrow V = \int_a^b \frac{1}{2\pi \epsilon_0 \cdot r} dr$$

$$= \frac{1}{2\pi \epsilon_0} \ln \frac{b}{a}$$

$$= \frac{1}{2}$$

Calculations:- Applications

(i) For a straight cylindrical wire



$$\lambda = \frac{q}{l}$$

Applying Gauss law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$\Rightarrow E \cdot \oint dA = \frac{q}{\epsilon_0}$$

$$\Rightarrow E \cdot 2\pi r l = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{2\pi r l \epsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi r \epsilon_0}$$

if (i) $r < a$ $E = 0$

(ii) $r > a$ $E = \frac{\lambda}{2\pi r \epsilon_0}$

(iii) $r = a$ $E = \frac{\lambda}{2\pi a \epsilon_0}$

$$\phi_s = Q_p \cdot n_s$$

$$= \mu_0 n_p \cdot A_p \cdot I \cdot n_s$$

$$\left\{ \begin{array}{l} B = \mu_0 n I \\ \phi = BA \end{array} \right.$$

$$\therefore \mathcal{E} = \frac{d\phi_s}{dt} = \mu_0 n_s n_p A_p \cdot \frac{dI}{dt}$$

$$\therefore M = \mu_0 n_s n_p A_p$$

We know,

$$\vec{E} = -\vec{\nabla} V$$

$$\therefore \vec{\nabla} \cdot \vec{\nabla} V = -\frac{\rho}{\epsilon_0}$$

$$\Rightarrow \nabla^2 V = -\frac{\rho}{\epsilon_0} \quad \text{Poisson's equation}$$

if $\rho = 0$

$$\nabla^2 V = 0 \quad \text{Laplace equation}$$

Class-09

9-02-2020

Capacitor:

$$C = \frac{Q}{V} \quad \text{also } F = CV^{-1}$$

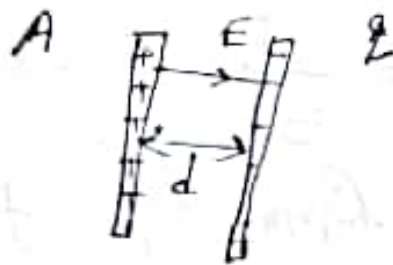
V.V.I * * *

Capacitance of two parallel plates:-

potential Electric field

$$V = Ed$$

$$\begin{aligned} \Rightarrow V &= \frac{\sigma}{\epsilon_0} d \\ &= \frac{Qd}{\epsilon_0 A} \end{aligned}$$



Again

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \int (\rho + \rho_p) dv$$

$\rho_p = \text{Polarization density}$

$$[\rho_p = -\vec{\nabla} \cdot \vec{P}]$$

$$\Rightarrow \epsilon_0 \oint \vec{E} \cdot d\vec{A} = \int \rho dv - \int \vec{\nabla} \cdot \vec{P} dv$$

$$\Rightarrow \epsilon_0 \oint \vec{\nabla} \cdot \vec{E} dv + \oint \vec{\nabla} \cdot \vec{P} dv = \int \rho dv$$

$$\Rightarrow \oint \vec{\nabla} \cdot (\epsilon_0 \vec{E} + \vec{P}) dv = \int \rho dv$$

Displacement
current = \vec{D}

$$\Rightarrow \oint \vec{\nabla} \cdot \vec{D} dv = \int \rho dv$$

$$\Rightarrow \boxed{\vec{\nabla} \cdot \vec{D} = \rho}$$

capacitors and its applications

Gauss's law and its "

Coulomb's law

Formulae

Application of Ampere's law
for a current carrying wire
Application of Gauss's law
for electric field
of a point charge
of a line charge
of a surface charge
of a volume charge